

# HYDRAULIC VULNERABILITY OF ELEMENTARY URBAN CELL

By Benoît Hingray,<sup>1</sup> Bernard Cappelaere,<sup>2</sup> Christophe Bouvier,<sup>3</sup> and Michel Desbordes

**ABSTRACT:** During urban storms, the overloading of a sewer system or a riverbank overflow produces flows that are routed by the road infrastructure and causes flooding of adjacent built-up areas. The evaluation of the damage inflicted in these built-up areas requires the determination of important hydraulic parameters of the inundation, such as the maximum water depth or the inundation duration. For each independent hydraulic urban cell, these parameters depend upon its intrinsic hydraulic properties (perviousness and storage capacity) and on the characteristics of the flood outside the cell. This paper proposes a quantitative formulation of the concept of cell hydraulic vulnerability based on the cell's hydraulic properties and on the basic characteristics of the flood (peak flow and time to peak). This study is based on observations made in three districts of Ouagadougou, Burkina Faso, and on a 23-year record of rainfall events in this region. The hydraulic vulnerability is discussed according to the district hydraulic properties and to the flood event characteristics. The developed methodology could be used in principle in every city where built-up areas are highly subpartitioned into walled properties.

## INTRODUCTION

Examples of important flood damage suffered by populations in built-up areas are numerous, especially in tropical regions where the violence and rapidity of tropical storms often lead to the overloading of drainage systems and to the inundation of adjacent built-up areas. In developing African countries, the population explosion and the rural depopulation result in rapid urbanization that is difficult to control. In most recent urban districts, drainage systems are often insufficient if not absent; roads thus serve as storm drainage water courses, which increases the risk of significant damage to the adjacent built-up areas.

It is generally considered that the potential damage a flooded zone may suffer results from the magnitude of the hydrological event and from its own vulnerability to inundation (Debo 1982). The phenomenon may be characterized with a flood flow model that should provide the essential characteristics of water flows. Because of the geometric complexity of the urban environment and of the rapidity of urban storms and their associated flooding, most of the current models have difficulties in correctly representing the dynamics of flooding in urban districts when the drainage system is failing. Nevertheless, such models that often neglect the local heterogeneity of the urban environment may become practical in a few years time (Kinoshita et al. 1996; Riccardi 1997).

The vulnerability of an area is usually said to depend on its socioeconomic value, which is related to land use, and increases in the following order: wooded zones, agricultural areas, residential urban areas, commercial, or industrial areas. Nevertheless, the vulnerability of each elementary urban cell of a given urban zone depends not only on its socioeconomic value but also on its hydraulic vulnerability to flooding (U.S. Army Corps of Engineers 1996). This hydraulic vulnerability, which determines the inundation parameters inside the cell, is governed by the cell's hydraulic properties and by the characteristics of the passing flood wave observed in the flood-

conveying adjacent road. It may also vary from one cell to the other.

For practical computing reasons, the heterogeneity of the intrinsic hydraulic properties within an urban zone, and consequently of the hydraulic behavior of each individual cell, is most often neglected in flood modeling at the city scale. Models that take into account some hydraulic properties of individual cells (geographic information system supported) are cited (Proc. 1996; *Hydroinformatics* 1998), but they are not yet widely applied. The flood characteristics provided by most flooding models are then too rough to quantify damage suffered by each elementary urban cell. The inundation parameters of each cell may be evaluated with more accuracy through the study of its hydraulic vulnerability. In the following sections, this concept of hydraulic vulnerability will be developed and applied to the case of three residential built-up areas in the city of Ouagadougou, Burkina Faso.

## ELEMENTARY HYDRAULIC URBAN OBJECT: INDIVIDUAL PLOT

Ouagadougou is the capital of the Sahelian state of Burkina Faso. In this city, as in a lot of other cities, the structure of the residential districts is repetitive and organized as blocks of plots. Each block is delimited by roads leading into the district and comprises  $N$  individual plots as shown in Fig. 1. All of these individual plots are constructed according to the same rural model: they contain several dwellings accommodating one or several families and a yard, the life center of the plot. The individual plot is isolated from other plots and from the roads by a 1.5-m-high surrounding wall, which contains only one or two openings onto the road, exclusively; the individual plot thus constitutes one of the elementary hydraulic objects of this urban environment (Fig. 2).

The hydraulic behavior of such an urban cell can be mod-

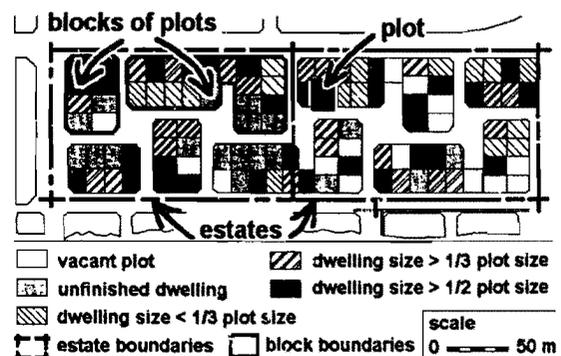


FIG. 1. Individual Plots and Blocks of Plots of Typical African Built-Up Area: Patte d'Oie District in South Ouagadougou

<sup>1</sup>Res., ORSTOM, 911 Ave. Agropolis, B.P. 5045, 34032 Montpellier, France.

<sup>2</sup>Res. Engr., ORSTOM, 911 Ave. Agropolis, B.P. 5045, 34032 Montpellier, France.

<sup>3</sup>Res., ORSTOM, 911 Ave. Agropolis, B.P. 5045, 34032 Montpellier, France.

<sup>4</sup>Prof., Dept. of Hydro., UM II, Place E. Bataillon, cc056, 34095 Montpellier Cedex 5, France.

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## MODELING HYDRAULIC BEHAVIOR OF URBAN PLOTS

As far as the hydraulic behavior of Ouagadougou's individual plots is concerned, some important observations have to be made:

- There are no exchanges between adjacent plots because of the hydraulic impermeability of the partition walls: the hydraulic behavior of each individual plot may therefore be described just by its perviousness and its storage capacity.
- The perviousness is easy to determine because of the simple geometry of the apertures found in the surrounding walls. The apertures correspond to the plot entrances and are either of a simple open doorway design, which makes them behave as a hydraulic weir, or of a closed doorway design with a rectangular residual gap between the door and the doorstep (i.e., an orifice) (Fig. 4). Moreover, the entrance orientation is usually orthogonal to the road and consequently to the main flood flow direction.
- The land inside a plot is usually flat; the slope inside the plot may therefore be neglected. Moreover, the internal geometry of the plot is usually very basic so that its storage capacity can be described simply with its "floodable" surface area  $S_p$ . As this floodable surface area is generally small (from 50 to 300 m<sup>2</sup>), it may be considered that the water table in the plot always remains horizontal and that the internal storage of the plot does not have any influence on the flood dynamics in the adjacent road.
- If the land inside the plot is not covered with concrete, the soil is made of compacted laterite with small infiltration capacity. Water losses due to ground infiltration may therefore be ignored.
- The bottom elevation of a plot and the elevation of the adjacent flood-conveying roadway are usually the same. The water height in the plot  $h_p$  and the water depth in the adjacent road  $h_r$  will therefore be used to express the respective water elevations.

