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Journal of Hydrology 263 (2002) 114–130

Journal of Hydrology

www.elsevier.com/locate/jhydrol

Evaluation of an antecedent precipitation index to model runoff yield in the western Sierra Madre (North-west Mexico)

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Received 26 October 2000; revised 15 August 2001; accepted 27 February 2002

Abstract

In a subtropical mountain of northern Mexico, soils and slopes seem to have a hortonian functioning, as it is commonly observed in other tropical or subtropical areas. So it is important to know previous soil moisture before any rainfall event in order to properly estimate the runoff coefficients and to get a better understanding of the hydrologic behaviour of the catchments.

A very simple deterministic model, named NAZASM, is developed and tested for 6 microplots (1 and 10 m^2), 13 plots (50 m^2) and 5 catchments (1–50 km^2). It is based on the 'antecedent precipitation index' with an exponential decay parameter (α) into the rainfall–runoff relationship. Measurements of rainfall and runoff were made during four rainy seasons. Between 22 and 157 rain events were considered in the analysis, depending on the sizes of the plots and catchments.

'NAZASM' gives reasonable estimations of runoff if the calibration data set includes events occurring during wet and dry years, due to the difference of hydrological processes involved in these two situations.

All the results have shown that the use of such a model significantly improves the explained variances between measured runoff and rainfall as compared to classical regression analysis, whatever the size of plots and catchments are.

The α soil water content decay parameter which in theory is a soil characteristic, appeared in fact to be scale dependent.

Keywords: Rainfall–runoff relationship; Deterministic model; Hortonian runoff; Antecedent precipitation index; Western Sierra Madre

1. Introduction

Numerous experimental and numerical studies have been devoted worldwide to the formation of runoff on slopes at both catchment and plot scales. They clearly show that the initial soil water content has a direct influence on the infiltration capacity and as a result on surface runoff. The principle of the infiltration excess runoff was presented by Horton (1933) and applied by Sherman (1932) with the unit hydrograph theory.

A slope or catchment area is said to have a hortonian behaviour if runoff starts, when the rainfall intensity exceeds the soil infiltration capacity (Hillel, 1988).

In semi-arid and arid regions, presence of surface crusts have been observed to play a major role on runoff yield by reducing the entry of water into the soil (i.e. Duley (1939), McIntyre (1958), Morin and

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PII: S0022-1694(02)00047-1

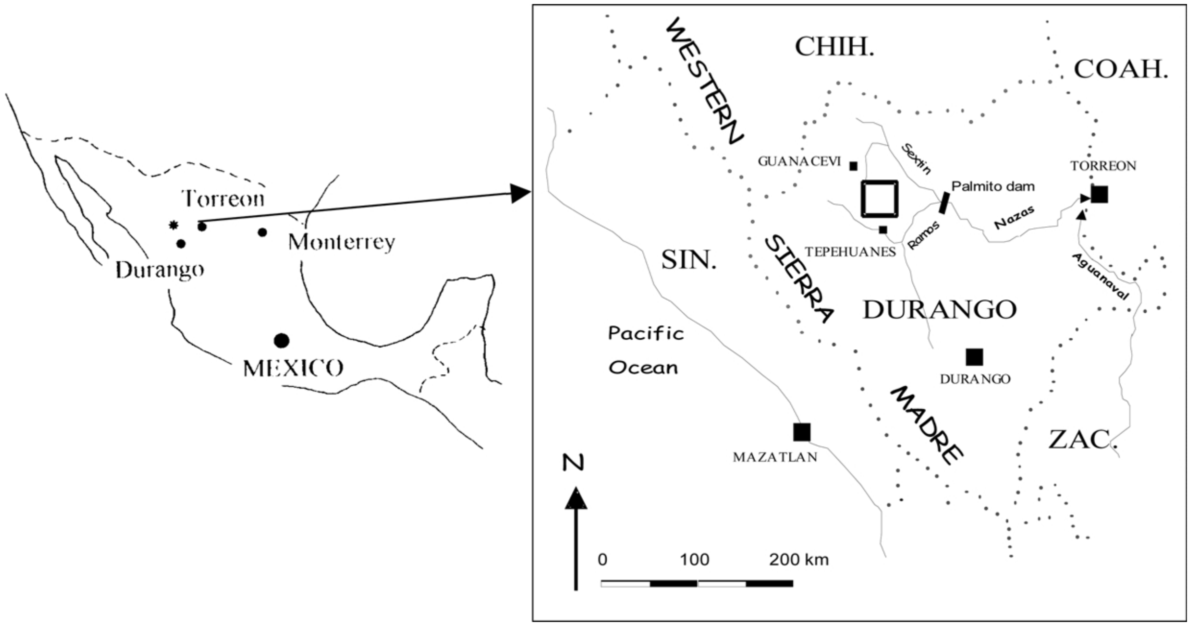


Fig. 1. Location of the study area (bold square).

