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MAPPING THE PRECIPITATIONS AND DETERMINING THE DESIGN STORMS IN WEST AFRICA

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MAPPING THE PRECIPITATIONS AND DETERMINING THE DESIGN STORMS IN WEST AFRICA

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The Hydrology Service was given the responsibility of making a statistical study of daily precipitation in West Africa based on the results of observations made in the following countries:

SENEGAL, MAURITANIA, MALI, UPPER-VOLTA, NIGER, CHAD, IVORY COAST, TOGO DAHOMEY, CAMEROONS (Northern part).

The studies commenced with an attempt to fit the readings to a graph for a truncated Galton distribution in accordance with the procedure given by M. ROCHE (1). Application of the station-year method, however, showed that the observed number of days for which the daily precipitation was greater than the total measured rainfall (of 100 year recurrence) as determined for each station by the procedure was too low. Fitting the readings to a graph for a truncated Galton distribution resulted in excessive estimates for the rarely exceeded levels of precipitation.

Because of the difficulties described above a completely new start was made with an attempt to fit the readings, using the moments method, to a graph for an incomplete truncated gamma distribution (PEARSON III). The procedure which was based on the use of a desk top calculator is detailed in reference (2). As will be seen later this distribution law appears to be satisfactory. It is probable, however, that a procedure of dealing with the readings based on a maximum probability procedure, involving the use of a large computer, would result in a better determination of the values for the distribution parameters and hence a better determination of the rarely occurring daily precipitations.

RESULTS OF OBSERVATIONS

1.1 Fortunately we had access to nearly all the original observations written by the observers. As a result of this we were able to note the large number of errors of interpretation and copying which arose when reducing a printed (or roneo typed) document based on the original observation.

For each station we used the largest possible number of complete calendar years of daily precipitation. Where years have been omitted (in series of several years), sometimes as many as five this was because in general the readings were either clearly incorrect (the observer always adding zeros for millimetre units) or obviously invented (for example the observer without any imagination recording readings in multiples of six).

We conserved the years of "rounded" readings providing that there were not too many in centimetres, and the years of cumulative readings on condition that they are not in groups of five. Very often the observer does not record light falls of rain and does not take readings on public holidays. As a result of this the total number of rainy days recorded for the year is appreciably reduced and there is a corresponding increase in the proportion of days for which there was heavy precipitation.

We did not verify that the readings were consistently made over periods of time. We considered that at the time when this study was made that there were not many errors due to a lack of such consistency. The errors here fall into two categories.

In the first place it often happened that the rain gauge in a particular area was moved when the observer was changed. Such displacements amounted to or exceeded a kilometre and the position of the rain gauge was accordingly changed with respect to the local relief and the prevailing wind. We had no detailed information on the previous locations and could not therefore take account of these changes. However, we consider that the errors here can be neglected in studying daily rainfall since the relief is in general not very great in the regions concerned.

In the second place inconsistent readings could have resulted from the use of a sampling receptacle which did not correspond to the collecting area of the rain gauge. In some cases the value of the correcting coefficient was wrong or the measured values were not corrected at all. We now consider that this practice was fairly common and that it continues. In general the fault involves the use of a sampling receptacle, for the rain gauge, having an area of 314 cm^2 for a rain gauge collecting area of 400 cm^2.

1.2 In conclusion, in considering the validity of the observed readings, we think that they are in general acceptable otherwise it would not have been possible to recognise the general tendencies in the distribution as detailed below. The readings that we used were those taken from stations for which we had at least ten years of daily records:

SENEGAL	41	stations, hence 1195	years	(1961 included)
MAURITANIA	18	546		(1963 ")
MALI	60	1569		(1961 ")
UPPER-VOLTA	30	714		(1960 ")
NIGER	37	958		(1965 ")
CHAD	65	1074		(1963 ")
CAMEROONS (Northern part)	11	165		(1965 ")
IVORY COAST	39	1063		(1964 ")
TOGO	47	968		(1964 ")
DAHOMEY	35	1021		(1964 ")
Total	363 stations	9273 station years		

The above list does not include:

- the coastal stations, that is those which are less than 10 km from the sea coast or from a lagoon.
- stations further north than 19° N of latitude or stations where the annual average rainfall is less than 100 mm.

In the first case the readings having been examined the data was rejected since it was clear that the distribution of daily rainfall for these stations could not be related to an incomplete gamma distribution or such relations could only be established with values of parameters for the distribution which did not correspond to the same laws as those applying to the inland stations.

In the second case the readings were automatically rejected since the spacial density of these stations is too low and because at these latitudes the amount of winter rainfall is too great compared with that occurring in the summer.

2. STUDY PROCEDURE

2.1 The equation used for the law of probability for the distribution of daily precipitation was that for a truncated incomplete gamma distribution (PEARSON III):

$$F_1(x) = F_1(o)\frac{1}{\Gamma(\gamma)}\int_x^{\infty}\left(\frac{x}{s}\right)^{\gamma-1} e^{-x/s}\,\frac{dx}{s}$$

$F_1(x)$ is the probability that the amount of daily precipitation is equal to or greater than x

$F_1(o)$ is the probability that the precipitation is zero - truncating parameter.

