The pore solid fractal model and soil density scaling

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Variations of soil density and soil porosity with scale have been reported by soil scientists for many years. Modellers have sought to explain these observations in terms of fractal models of soil structure. Two forms of mass fractal have been proposed yielding opposing scaling behaviour. The solid mass fractal describes a porous material with a density which decreases and a porosity which increases with increasing sample size. In particular mass and density scale as powerlaws. This model has found favour as a descriptor of soil aggregates (Young & Crawford; Rieu & Sposito, 1991; Anderson & McBratney, 1995). Conversely, the pore mass fractal describes a porous material with a density which increases and a porosity which decreases with increasing sample size. In this case the porosity scales as a powerlaw. The solid mass fractal exhibits a powerlaw pore-size distribution whereas the pore mass fractal exhibits a powerlaw particle size distribution but neither can represent both solid and void scaling distributions. The two models need to account for a lower cut-off of scale since they are seen to fail immediately if this lower bound is not present, yielding porosities of one and zero respectively.

The Pore Solid Fractal (PSF) model (Perrier, Bird and Rieu, 1999; Bird, Perrier and Rieu, 2000) of soil structure is an extension and generalisation of the fractal approach to modelling soil structure, in which a range of particle sizes and a range of pore sizes are incorporated in a common geometric model. Solid and pore mass fractal models appear as special cases of the PSF. We have already shown that the PSF can be used to model several other scaling properties such as fractal pore-solid interfaces (Perrier et al., 1999) as well as observed distributions of aggregates in a fragmentation process (Perrier and Bird, 2001). The PSF can be developed at arbitrarily small scales without exhibiting unrealistic bulk densities and porosities but, as for the mass fractal models when developed ad infinitum, the latter properties are scale independent. Scale variant bulk densities can be modelled by simple modifications to the PSF. One is to relax self-similarity (Rieu, unpublished work). In this communication we simply consider the existence of a lower cut-off of scale. Any soil system exhibiting scaling of structure must exhibit a lower bound to this scaling. We may take the smallest particle size as an absolute lower bound. By incorporating a lower bound we create a model with either increasing, constant or decreasing bulk density with increasing sample size, depending on the density of structure at scales smaller than that of the PSF regime. This provides a unified approach to modelling density and porosity scaling within the framework of the PSF, and we derive a new expression for density scaling, which reverts to existing forms with selection of parameter values associated with mass fractal models.

In the second part of this communication we consider the link between structural and hydraulic properties. We show that the general expression obtained to describe density scaling is closely related to the general expression for the retention curve already derived for the PSF model (Bird et al., 2000) or for any fractal pore size distribution (Perrier at al., 1996). It involves the same parameters, thus through this link we may infer the water retention function from bulk density scaling and vice versa. In particular we show that the widely adopted Brooks-Corey equation for water retention is associated with a scale invariant density. Conversely, if density varies with scale, the retention curve cannot follow a simple powerlaw,

and one has to account for the porosity of the medium at scales smaller than that of the fractal or PSF regime.
The third part deals with application of our theoretical concepts to data. We show that the general equations easily fit either aggregate bulk density data or retention data; but no data are available both on bulk density and retention over a large range of scale. Density data are available by means of aggregate weighing only at rather large scales. How actually does soil bulk density scale in soils? Further experimental research should be carried out using image analysis, but several conceptual errors may arise.
In conclusion the PSF approach appears now as a general, simple way to represent in the same framework several major structural soil scaling properties and to go beyond conventional fractal models of multiscale soil structure. Further generalizations of the PSF are possible and attractive with a view to developing an operational model. In particular we may relax self-similarity imposed on the system thus allowing bulk density to pass through different scaling regimes and exhibit non-monotonic scaling behaviour.

References

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