Benyamini (1977), Valentin (1985), Planchon (1991) among others).

In other environments, such as temperate zones or forest areas, floods (even attenuated or delayed ones) have been recorded without any apparent significant overland flows. This observation led Bonell and Gilmour (1978), Bonell (1993) among others to suggest that subsurface flow could be the main generator of flow in thalwegs, due to the presence of an almost impermeable formation lying at a shallow depth. It was hypothesised that 'contributing areas' with high runoff coefficients in certain parts of a catchment, were the main reason for the formation of floods (Cappus, 1960; Dunne and Black, 1970).

Hydrological models have to take into account the main physical processes involved in the area of interest. Clarke (1973), Freeze (1978), Bowles and O'Connell (1991), Beven (2000) among others, provided extensive reviews of different modelling approaches. Some of them (Sittner et al., 1969; De Vries and Hromadka, 1993; Rose, 1998) are based on the antecedent precipitation index (API) concept to model the rainfall–runoff relationship.

So the main objective of this paper is to evaluate such a simple approach against field observations acquired on different sites of the western Sierra Madre, northern Mexico.

2. Study area

The western Sierra Madre (Fig. 1) is a volcanic chain of medium altitude (max. 3300 m). It is one of the biggest volcanic systems in the world. Almost the whole range is constituted by rhyolitic ignimbrites, an igneous type of rocks, which appeared at the end of the tertiary. Its behaviour in terms of erosion, water runoff and infiltration is almost the same as the granite. In some cases (particularly at the bottom of the grabens), previous stages of erosion have constituted correlative formations, mainly conglomerates with a mineralogical composition similar to the ignimbrites, but with an hydrodynamic behaviour very different because of its different appearance. They are constituted of blocks and pebbles coated into acid cement and tuffs and are generally trapped in grabens. Consequently, the material is appreciably more permeable than the ignimbritic bedrock (Viramontes, 2000; Descroix et al., 2001a).

The climate is subtropical (Descroix et al., 2001b)