γ is a positive, non dimensional shaping parameter

s is a positive scale parameter expressed in the same unit as the precipitation (in mm)

$\Gamma(\gamma)$ is the complete gamma function (second order Euler term), that is

$$\Gamma(\gamma) = \int_o^{\infty} u^{\gamma-1} e^{-u}\, du$$

2.2 Determining the parameter values.

We could have assumed $F_1(o)$ equal to $\frac{M}{365.25}$, M being the number of rainy days per year. This latter number is not well known: number of days of precipitation less than 0.1 mm not recorded, light rainfalls not recorded, fall of dew counted as rain.

We determined the values of the parameters $F_1(o)$, (or rather the value of M), s and γ by an improved moments method. The detailed procedure involved is not described here but is given by reference (2).

It should be noted that there was a wide variation in the values of the parameters. For 3 600 observed days of precipitation (not zero) the variation amounted to:

for s of the order of 15%

for the product $s\gamma$ of the order of 19%

and for γ of the order of 30%.

However, variations in the value of γ do not have much effect on the result: a variation of 10% (assuming the value of the product $M s \gamma$ equal to the annual average rainfall) results in a difference of 2% in the value for the height of daily precipitation of 100 year recurrence. Furthermore, whatever equation is used, the variation in the calculated values of these parameters will be of the same order, as given by the form of the actual distribution.

2.3 Calculating the height of daily precipitation at a particular point and for a previously selected recurrence period.

For this calculation we established tables giving values of log 1/F as a function of u

where $F = \int_u^\infty \sqrt{\gamma}\,(u\sqrt{\gamma})^{\gamma-1}\; e^{-u\sqrt{\gamma}}\; du\,\sqrt{\gamma}$

with $u\sqrt{\gamma} = \frac{x}{s}$

The values were established as a function of u increasing in steps of one tenth and for values of γ ranging from 0.2 to 1.1 in steps of 0.05. Since the result is not very sensitive to the values γ, the error caused by using the nearest tabulated value γ_t, in place of the real value of γ, is negligible. It is sufficient to use the values M_t and s_t for the other parameters such that

$s - \gamma_t M_t = s\gamma m$ and $s_t(\gamma_t + 1) = s(\gamma + 1)$

3. GENERAL RESULTS OF THE ANALYSIS

The analysis based on the method described above involved a total of 383 stations corresponding to 9 273 years.

The following values were determined for each of these stations:

- the value of the parameters for the incomplete truncated gamma distribution: γ s and M
- the selected values of these parameters for inclusion in the tables
- from the tables: the average number of days in the year when the rainfall was ≥ 10.0 mm and the values for the height of the daily rainfall at particular points having recurrence frequencies of 1, 2, 5, 10, 20, 50 and 100 years
- from the observations: the number of times when the precipitation was equal to or exceeded the calculated heights for the different recurrence periods mentioned above.

The results for the complete analysis were as follows:

- with rare exceptions, the calculated value M, that is the average number of rainy days per year, is slightly greater than the observed number, which is quite normal

- the calculated value for the number of days when the precipitation was greater than 10.0 mm was, per station-year, on average 0.2 days greater than the observed number of days when the precipitation was equal to or greater than 10.1 mm. This is a good agreement

- the table below shows the number of days (observed and theoretical) where the calculated height, for different recurrence periods, was equalled or exceeded.

Recurrence period	Observed No.	Theoretical No.
	9229	9273
2 years	4630	4636.5
5 years	1881	1854.6
10 years	978	927.3
20 years	509	463.65
50 years	220	185.46
100 years	116	92.73

This station-year method appears to give lower theoretical compared with observed values commencing with recurrence periods of ten years. This is not apparent when each station is considered in isolation. Values calculated according to an incomplete truncated gamma distribution appear to be low with respect to the observations, the discrepancy amounting to:

1% lower for recurrence period of	10 years
1.5%	20 years
2%	50 years
2.5%	100 years

It should be noted that we have used a large number of years of cumulative observations (3.1) and the number of observations in the above table are greater than the number of observations that would have been made if the reading had not not been cumulative. So far as the end result is concerned the incomplete truncated gamma distribution appears to give a good and convenient representation for the distribution of daily precipitation at local points in West Africa (coastal regions excepted).

4. VARIATIONS IN THE PARAMETER VALUES

4.1 Variations in the shaping parameter γ .

The average and median value of γ amounts to 0.70. Two thirds of the calculated values of γ lie between 0.5 and 1.10. The effect of these variations in this parameter is less important than it would seem: for equal annual and average rainfalls the daily precipitation at particular locations for a recurrence period of 100 years, for values of γ = 0.7, amount to

14%	greater when	γ	equals	0.45
7%	"	"		0.55
6%	less	"		0.90
11%	"	"		1.10

and the differences here diminish with the recurrence period

4.2 We have not been able to find any relation between latitude, longitude or altitude and localised values of γ , except that the values fall on approaching the sea particularly in the case of the Atlantic Ocean and towards the Gulf of GUINEE.