For this study, the plot inundation is assumed to be only due to the flood observed in the road adjacent to the plot. Water inflows due to local rainfall or to runoff from impervious parts of the plot are ignored. Under these conditions, the evolution of the inundation in a plot adjacent to a flooded road is governed by the following inundation model:

$$Q_{ex}(h_r(t), h_p(t)) = S_p \frac{dh_p}{dt} \quad (1)$$

where  $Q_{ex}$ , determined by the plot perviousness, is the exchange flux between the road and the plot. The perviousness may be simply defined through the geometric characteristics of the apertures present in the wall as shown in Fig. 4: the type of aperture (weir or orifice), the base elevation of the aperture  $h_s$ , its width  $L_s$ , and its gap height if an orifice  $a$  (the cross-sectional area of the aperture is then  $aL_s$ ). Measures of exchanges between plots and adjacent roads were made during several flood events in Ouagadougou (Hingray 1999) and show that the common formulations for discharge over weirs or through orifices can be used to compute these exchanges.

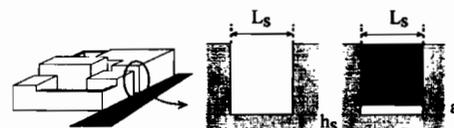


FIG. 4. Weir Aperture and Orifice Aperture Found in Surrounding Walls of Individual Plots: Characteristics Used to Describe Plot Perviousness

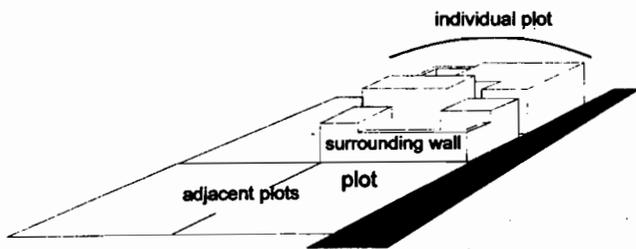


FIG. 2. Typical Individual Plot of Ouagadougou's Residential Areas

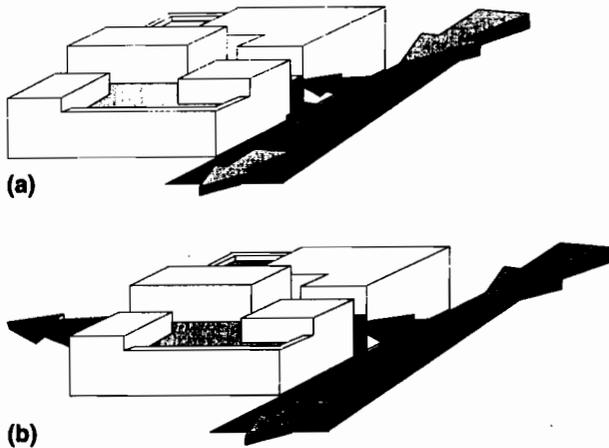


FIG. 3. Two-Plot Hydraulic Properties: (a) Plot Perviousness; (b) Plot Transmissivity

eled using the following three important physical properties (Hingray 1999), of which the first two are highly related to the structure of the surrounding walls:

- Perviousness—This is the ease with which the passing flood can enter or exit the plot. The plot's perviousness is a property of the plot's boundary (i.e., of the plot's surrounding wall). It depends upon the aperture density in this wall, which varies from one facade to the other [Fig. 3(a)]. It is a function of water elevation.
- Transmissivity—This is the ease with which water can pass through the plot. It is a global property of the plot. It depends on the perviousness of all external and/or internal facades of the individual [Fig. 3(b)]. If one facade is waterproof, the transmissivity is zero in the perpendicular direction. This property also depends on water elevation.
- Storage capacity—This is determined by the water quantity that may be stored by an individual plot. It is a property of the plot heart. It is a function of the internal surface area of the plot and of water elevation.

These three properties are essential and sufficient to describe the behavior of every basic or global urban object (plot, block of plots, etc.). The transmissivity and storage capacity properties have previously been introduced elsewhere in scientific literature: the storage capacity is described with the urban media porosity that depends on the density of dwelling houses or other buildings (Braschi et al. 1991), and the transmissivity appears as a roughness coefficient (Braschi et al. 1991; Liang et al. 1994). Its evaluation seems to be quite difficult and is frequently obtained through calibration of flood flow models. These two properties are suitable for open urban media but are still insufficient in the case of highly partitioned areas: in this latter case, the property of perviousness needs to be introduced. It will be developed in the following sections.

The hydraulic behavior of the individual plot is first studied when the aperture is a weir. The concept of plot hydraulic vulnerability is also introduced in this section, and an analytical expression is proposed allowing its evaluation in the case of the weir-type aperture. The case of the orifice-type aperture is discussed later.

### Discharge Model and Simulations of Plot Inundation—Case of Weir-Type Aperture

The discharge model for a broad-crested, submerged weir is as follows (Cunge et al. 1980):

Submerged flow

$$Q_{ex}(h_r, h_1, h_2) = mL_s(h_2 - h_r)\sqrt{2g(h_1 - h_2)} \quad (2)$$

where  $Q_{ex}$  = discharge rate on the weir;  $h_1$  (upstream water depth) and  $h_2$  (downstream water depth) are equal, respectively, to  $h_r$  (road water depth) and  $h_p$  (plot water height), if  $h_r > h_p$  or to  $h_p$  and  $h_r$  in the opposite case; and  $m$  = flux coefficient.

Eq. (2) is obtained from the energy conservation principle, when neglecting the upstream approach velocity because of the lateral disposition of the apertures with respect to the main flood flow direction, and when assuming that the water level over the weir is the same as the downstream level. The discharge model over a broad-crested, nonsubmerged weir may be obtained as the maximal value of (2) for decreasing downstream water depth  $h_2$  as follows:

Free flow

$$Q_{ex}(h_r, h_1) = \frac{2}{3\sqrt{3}} mL_s \sqrt{2g} [h_1 - h_r]^{3/2} \quad (3)$$

This free flowing case is obtained when  $(h_1 - h_r) \geq 3/2 \cdot (h_2 - h_r)$ , and the submerged case is obtained in the opposite case. Whether the flow over the weir is submerged or free, the combination of the appropriate discharge model [(2) or (3)] with the inundation model [(1)] highlights one single important parameter determining the rapidity and the attenuation of plot inundation for a given base height  $h_r$  of the weir, which is the ratio  $R_p = L_s/S_p$ . It is a measure of the ratio between the plot perviousness and the plot storage capacity. (The ratio  $1/R_p$  is analog to the characteristic relaxation time  $\tau = RC$  of the electric system defined by a circuit combining in series a capacitance  $C$ , a resistance  $R$ , and a generator producing an alternative current. Rapidity and amplitude attenuation of the signal transfer through the system is also controlled by  $\tau$ .)

