Can Sph be suitable methods for modeling shallow water flow ?

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The poster presents a smoothed particles hydrodynamics (abbrev. as SPH) system, modeling water flow in a linear canal of constant width. The problem is usually modeled by the Saint Venant equations for the 1-D shallow waters. A short historical survey of the approach, which appeared originally in the context of Astrophysics [Lucy 1977] [Monaghan 1977] is given first. In the context of incompressible fluids [Monaghan 1994][Morris 1996] and especially water flow [Zhu 1997], there have been several contributions, including recent work undertaken by D.Servat as part of a PhD thesis as well as in conferences and publications [Servat &al 1999, 2001]. The intent here is to provide a complement to Servat's work both in the modeling and analysis of SPH systems arising in the case of shallow waters in a canal.
A brief description of the contents is as follows:
First, the basic principles underlying the derivation of an SPH model from an original hydrodynamic or whatever model are summarized, and related to the context of meshfree particles methods [Belytschko & al, 1996, 1998]. Let us just mention that the principle of SPH is to move from a continuum (Eulerian) viewpoint to a discrete (Lagrangian) viewpoint: the water is viewed as a collection (finite or infinite) of particles, each of which consists of a packet of water whose shape changes with time. The end-product is a family of ordinary differential equations, governing the time evolution of the state of each particle.
This is the general scheme, it has been applied here to the PDE 1-D shallow water equations [Chow & al 1988]: a detailed description of the state variables and hypotheses is given, this would be the second part of the poster. The SPH system is stated both in the form obtained from general principles, and then in a form, which comes out from a partial integration,. From the latter form, it appears that the core of the SPH is a system of first order differential equations relating the position and velocity of the particles.
The third part of the presentation reports on the main theoretical results that have been obtained in the study of the SPH system. There are two aspects. The first one is about the way the SPH formulation copes with fundamental properties of the original equations and whether it preserves some invariants. Namely, it has been shown, and is reported here, that the SPH system preserves the mass, momentum and the energy of the system of particles. The other aspect is about the behavioral properties of the SPH system, notably, how it reacts to small perturbations of the uniform flow. A linear stability study is performed and shows the onset of waves of arbitrary spatial frequency, as a response to such perturbations. The main achievements, in our view, are formulas, expressed in terms of the parameters of the system and the wave spatial frequency, for the wave speed. We show in particular that under a scaling assumption, namely that the wave length be large compared to the depth, the wave speed does not depend on the spatial frequency and the formula, in this case, is close to the one derived from the continuous equations [Stoker 1958]
The fourth part of the poster reports on the reciprocal step, that is, to move from the approximation by SPH system to the exact model. Interpolation formulas for the state variables of the continuous model in terms of the state variables of the SPH system are proposed. These are indeed only approximations of the solutions of the full Saint Venant

equations: good agreement with the latter ones is achieved though, which is reflected by the fact, shown in the poster, that the continuity equation is exactly satisfied (by the interpolate solution) and the discrepancies in the momentum equations can be estimated.
Finally, the dynamical behavior of the SPH system is illustrated by numerical simulations, whose results are shown in several figures, and a discussion is made as to the time discretization scheme to be used in the simulator of the SPH system.

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