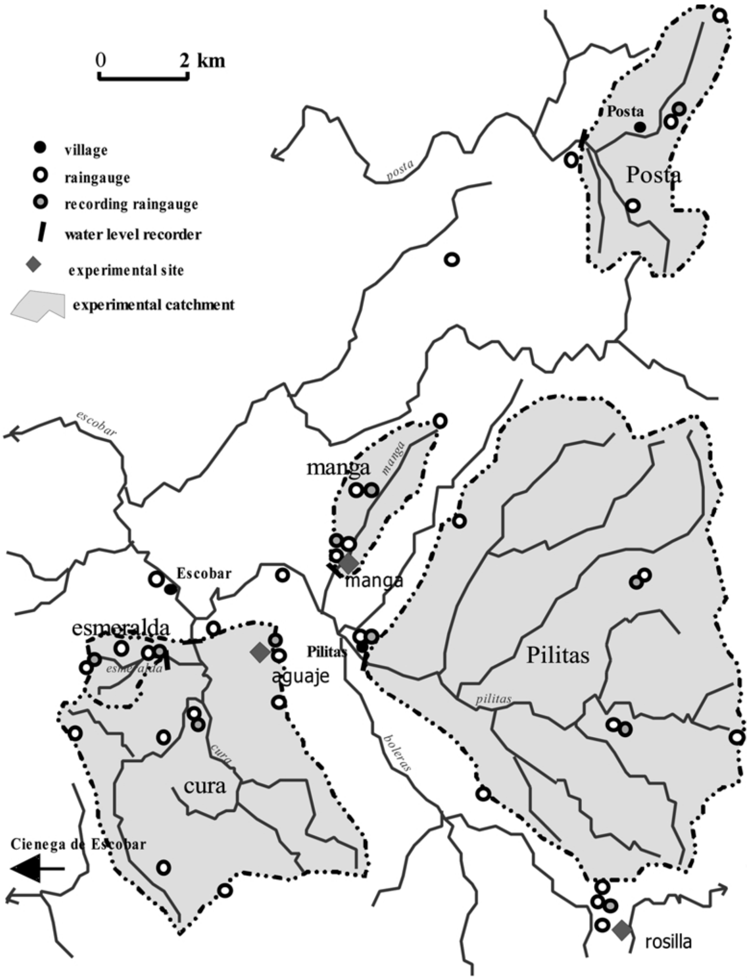


Fig. 2. The experimental network.

characterised by a 4 month-wet season (80% of annual rainfall amount) and a long dry season from 15 October to 15 June with cold winters due to both the altitude and latitude (25°N) effects. The mean annual rainfall varies from 450 mm at 1700 m to 850 mm at 2800 m of altitude. The annual evaporation (pan class A) is 1900 mm at 2200 m and the mean temperature is 14 °C with a minimum of 4 °C in January and a maximum of 25 °C in July.

Rainfall are moderately variable in time for a tropical region: previous studies (Descroix et al., 1997; Descroix et al., 2001b) showed that annual rainfall has a coefficient of variation (CV) ranging from 0.2 to 0.25 at 2000 m height, and from 0.18 to 0.2 at 2800 m height, but, given the rainfall range, these variations have a major impact on streamflows. Thus, for the upper rio Nazas basin (19,000 km^2) with an annual rainfall CV of 0.23, the corresponding CV of the average discharge is 0.67. In fact, during a ‘normal’ rainy season (total amount close to the average one), most of the small rivers have base flows for several weeks. For a season with a deficit of 40–50%, there is no base flow and runoff stops a few hours after the end of the rainfall event, even on catchments of several tens of km^2. Similarly, in spatial terms, this area is located at a boundary for significant runoff. For the entire Nazas basin, the average annual runoff coefficient is 0.13 for 600 mm of rainfall, but only 0.01 or

Table 1
Main features of the experimental catchments

Catchment	Area (km^2)	Mean slope (%)	Lithology	% Forest	% Grassland	Number of measurement (years)
Cura	21.8	6.6	Ignimbrites	43	57	4
Esmeralda	1.28	13.3	Ignimbrites	66	34	4
Manga	3.08	4.6	Conglomerates	16	84	4
Pilitas	52.3	6.1	30% conglomerates, 70% ignimbrites	43	57	3
Posta	8.62	8.5	Ignimbrites	56	44	1

0.02 for catchments of the same size located in the semi-arid part of the middle Nazas basin (rainfall between 400 and 450 mm), and flows are negligible below 350 mm.

The vegetation is adapted to the 8 month-dry season. Below 2000 m of altitude, typical formations of semi-arid zones are encountered (pasture and *Acacia schaffneri* scrub, *Prosopis glandulosa*, with succulents and cactaceae as *Fouquieria splendens*, *Opuntia megacantha* and *Opuntia streptacantha*). Between 2000 and 2400 m level pastures (with a predominance of *Bouteloua hirsuta*) and oak forests (*Quercus grisea* and *Quercus viminea*) dominate. Finally, pine forests (*Pinus cembroides* and *Pinus duranguensis*) are the predominant vegetation above 2400 m (Descroix et al. 2001a).

Soils are generally Phaeozems, 20–60 cm deep only (locally 100 cm) because of previous and present active erosion. They are red and include a surface horizon rich in organic matter (2–6%) with accumulations of clay below. This type of soil covers approximately 85% of the area, the remaining consists essentially of lithosols with occasional patches of vertisols. The soil texture is sandy clay with a relatively large amount of organic matter. Areal erosion due to overgrazing is widespread, explaining the thinness of the soil (Viramontes, 2000).

The whole Nazas–Aguanaval Basin (92,000 km^2) is divided into three geoclimatic regions: an arid area (below 300 mm of annual rainfall), a semi-arid zone (between 300 and 500 mm) and a subhumid area (above 500 mm). The present work deals only with the subhumid zone (the upper Nazas basin). The study area is mainly located between 25°N and 26°N in latitude, and between 105°W and 106°W in longitude.