Fig. 5 shows the simulated plot inundation  $h_p(t)$  versus the  $R_p$  ratio for the August 14, 1996, flood stage record [ $h_r(t)$ , bold curve] in a road of the Patte d'Oie district (South Ouagadougou): as  $R_p$  decreases, the time to peak and the attenuation of the plot inundation increase.

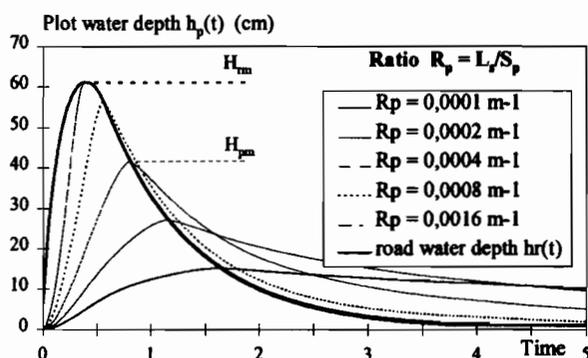


FIG. 5. Plot Inundation Scenarios for Various  $R_p$  Ratios—Case of a Weir of Bottom Height  $h_r = 20$  cm; Flood Event of August 14, 1996

This simulation is produced by the numerical solution of (1) and (2) [or (3) in the case of a nonsubmerged weir]. An implicit finite-difference scheme is applied to discretize the ordinary differential equations [(1) and (2)] [or (3)] with a time step  $\Delta t$ , and the Newton method is then used to linearize and solve the discretized set of equations. The initial value for  $h_p$  is zero, and the time step  $\Delta t$  is 1 mn.

### Hydraulic Vulnerability

#### Case of Weir-Type Aperture

The stage attenuation ratio  $\phi$  is defined as the ratio between the maximal water height observed in the plot,  $H_{pm}$  and the peak water depth  $H_{rm}$  of the flood event observed in the adjacent road. The maximal stage attenuation ratio of the plot inundation for a given flood event is equal to 1. It is obtained when the ratio  $R_p = L_s/S_p$  is greater to or equal to a limiting value  $R_o$  dependent on the characteristics of the passing flood wave. For values less than  $R_o$ , the smaller  $R_p$  gets, the more attenuated is the plot inundation, and the plot is increasingly protected against flooding. Later in this paper it will be shown how the stage attenuation ratio  $\phi$  can therefore be related to the hydraulic vulnerability of the plot defined as the ratio

$$\phi = R_p/R_o \quad (4)$$

An analytical expression of  $R_o$  may be obtained if the following simplifying assumptions are made on the plot inundation model [(1)]:

- The exchanges between the plot and the adjacent road occur only through one aperture.
- The flow through this aperture is nonsubmerged throughout the inundation period.
- During the rising phase, the floodwater depth on the road is a linear function of time:  $h_r(t) = At$  for  $t \in [0; T_{rm}]$ , where  $T_{rm}$  is the time to flood peak on road, and  $A$  is the slope of the rising curve limb (m/s), which is given by following equation:  $H_{rm} = AT_{rm}$ .

Under these assumptions, the simplified equation governing the filling of a plot is

$$Q_{ex}(h_r, h_p) = \frac{2}{3\sqrt{3}} mL_s \sqrt{2g} [h_r(t) - h_p]^{3/2} = S_p \frac{dh_p}{dt} \quad (5)$$

where the exchange flux  $Q_{ex}$  only depends on the road water height  $h_r(t)$ . The plot inundation water depth  $h_p(t)$  can be calculated as a function of time with a simple integration of the preceding equation. As the plot inundation begins when  $h_r(t) > h_r$  (i.e., when  $At > h_r$ ), this function of time is

$$h_p(t) = 0 \quad \text{if } t \leq h_r/A \quad (6a)$$

$$h_p(t) = h_p \left( t, \frac{L_s}{S_p} \right) = K \frac{L_s}{S_p} \sqrt{gA}^{3/2} \left( t - \frac{h_r}{A} \right)^p \quad (6b)$$

if  $t > h_r/A$

where  $K = 4/15m\sqrt{2/3}$ ; and  $p = 5/2$ . The limiting value  $R_o$  is the ratio  $L_s/S_p$  when  $h_p(T_{rm}, L_s/S_p) = H_{rm}$ . The analytical expression of  $R_o$  for a weir of elevation  $h_r$  is then

$$R_o = \left( \frac{L_s}{S_p} \right)_o = \frac{1}{K} \frac{1}{T_{rm} \sqrt{gH_{rm}}} \left( 1 - \frac{h_r}{H_{rm}} \right)^{-p} \quad (7)$$

#### Case of Orifice-Type Aperture

The same analysis can be performed for an orifice-type aperture. With the additional assumption that the gap height  $a$  is small compared to the peak water depth of the flood event

$H_{rm}$ , the inundation model (Carrier 1972) and the plot water depth time-function are as follows:

Inundation model

$$Q'_{ex}(h_s, h_p(t)) = mL_s a \sqrt{2g(h_p(t) - (h_s + a/2))} = S_p \frac{dh_p}{dt} \quad (8)$$

Water depth time-function

$$h_p(t) = 0 \quad \text{when} \quad t < \frac{h_s + a/2}{A} \quad (9a)$$

$$h_p(t) = K' a \frac{L_s}{S_p} \sqrt{gA}^{1/2} \left( t - \frac{(h_s + a/2)}{A} \right)^{p'}$$

$$\text{when} \quad t \geq \frac{h_s + a/2}{A} \quad (9b)$$

where  $K' = 2/3m\sqrt{2}$ ; and  $p' = 3/2$ . In this case, the only important parameter controlling the rapidity of plot inundation for a given bottom elevation  $h_s$  of the orifice is the dimensionless perviousness-to-storage capacity ratio  $R'_p = (aL_s)/S_p$  (the perviousness here is characterized by the cross-sectional area of the aperture  $aL_s$ ). During a plot inundation, the  $R'_p$  parameter plays the same role as the  $R_p$  parameter. Following the same process as before, the limiting value  $R'_o$  for the  $R'_p$  ratio, which results in a nonattenuated plot inundation, may be calculated

$$R'_o = \left( \frac{aL_s}{S_p} \right)_o = \frac{1}{K'} \frac{1}{T_{rm}} \sqrt{\frac{H_{rm}}{g}} \left( 1 - \frac{h_s}{H_{rm}} \right)^{-p'} \quad (10)$$

