Which theory for infiltration-excess runoff on rough surfaces ?

P. Davy[1], F. Darboux[2], C. Gascuel-Odoux[3], O. Planchon[4]

[1]Géosciences Rennes, Campus de Beaulieu, 35042 rennes Cedex
[2]INRA, 65 Route de Saint Brieuc, 35042 Rennes Cedex
[3]Purdue University, NSERL, 1196 SOIL Bldg, West Lafayette IN 47907-1196, USA
[4]IRD, 1386 Dakar, Sénégal

There are several conceptual models that aim at describing runoff processes and the role played by soil characteristics – mostly roughness, retention and permeability. For infiltration-excess overland flow, one of the processes that may occur on hillslopes, runoff is supposed to result from the overflowing of ponds that form due to soil roughness, and the general dynamics is determined by the dependency of the three main fluxes, runoff, infiltration and the time derivative of soil water storage, upon input and structural variables. The simplest model consists in assuming that soil is a reservoir that fills up at a rate equal to the difference between rainfall rate and infiltration and that abruptly overflows when its water content exceed a specific volume. This vision actually extrapolates at a field scale processes that occur at a pond scale. The relevance of this schematic scaling transfer is however questionable since the global dynamics results in a time-dependent spatially-heterogeneous organisation of infilling and overflowing depressions, and of channel flows between ponds or down to the outlet. Determining a theoretical framework for the large-scale dynamics and assessing the control played by soil roughness and infiltration is thus crucial to well pose simplified runoff models.

A key-factor in predicting runoff efficiency is flow connectivity that is obviously strongly related to surface roughness. Significant runoff occurs if, and only if, the length scale of water flow connectivity is of the order of system size. In rough permeable surfaces, connectivity is due either to pond overflowing or to an organised structure of soil topography that define a drainage network. In the former case, connectivity evolves with the amount of water stored in topographic depression. The process is cooperative in the sense that the pond infilling rate can be increased by the overflowing of upstream ponds. In the latter case, connectivity is achieved as the first rain drop hits the surface, and runoff depends only on flow velocity. These two end-member cases exist in nature. Soil aggregates and clods that form depressions can be considered as random with respect to flow, at least at large scale (larger than several decimetres). In contrast, large-scale slopes, rills and even seedbed lines form a drainage network that ensures large-scale correlations to flow. In general, soil erosion tends to increase the "structural" flow connectivity, and thus to decrease the role of pond overflowing.

In this paper, we argue that these two types of connectivity are basically different, and can be related to existing percolation theories. Qualitative arguments were derived from rainfall-simulated experiments on natural soils [*Planchon* et al., 2000; *Darboux et al.*, 2001]. In some laboratory experiments, soils were tilled so that topography can be considered uncorrelated for scales larger than about 10 cm. For the very first rainfalls, runoff shows a classical 'S-shape' with a sharp increase around a threshold. This emphasises cooperative pond overflowing as a dominant process, as it is expected in the classical percolation theory. For the last rainfalls, soil topography is significantly eroded with a visible drainage organisation due to rill development. A significant runoff occurs much faster from this drainage network, but the runoff increase with rain is much smaller than in the random case.

To rationalize these qualitative results, we expect theoretical arguments to be derived from percolation theory. Indeed if the average distance between successive depressions is small enough, the time scale of the problem is given by precipitation in relation to hydrodynamic soil properties, and the runoff dynamics is fully determined by pond connectivity. This process is closely related to percolation problems where the macroscopic behaviour is due to cluster connectivity. For infiltration-excess overland flow, clusters are individual drainage basins defined as the ensemble of points which eventually flow into a pond. The horizontal extension of drainage basins grows when adding water by connection of overflowing ponds.

The analogy to classical percolation problem is intuitively sound but has never been really tested. Natural systems cannot be only considered as running water on top of random impervious surfaces; the consequence of infiltration as well as the existence of long-range correlations are natural conditions that have got to be taken into account. We have analysed this problem by using a "walker" numerical model that simulates runoff on any permeable surface [*Crave and Davy*, 2001; *Darboux et al.*, 2001]. Each walker is supposed to represent a droplet which runs on top of the upper surface (soil or pond) and looses their water content to fill up local holes. Except in holes, walker runs following the steepest slope. It stops when it is empty or it reaches a predefined system boundary.

The simplest case of a random (uncorrelated) impervious surfaces is clearly analogue to percolation problems. Runoff curves have the classical 'S-shape' with a sharp increase of runoff around a threshold rainfall r_c that is about equal to the volume of water potentially storable on the surface. The key process is pond overflowing, a mechanism which is controlled by pond infilling rate equals to rainfall rate multiplied by the ratio between drainage area and pond area (upper surface of water). Overflow occurs when water height reaches the lowest pass in the drainage divide. Because of the potential variations of pond drainage area, this mechanisms of cluster growth is slightly different from classical percolation theory for which it would be assumed a random distribution of the overflow conditions. Also classical percolation theory predicts that percolation clusters are fractal while drainage basins are clearly space filling. Despite these differences, we demonstrate that that the process contains the basic ingredients of the percolation theory with a divergence of the correlation length around threshold characterised by a scaling exponent of about 4/3 that is close to 2D percolation problems. Note that for infinitely large systems, runoff is an all-or-none process as pictured in the overflowing "box" model.

Infiltration does not modify this theoretical scheme, except in two respects:

The percolation threshold is obtained for an amount of water larger than in the impervious case, trivially showing that the infiltration flux do not participate to runoff. But we found that percolation threshold is independent of infiltration rate if the efficient added water (total rainfall – infiltrated water) is considered.

The eventual runoff R decreases with infiltration rate such as $R(t=\infty, I) = 1 - \frac{I}{1.7 * p}$ with t the time, I the infiltration rate, and p the precipitation rate. Runoff occurs even when infiltration rate is larger than rainfall rate (up to 1.7 times the rainfall rate) for these flat random topographies. This reflects the fact that the increase of pond height depends on the ratio between drainage basin area and pond area, making possible an increase of the pond height for basins whose drainage area is larger than pond area. If the number of such basins is sufficient, they can eventually form a large connected basin that significantly contribute to runoff.

The case of soil topography with long-range correlations is clearly different from uncorrelated surfaces. We have especially studied the effect of a general slope as an illustrative example of such long-range correlation. A noteworthy result is that the threshold width – that is the amount of water necessary to achieve significant runoff – tends to a non-nil constant value for large sloping systems. This is a strong argument to state that such system does not belong to the same class of universality than percolation theory. It is rather related to directed