3. Material and methods

Five catchments of areas ranging from 1 to 50 km^2, were equipped with discharge (float recording devices) and rain gauges (Fig. 2): Esmeralda, La Manga, La Posta, El Cura and Pilitas. Their main characteristics are given in Table 1. Forty-five raingauges (including nine recording ones) were installed; so there was around one raingauge per 4 km^2 in average. Measurements of rainfall and runoff were made during four rainy seasons (from 1995 to 1998). Between 22 and 157 rainfall events were considered in the analysis, depending on the catchment, and the measuring duration.

Data were also collected at the plot scale. Three sites were selected according to vegetation cover and lithology: one in pine forested land (Rosilla), two in areas of pasture and oaks (Manga and Aguaje) (Fig. 2); one located on rhyolite and the others on conglomerates. Because conglomerates are present only in the grabens and forests above 2300 m of altitude, no configuration with forest on conglomerates was encountered. All the sites were equipped with at least one recording raingauge and with plots of 1, 10 and 50 m^2, in order to be able to consider as many spatial variables as possible, such as catchment area, slope, vegetation, roughness, soil texture, soil surface features, percentage of organic matter, etc. Thirteen plots of 50 m^2 were installed to consider the main site variables (lithology, vegetation, stoniness, type of soil surface feature and slope); the small plots (3 of 10 m^2 and 3 of 1 m^2) were dedicated also to study the scale effect. Each plot was equipped with devices to collect runoff and soil losses. Each device was constituted by a gutter, a pipe and a collecting capacity ranging from 0.05 m^3 (for 1 m^2 plots) to 3 m^3 (for 50 m^2 plots). The

Table 2
Main characteristics of the experimental sites (OM is the organic matter content)

Site	Altitude (m)	Lithology	Vegetation type	% Sand	% Silt	% Clay	% OM	Annual rainfall (mm)			
								1995	1996	1997	1998
Rosilla	2500	Ignimbrites	Forest	63	19	18	4.08	360	612	339	
Aguaje	2200	Ignimbrites	Pasture	52	23	25	6.3	324	690	298	320
Manga	2150	Conglomerates + tuffs	Pasture	70	16	14	3.37	284	602	322	272
Cienega de Esc.[a]	2100							490	694	460	410

[a] The nearest climatologic station, characterised by a mean annual rainfall value of 584 mm (with a standard deviation of 141 mm) calculated for the 1965–1998 period.

main characteristics of each plot are given in Tables 2 and 3. For both plots and catchments, it seems important to distinguish two main types of lithology: ignimbrites and rhyolite in one side, tuffs and conglomerates in the other one.

Cumulative flow values (total per event) were measured in capacities at the end of each rainfall event. Between 59 and 111 rain events were considered, depending on the site, and for a minimum of 2 years of measurement: 1995 and 1996 in Rosilla site, 1996, 1997 and 1998 in the other two (Manga and Aguaje). Annual rainfall values measured at the three sites, as well as at the nearest meteorological station of Cienega de Escobar located at 2100 m height are reported in Table 2 for the four rainy seasons.

4. A simple rainfall–runoff model

It is well known that the previous soil moisture content is an important parameter to explain the runoff yield function. However, it was uneasy to monitor it continuously, because appropriate devices, such as

Table 3
Main features of the plots selected within the 3 sites (OM is the organic matter content. The total of cover types can exceed 100%, because tree cover is superimposed on the other ones. In Rosilla site (plots 31 and 32), the soil cover under and around the trees is constituted of litter. – indicates no data)