As will be shown later, the stage attenuation ratio  $\omega$  may again be related to the plot hydraulic vulnerability

$$\phi' = R'_p/R'_o \quad (11)$$

## MODEL DISCUSSION

The analytical expressions [(4)–(7)] or (10) and (11) of the plot hydraulic vulnerability  $\phi$  (or  $\phi'$ ) highlight two essential plot parameters: the first one ( $R_p$  or  $R'_p$ ) expresses the relative importance between the plot perviousness and the plot storage capacity. The second one, the nondimensional ratio  $h_s/H_{rm}$  [ $(h_s + a/2)/H_{rm}$  if an orifice] controls the nondimensional lag time  $\delta t/T_{rm}$  between the start of the flood on the road and the initial appearance of the inundation inside of the plot. The plot inundation actually begins when  $h_p(t) > h_s$ . In the case of a linear rising limb of the road flood, this occurs for the time  $\delta t$  when  $h_s(\delta t) = A\delta t = h_s$  [i.e., when  $\delta t/T_{rm} = h_s/H_{rm}$  (for a weir-type aperture)]. For the two types of aperture, the hydraulic vulnerability is an increasing function of the first parameter  $R_p$  (or  $R'_p$ ) and a decreasing function of the nondimensional ratio  $h_s/H_{rm}$  [or  $(h_s + a/2)/H_{rm}$  in the case of an orifice].

The following less obvious observations need to be made concerning the passing flood wave characteristics. Two limiting values  $R_o$  and  $R'_o$  are decreasing functions of the passing flood wave rising time  $T_{rm}$ . If the time to peak of the passing flood wave is small, the hydraulic vulnerability of the plot  $\phi$  (or  $\phi'$ ) is lessened. The time to peak for fluvial floods taking more than several hours is sufficient to result in  $\phi$  or  $\phi'$  ratios  $>1$ . On the other hand, pluvial floods with a short time to peak ( $<1$  h) may lead to ratios  $<1$ .

Finally, the limiting ratios for apertures with a zero bottom elevation ( $h_s = 0$ ) are studied. For weirs, the limiting ratio  $R_o$  is a linear decreasing function of the square root of the road flood peak depth  $H_{rm}$ . The higher the maximal road water level is, the greater is the  $\phi$  (or  $\phi'$ ) ratio and the more the plot is vulnerable. On the other hand, the limiting ratio  $R'_o$  for orifices is a linear increasing function of the square root of  $H_{rm}$ . In this

case, the hydraulic vulnerability is then a decreasing function of the magnitude  $H_{rm}$  of the road passing flood wave: the greater  $H_{rm}$ , the smaller the  $\phi$  (or  $\phi'$ ) ratio.

## Reliability of $R_o$ and $R'_o$ Analytical Expressions [(7) and (10)]

The assumptions made for the determination of the analytical expression for the limiting values  $R_o$  and  $R'_o$  (continuous free flow regime and linear shape of the rising flood stage) may produce very approximate  $R_o$  or  $R'_o$  values. For eight different floods and for different values of  $h_s$  or  $a$ , the  $R_o$  or  $R'_o$  calculated according to the analytical expressions [(7) or (10)] were compared with the limiting values of  $R_p$  and  $R'_p$  obtained by numerical simulations with an iterative search method. [The events used are eight flood events chosen among the 546 events presented in the later section entitled "Characteristics of Flood Events." They differ from one another in their basic characteristics (time to peak  $T_{rm}$  and maximum water depth  $H_{rm}$ ) and in the shape of their falling limb (short or slow falling limb), and they are fairly representative of the range of characteristic values and shapes observed. The apertures used are either a weir, or an orifice with a gap height  $a$  of 2, 5, or 10 cm. The ratio  $h_s/H_{rm}$  varies from 0 to 0.9.] The relative errors between the calculated  $R_o$  (or  $R'_o$ ) values and those produced by the simulation vary from 0 to 50% when the aperture elevation  $h_s$  is zero according to the flood event and may significantly increase when  $h_s/H_{rm}$  increases (the errors vary from 200% to more than 400% when  $h_s/H_{rm} = 0.9$ ). These errors may be considerably reduced when modifying the  $p$  and  $p'$  exponents in the  $R_o$  and  $R'_o$  analytical expressions [(7) and (10)]: taking  $p = 2$  and  $p' = 1$  always makes them smaller than 50% whatever the flood shape and the  $h_s$  and  $a$  values. A 50% error may be neglected if compared with the natural variation of the  $R_o$  (or  $R'_o$ ) value versus  $h_s/H_{rm}$ : when  $h_s/H_{rm}$  varies from 0 to 0.9, the limiting value  $R_o$  actually varies from 1 to 320, whereas  $R'_o$  actually varies from 1 to  $>32$ . These results may probably be improved to an even greater degree by using a shape more realistic than the linear shape used in the last section for the rising stage of the road flood. From now on in this paper, all  $R_o$ ,  $R'_o$ , and hence,  $\phi$  and  $\phi'$  values will be computed with the approximate analytical models [(7) and (10)] with the improved exponent values  $p = 2$  and  $p' = 1$ .

## Relationship between Stage Attenuation Ratio $\omega$ and Hydraulic Vulnerability $\phi$ or $\phi'$

Because of the  $\phi$  and  $\phi'$  definitions, the stage attenuation ratio  $\omega$  is dependent on the hydraulic vulnerability  $\phi$  or  $\phi'$ :

- If  $\phi$  (or  $\phi'$ )  $\geq 1$  the plot inundation is fast enough to produce a maximum plot inundation depth equal to the peak depth of the passing flood wave on the road. The hydraulic vulnerability and the stage attenuation ratio are maximal ( $\omega = 1$ ).
- If  $\phi$  (or  $\phi'$ )  $< 1$ , the inundation in the plot is attenuated. A study performed with the numerical simulations of the eight previous flood events for various plot properties shows (Fig. 6) that the value of this stage attenuation ratio  $\omega$  is strongly nonlinearly correlated with the value of the hydraulic vulnerability  $\phi$  (or  $\phi'$ ) computed according to (4)–(7) [or (10) and (11)].