On the other hand geographical conditions in the vicinity of the rain gauge station certainly account for variations in the value of the shaping parameter:

- if the relief which overlooks the station is to the South or West, γ is greater than 0.7 to the extent that the relief is close
- if the relief is to the East or North, γ is less than 0.7 to the extent that the relief is close but it seems that the extent of this effect is less in the case of East or West reliefs (5 km ?) than in the case of reliefs to the South or West (15 km ?).

The presence of a major river (NIGER, SENEGAL) in the immediate vicinity of the station is equivalent to a relief on the opposite side of that station.

If there are reliefs in different directions from the station, that to the South dominates the effects of relief to the North or East, effects of relief to the North dominates the effects of relief to the West, effects of relief to the West dominates the effects of relief to the East, account being taken of the relative distances involved.

Location in wet forests or extensive flooded areas results in lower values of γ.

4.3 The values of the other parameters are related to the amount of average annual precipitation $\overline{P}$ and the value of γ . The value of the parameter M is given by

$$M = P / (s\ \gamma)$$

and the value of the product ($s\ \gamma$) allows s to be determined when γ is known, the values of this product being given in the following table:

$\overline{P}$ in mm	100	200	300	400	500	600
$s\ \gamma$	7.15	9.51	10.61	11.28	11.76	12.12
$\overline{P}$ in mm	800	1000	1200	1500	2000	2500
$s\ \gamma$	12.62	12.93	13.11	13.22	13.23	13.24

The values of the product ($s\ \gamma$) given above are of course average values.

5. PRESENTATION OF THE RESULTS

The results of the study of localised daily precipitation in West Africa is given in the report (2) in the form of $1/5.10^6$ scale maps. These maps show:

- the inter-annual isohyets for 100 mm intervals and values of the parameter γ , no account being taken of relief effects.

- the daily precipitation at intervals of 5 mm and for a one year recurrence period

- the daily precipitation at intervals of 10 mm and for a two year recurrence period

- the daily precipitation at intervals of 10 mm and for a five year recurrence period

- the daily precipitation at intervals of 10 mm and for a ten year recurrence period

- the daily precipitation at intervals of 20 mm and for a twenty year recurrence period.

6. DESIGN STORM

The study of storms in order to define a "design storm" is more difficult than that of daily precipitation. We have only been able to complete the study for storms at a single point of intensity.

We do not discuss the observed data used for the graphical method employed, these points being dealt with in detail by reference (3), we simply consider the results obtained.

In the case of a storm consisting of a single fall of rain around a single point of intensity, the most common type of precipitation in tropical regions when the average annual rainfall is less than 1000 mm, we can make a distinction between:

- a pre-storm period, very often absent, of short duration and of intensity less than 18 mm/hour

- the storm period itself, consisting of a single fall of rain at the rate of more than 18 mm/hour, the rate or intensity increasing very rapidly up to a peak point and then falling

- a post-storm period, rarely absent and lasting several hours, the intensities being variable and less than 18 mm/hour.

6.1 The interesting part of the storm is the main rainfall period. In the case of a single storm during the day, the height of precipitation C corresponding to this main period is:

$$C \text{ mm} - 0.9 \left[\text{precipitation for the day} - 5 \text{ mm} \right]$$

The duration of this period amounts to

$$D \text{ min} = 14.9 \, (C + 1.82)^{1/3} - 18.2$$

The rated intensity of this main fall of rain as a function of t in minutes is

$$I\,(t) \text{ mm/h} = 6 \left[\frac{D - t + 18.2}{10.5} \right]^2$$

and therefore the maximum rate after 5 minutes, about the point of intensity, is

$$I_s \text{ mm/h} = 12.06 \left[C + 1.8\right]^{2/3} - 4.05 \left[C + 1.8\right]^{1/3} + 0.45$$

For example, with a daily precipitation of 184 mm

$C = 161$ mm

$D = 63.2$ min

$I_s = 338$ mm/h

I_{max} (instantaneous) = 360 mm/h

With this example, however, we are beyond the practical limits for applying this equation. The maximum intensity is over-estimated since storms of this magnitude do not generally consist of a single fall of rain around one point and for such a daily precipitation there would probably be several storms.

Given a graph for the rated intensities for the main fall of rain of the storm it is easy to produce a hyetograph for this storm by assuming a linear increase in intensity rising to a peak value over a period of 8.8 minutes (whatever the height of precipitation of the storm).

6.3 It should be noted that hyetograph established in this way and based on the daily precipitation for a given recurrence period will necessarily indicate less frequent occurrences than those of the daily precipitation.

References :

(1) M. ROC E - Surface Hydrology - Paris, 1963.

(2) Y. BRUNET-MORET - General study of exceptional storms in West Africa. ORSTOM Paris, 1968.

(3) Y. BRUNET-MORET - A supplement to the general study of exceptional storms in West Africa. CHAD Republic. ORSTOM Paris, 1966.