The variability of the  $\omega$  ratio is partly due to assumptions made for the determination of the analytical expression for the limiting values  $R_o$  and  $R'_o$ . It is also due to the varying shapes of the falling limb of the flood events, which can be very different depending on the rainfall and catchment character-

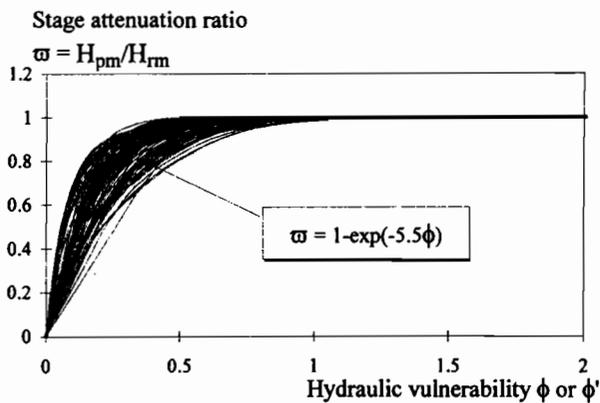


FIG. 6. Relationships between Stage Attenuation Ratio  $\varpi$  and Plot Hydraulic Vulnerability  $\phi$  (or  $\phi'$ ) for Various Aperture and Flood Event Characteristics

istics. Moreover, it should be noticed that the maximum  $\varpi$  ratio may often be reached for a  $\phi$  ratio smaller than 1. These situations are obtained for flood events for which  $h_s(t)$  is close to the peak value  $H_m$  over a rather long period of time.

The relationship between calculated  $\phi$  ratio and simulated  $\varpi$  ratio may nevertheless be fitted to the following analytical expression:

$$\varpi = 1 - \exp(-5.5\phi) \quad (12)$$

where the 5.5 coefficient was obtained with a least-squares adjustment to 1,400 simulated couples ( $\phi$ ,  $\varpi$ ). For all  $\phi$  values smaller than 1, the Nash-Sutcliffe coefficient (Nash and Sutcliffe 1970) for this adjustment is 0.91. The relative deviation between the stage attenuation ratio  $\varpi$  evaluated with (12) and the analytical  $\phi$  (or  $\phi'$ ) model on one side, and the ratio  $H_{pm}/H_m$  produced by numerical simulations of plot inundation on the other side, varies from 40% when  $\phi \approx 0$  to 0%, when  $\phi \approx 1$ .

All of the previous general observations will be illustrated in what follows with real cases observed in the city of Ouagadougou.

#### HYDRAULIC VULNERABILITY OF INDIVIDUAL PLOTS IN OUAGADOUGOU

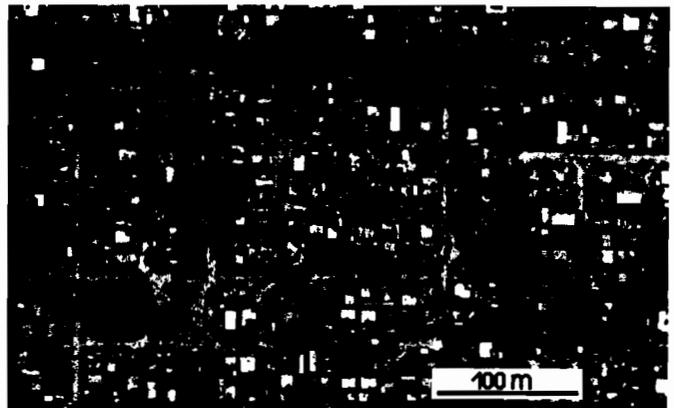
In Ouagadougou, the insufficiency of the artificial drainage infrastructure due to rapid and difficult-to-control city growth, leads to frequent flooding of built-up areas. Most of these flood events are headwater events; they are caused by runoff from small upstream contributory areas. They correspond to the type of "tributary flooding" described by Lee and Essex (1983): "the result of siting [urban] structures on or adjacent to natural well-defined drainage channels." In Ouagadougou, many plots are actually established in or close to ephemeral streams and are affected whenever the stream is making use of its natural floodway that has been colonized with roads and buildings. Headwater flooding events are one of the greatest worries mentioned by inhabitants of these districts (Morel A l'Huissier 1997): they fear death by drowning, are victims of health degradation in the stricken districts, and suffer from damage to the public infrastructures and to their personal properties.

One of the aims of the study carried out in this city during the 1996 rainy season was the determination of the hydraulic vulnerability of the individual plots of these floodable districts (Hingray 1999). The perviousness of these plots has been determined thanks to a systematic survey of the apertures found in their surrounding walls. This investigation has been made on three different districts, which are described below.

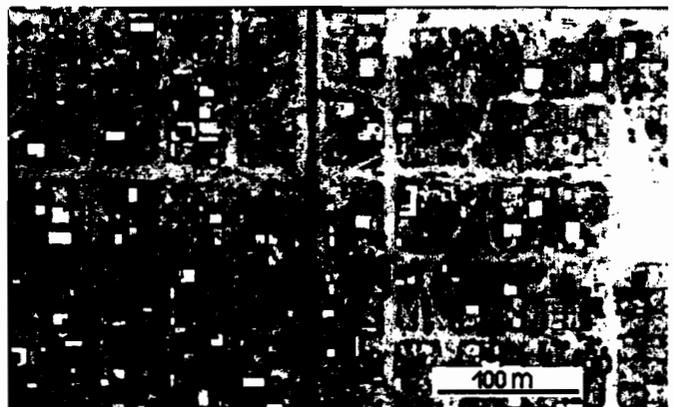
#### Results of Field Survey—Floodable Surface Area and Perviousness of Plots

The three different districts have a relatively homogeneous land use. The first two are residential traditional districts. The Patte d'Oie district (PO) (South Ouagadougou) is fairly old and was established in the 1970s: many individual plots are well developed [Fig. 7(a)]. The second one, Wemtenga 1 (W1) (East Ouagadougou), is a more recent housing estate established in 1988 and is not yet completely developed: there are many unfinished or empty plots left [Fig. 7(b)]. The last one, Wemtenga 2 (W2), is a very recent district of spontaneous development bordering W1 with a disorganized built-up area structure and irregular habitation density [Fig. 7(c)].

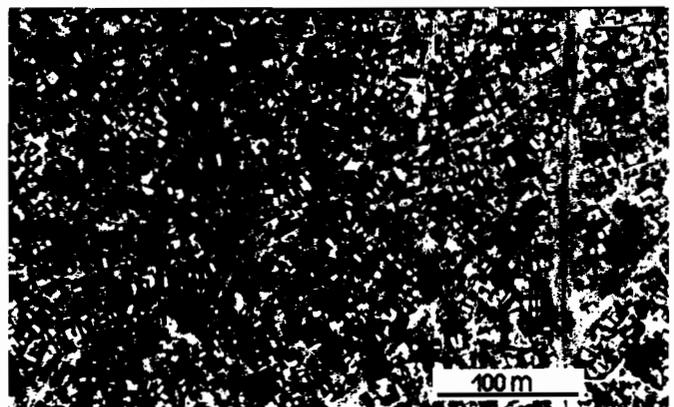
For each district, eight road facades have been surveyed



(a)



(b)



(c)

FIG. 7. Aerial Photographs of Part of Three Different Districts: (a) Old Patte d'Oie District; (b) Wemtenga District: Recent Housing Estate; (c) Wemtenga District: Spontaneous Development

over an average distance of 600 m (Belem and Moyenga 1996), and the perviousness of more than 1,000 individual plots has been described. The results are summarized below.

In the two legally established districts, all plots have the same 300-m<sup>2</sup> surface area. The floodable surface area  $S_p$  still depends on the level of plot valorization, which is usually related to its age and to its standard level. The floodable surface area  $S_p$  varies from <100 m<sup>2</sup> for highly developed plots to >250 m<sup>2</sup> for the others with an average value of 200 m<sup>2</sup>. The plots in the spontaneous district are smaller, and the floodable surface area  $S_p$  varies from 50 to 100 m<sup>2</sup>.

Furthermore, the aperture proportion between orifices and weirs depends on the district: the older the district is, the more numerous the orifices and the less numerous the weirs (75% orifices on PO, 40% on W1, and 10% on W2). Gaps found in walls of new districts gradually disappear: they are filled in by residents, who want to protect themselves from the external world with portals or front doors because of Muslim and rural influence.

The weir length  $L$ , varies on the three districts from <1 m to >10 m, but when the plot is inhabited, it never exceeds 4 m: this width is actually the maximal size of the plot entrances that correspond either to the pedestrian entrance or to the car access-way into the plot. Next, the distribution of orifice lengths is district-dependent. In the spontaneous district W2, the only doors found are for pedestrians (1- or 2-m width). The same observation has been made on the recent built-up residential district W1. In the older residential district PO, orifices with longer length (3 to 4 m) appear: they are often found in plots of car-owning residents. The gap height of all of these orifices  $a$  is always <5 cm.

The last observation concerns the base elevation of these apertures  $h_s$ . More than 70% of the apertures have a zero bottom elevation. In the general case, this elevation value is not dependent upon the district but is closely related to the situation of the plot in the watershed. It may correspond to the protection level of the plot and could be seen as an indication of its inundation frequency: inhabitants of plots situated in lowlands, and therefore often confronted with inundation problems, have actually raised the apertures of their plots. Nevertheless, these protections are not sufficient for extreme flood events.

### Characteristics of Flood Events

The basic characteristics,  $T_m$  and  $H_m$ , of the headwater flooding events were determined by the study of a complete set of 1,316 rainfall events, from the 23-year record at the meteorological station of Ouagadougou Airport (i.e., close to the city center). All storms with a rainfall depth >10 mm (i.e., 546 events) were used to compute the corresponding hydrographs according to the linear reservoir model [(13)] (Nash 1958). For this model, which is based on a lumped parametric approach, the studied watershed is considered as a linear time-invariant system. The direct runoff hydrograph  $Q(t)$  is expressed as the convolution [(13)] between the effective rainfall hyetograph  $i_n(t)$  and the instantaneous unit hydrograph  $u(t) = \exp(-t/K_{IUH})/K_{IUH}$ . This model, widely used in urban hydrology (Proc. 1996), was calibrated for the watersheds studied in Ouagadougou thanks to local hydrological data (Bouvier and Desbordes 1990).

The peak discharge  $Q_p$  extracted from each hydrograph was then used as input to the channel flow rate equation [(14)] to determine the maximum road water depth  $H_m$ . The major involved assumptions are runoff conveyed by only one road, uniform flow regime, and no backwater effect downstream to the watershed exit

$$Q(t) = A_d \int_0^t i_n(t - \tau) \frac{1}{K_{IUH}} \exp\left(\frac{-\tau}{K_{IUH}}\right) d\tau \quad (13)$$

$$Q_p = \frac{S(H_m)R_H(H_m)^{2/3}}{n} \sqrt{s} \quad (14)$$

in which  $Q(t)$  = flow rate at each time  $t$  (m<sup>3</sup>/s);  $Q_p$  = peak flow;  $H_m$  = peak flow water depth;  $S(H_m)$  = cross-sectional area of flow (m<sup>2</sup>);  $R_H(H_m)$  = hydraulic radius (m);  $s$  = slope of road (m/m);  $n$  = Manning coefficient roughness;  $i_n(t)$  = effective rainfall intensity;  $K_{IUH}$  = the instantaneous unit hydrograph parameter; and  $A_d$  = drainage area (ha). A statistical study of the basic characteristics of the flood (i.e., peak flow water depth  $H_m$  and time-to-peak  $T_m$ ) has led to the selection of four flood events corresponding to the combination of two time-to-peak values and two return periods of the peak flow (Table 1). The two rising time values correspond to a range that encompasses 90% of the rising times obtained for the 546 selected hydrographs.

To assess the influence of the district hydraulic properties on its hydraulic vulnerability, all of the following computations are made for the three districts with the same road and watershed characteristics. Average values of 10 m and of 0.01 m/m were used for the road width and the road slope, respectively. An estimated value of 0.025 was used for the Manning roughness coefficient of the earth roads. For an arbitrary drainage area of 100 ha, the peak depth values  $H_{m1}$  and  $H_{m50}$  are 30 and 70 cm for the 1- and 50-year return period events, respectively, and the corresponding flow peaks are  $Q_{p1} = 12$  m<sup>3</sup>/s and  $Q_{p50} = 28$  m<sup>3</sup>/s.

### Hydraulic Vulnerability $\phi$ or $\phi'$ versus Plot Characteristics

Fig. 8 represents the evolution of the hydraulic vulnerability  $\phi'$  versus  $R_p'$  for an orifice and for the four flood events, and Fig. 9 represents the evolution of the hydraulic vulnerability  $\phi'$  versus  $R_p'$  for an orifice and the four flood events. The  $\phi'$  ratio was calculated according to (4)–(7), and the  $\phi'$  ratio was calculated according to (10) and (11). The flux coefficient  $m$  was calibrated with the measures of the flux exchanges be-

TABLE 1. Characteristics and Codes of Four Flood Events Chosen

Return period of peak flow (1)	Time to Peak of Flow $T_m$	
	10 mn (2)	45 mn (3)
$T(Q_{p1}) = 1$ year	Flood event F1	Flood event F2
$T(Q_{p50}) = 50$ years	Flood event F3	Flood event F4

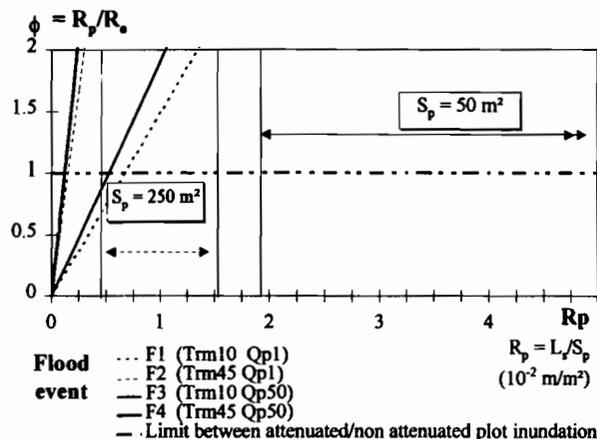


FIG. 8. Hydraulic Vulnerability  $\phi$  in Case of a Weir for Four Different Flood Events (Weir Base Elevation  $h_s = 0$  cm)

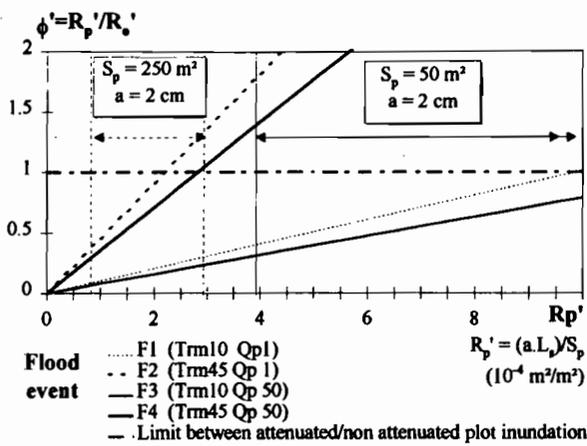


FIG. 9. Hydraulic Vulnerability  $\phi'$  in Case of an Orifice for Four Different Flood Events (Orifice Base Elevation  $h_s = 0$  cm; Gap Height  $a = 2$  cm)

tween road and plot; it is taken constant, equal to 0.6, for all plots. A sensitivity study of the  $R_p$  and  $R_p'$  ratios shows that the plot hydraulic behavior is much more sensitive to the  $R_p$  (or  $R_p'$ ) ratio than to the dimensionless ratio  $h_s/H_m$  for the Ouagadougou plot characteristics. Results are therefore presented for a base elevation  $h_s$  of the apertures equal to zero.

The two intervals plotted in Figs. 8 and Fig. 9 show the values taken by the  $R_p$ ,  $R_p'$  ratios, respectively, for two different floodable surface areas, which are the  $S_p$  extreme values observed in the three different districts ( $S_p = 50$  and  $250$  m<sup>2</sup>). The boundaries of both intervals correspond to two extreme values observed for  $L_s$  ( $L_{s,min} = 1$  m and  $L_{s,max} = 4$  m). A gap height  $a = 2$  cm was taken for the orifice case of Fig. 9.

Fig. 8 shows that the hydraulic vulnerability of every individual plot found in the three districts, with the weir apertures, is almost always  $>1$ , whatever the length  $L_s$ , the floodable surface area  $S_p$ , or the flood event characteristics. For these district, the corresponding plot inundation events are therefore almost never attenuated.

In the case of an orifice-type aperture (Fig. 9), plots are to some extent better protected against flooding. When the flood events are short (F1 and F3), which is often the case in this Sahelian region, the plot inundation is nearly always attenuated. On the contrary, slow flood events (F2 and F4) lead to a  $\phi'$  ratio  $>1$  except when the  $R_p'$  ratio is smaller than  $3.10^{-4}$  m<sup>2</sup>/m<sup>2</sup>. It is the case for example when the aperture width is smaller than 3 m for a floodable surface area of 200 m<sup>2</sup> and for a gap height of 2 cm.

Finally, it should be noticed that the results are much more influenced by the time-to-peak  $T_m$  than by the peak flow  $Q_p$ . The explanation is obvious: while  $\phi$  (or  $\phi'$ ) is an increasing linear function of  $T_m$ , it is only a function of the square root of  $H_m$ , where  $H_m$  is approximately a linear function of  $Q_p^{3/5}$  because of the rectangular shape of the road cross section as in road flow equation [(14)].

### Hydraulic Vulnerability $\phi$ or $\phi'$ versus District Type

Fig. 10 presents histograms of computed hydraulic vulnerability for all plots surveyed in the three different districts. Because the influence of the peak flow return period on the  $\phi$  ratio is not very significant, results are shown only for the first two flood events, F1 and F2. Plot frequencies are given in percentages for three groups of hydraulic vulnerability defined as follows: a zero  $\phi$  ratio corresponding to the plots without effective apertures (no gap between door and weir base), a  $\phi$  ratio leading to a theoretically attenuated plot inundation ( $\phi < 1$ ), and a  $\phi$  ratio leading to a rapid and nonattenuated plot inundation ( $\phi \geq 1$ ).

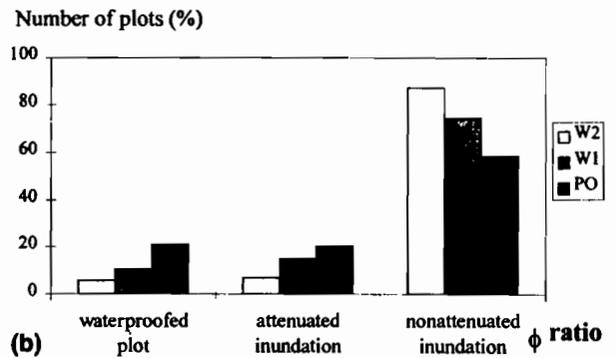
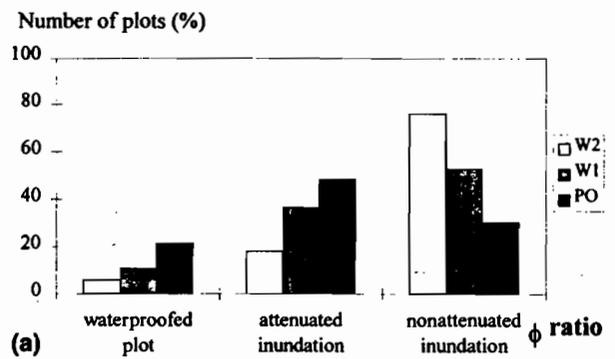


FIG. 10. Distribution of Individual Plots (in Percentages) versus Hydraulic Vulnerability  $\phi$  (or  $\phi'$ ), for Three Ouagadougou Residential Districts (Increasing Age Order): (a) Short Flood Event (F1); (b) Slow Flood Event (F2)

The differences between hydraulic vulnerability distributions for the three district types are quite obvious: the older the quarter is, the more numerous are the waterproofed plots and the less vulnerable are the other plots for short flood events [Fig. 10(a)]. Fig. 10(b) shows that these differences between districts diminish when the time-to-peak value of the road flood increases. The hydraulic vulnerability distributions for each district type are very dependent on the rapidity of the passing flood wave: for the well developed district (PO), while  $>70\%$  of the plots are relatively protected (attenuated inundation) in the case of a fast flood event such as F1, nearly 60% of them are completely inundated during a slow flood event such as F2.

### Stage Attenuation Ratio versus District Type

To have more accurate information on plot inundation, the stage attenuation ratio  $\omega$  has finally been evaluated. The  $\omega$  ratio has been computed versus  $\phi$  (or  $\phi'$ ) according to the approximate relationship [(12)]. Fig. 11 presents histograms of  $\omega$  ratio for all surveyed plots and for the two flood events F1 and F2. Plot frequencies are given for six groups of stage attenuation ratio defined in the figure.

Fig. 11(a) shows that for short flood event F1, a significant proportion of the plots are affected with well attenuated inundation events: from 18% (for the more recent, W2 district) to 62% (for the older, PO district) of the  $\omega$  ratios are smaller than 0.75. These frequencies are significantly lessened in the case of the slow flood event F2: from 9% (for W2) to 34% (for PO) of the  $\omega$  ratios are smaller than 0.75 (Fig. 11b).

Errors on these histograms due to the approximate computing procedure were estimated by combining the maximum relative deviations recorded earlier between the analytical and the numerical model components that link together the input parameters  $R_p$ ,  $\phi$ , and  $\omega$ , successively. The maximum absolute deviation thus obtained on frequency values vary from  $\pm 1\%$  for low  $\phi$  or  $\omega$  values to  $\pm 5\%$  for the higher  $\phi$  or  $\omega$  values.

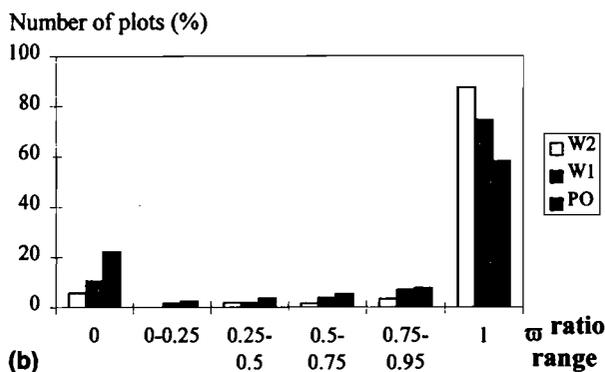
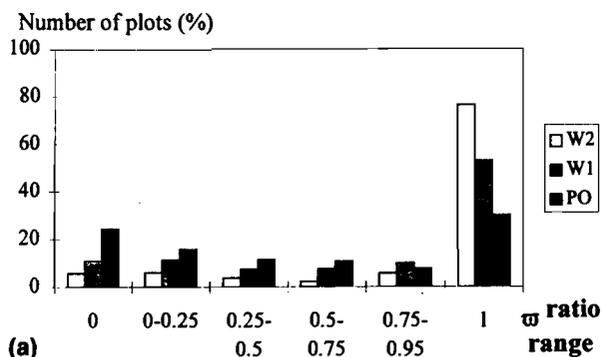


FIG. 11. Distribution of Individual Plots (In Percentages) versus Stage Attenuation Ratio  $\omega = H_{pm}/H_m$ , for Three Ouagadougou Residential Districts (Increasing Age Order): (a) Short Flood Event (F1); (b) Slow Flood Event (F2)

As a conclusion, the hydraulic vulnerability of individual residential plots may be related to the development level of the districts or to their age. Observations made in very old districts of the Ouagadougou city center showed that the plots are much more enclosed than in the Patte d'Oie district: they may therefore be less vulnerable. The hydraulic vulnerability is extremely variable according to the rapidity of the passing flood wave in the adjacent roads. It also varies with the plot hydraulic properties, but, aside from the waterproofed plots, this variability is largely attenuated in the case of slow flood events. For such events, it may reliably be assumed that, in all pervious plots, the inundation has the same characteristics as the flood outside the plot.

These results may be combined with appropriate stage-to-damage relationships in order to assess the economic losses suffered by the district residents.

## CONCLUSIONS

A methodology for quantifying the hydraulic vulnerability of individual plots separated from the external world by a surrounding wall has been developed and applied to plots of several residential areas of the Sahelian city of Ouagadougou. This hydraulic vulnerability is related to the characteristics of the flood event and to the hydraulic properties of the plot (perviousness and storage capacity). It can be simply estimated from analytical formulas, together with the stage attenuation ratio between the maximum water height in the inundated plot and the peak water depth of the flood on the road.

In the case of residential plots found in Ouagadougou, two types of apertures (weir and orifice) lead to totally different hydraulic vulnerabilities. The weirs produce a total inundation of the plots in most of the flooding circumstances, whereas orifices lead to an attenuated inundation of the plots provided the passing flood wave on the road is not too slow. Moreover, knowing the distribution of the plot hydraulic properties and the characteristics of the flooding events is sufficient to eval-

uate the hydraulic vulnerability distribution and the stage attenuation ratio distribution for a given urban area. Plot perviousness data could seem to be rather difficult to obtain; hopefully, they may simply be related to some more readily available characteristics of the district, such as its type, its age, or the average social standard of its inhabitants.

Finally, this simple analysis should allow a more accurate assessment of the damage suffered by a flooded area and may be generalized to every urban configuration where the urban environment is highly partitioned.

## ACKNOWLEDGMENTS

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## APPENDIX II. NOTATION

The following symbols are used in this paper:

$a$  = gap height (in case of orifice-type aperture) (m);  
 $H_{pm}$  = maximum plot water depth (m);  
 $H_{rm}$  = maximum road water depth (m);  
 $h_p(t)$  = plot water depth (m);  
 $h_r(t)$  = water depth of flood on road (m);  
 $h_s$  = elevation of base of aperture (m);  
 $L_s$  = width of aperture (m);

$R_o$  =  $R_p$  limiting ratio leading to nonattenuated inundation of plot ( $m^{-1}$  or dimensionless);  
 $R_p$  = ratio between plot perviousness and plot storativity ( $m^{-1}$  in case of weir or dimensionless in case of orifice);  
 $S_p$  = floodable surface area of plot ( $m^2$ );  
 $T_{rm}$  = rising time to peak of road passing flood wave(s);  
 $\phi$  =  $R_p/R_o$  = hydraulic vulnerability (dimensionless); and  
 $\omega$  =  $H_{pm}/H_{rm}$ , stage attenuation ratio (dimensionless).

Note: When aperture type is an orifice, the prime symbol (') is used with  $K$ ,  $p$ ,  $Q_{ex}$ ,  $R_p$ ,  $R_o$ , and  $\phi$ .