Number	Site	Area (m^2)	Slope (%)	% Tree cover	% Bare soil	% Grass cover	% Pebbles and stones cover	% Clay	% OM
11	Manga	50	20.7	39.4	3.1	59.7	37.2	14.8	4.7
12	Manga	50	13.6	63.2	4	27.6	68.4	13.1	3.8
13	Manga	50	12.9	0	1.5	21.6	76.9	12.2	2
14	Manga	50	23.2	0	0	22	78	15.4	3.8
15	Manga	50	10.2	0	0	41.4	58.6	11.7	2.5
16	Manga	1	12.8	0	3.7	24.9	69.1	–	–
17	Manga	10	15.2	0	8	42.4	49.6	–	–
18	Manga	10	16.8	0	3.9	43.6	52.5	–	–
19	Manga	10	16.8	0	8.7	36.3	55	–	–
21	Aguaje	50	13.2	65.3	20.3	54.5	25.2	37.4	8.4
22	Aguaje	50	14.2	0	4.9	64.2	30.9	20.2	3
23	Aguaje	50	25.4	60.3	3.9	19.1	77	18.2	7.9
24	Aguaje	50	27.9	33.2	5.7	24.2	70.1	35	4.7
25	Aguaje	50	27	27.7	1	28.5	70.5	15.5	7.5
26	Aguaje	1	14.2	0	0.6	45.3	54.1	–	–
27	Aguaje	1	23	10	12	48	40	–	–
30	Rosilla	50	23.1	0	9.9	10	7	19.1	–
31	Rosilla	50	23.1	0	3.1	5	7	–	–
32	Rosilla	50	36.4	100	0	10	5	22.2	–

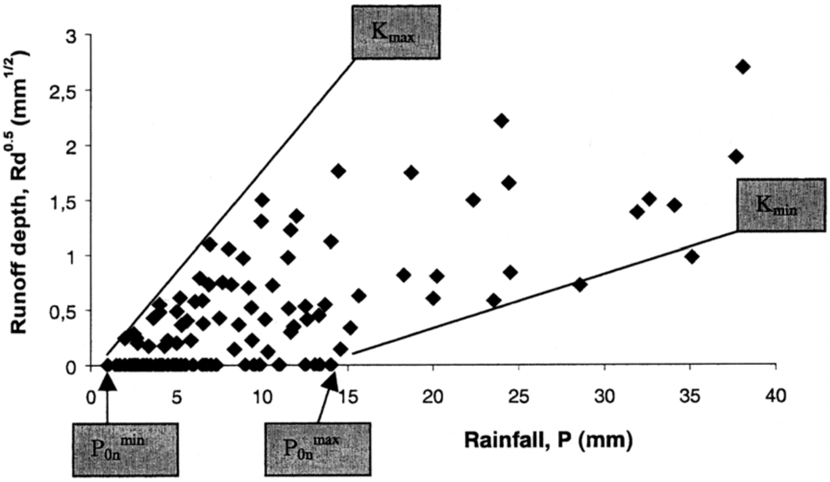


Fig. 3. Boundaries of the experimental relationship $\sqrt{\mathrm{Rd}_n} = K_n(P_n - P_{0n})$ (Eq. (1)) for the El Cura catchment (area 21.8 km^2, years 1995–1998).

neutron probe, TDR system or tensiometers were not available in the Sierra Madre context. So, the API has been used as a reasonable palliative to estimate the water status of the soil reservoir (Sittner et al., 1969; Chevallier, 1983; Seguis, 1987; Albergel, 1988; Casenave and Valentin, 1989; Molinier et al, 1993; Nouvelot, 1993; Grésillon, 1994; Descroix and Nouvelot, 1997). Recently, Rose (1998) used an API over 3-month periods to explain runoff variations over catchment areas ranging from 100 to 10,000 km^2, located in the east coastal plain of Georgia, USA.

A lumped deterministic API-type model, named 'NAZASM' model has been developed and used here to evaluate runoff on both plots and catchment areas. The model is inspired from those used by Girard (1975) for sahelian regions. It relies in the following assumptions:

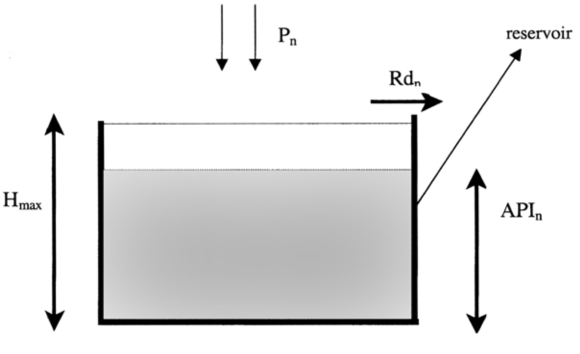


Fig. 4. Schematic representation of the soil in the API-type model.

(i) The rainfall–runoff relation (Eq. (1)) is assumed to hold for any rainy event, n:

$$\sqrt{\mathrm{Rd}_n} = K_n(P_n - P_{0n}) \qquad \text{with } P_n > P_{0n} \tag{1}$$

where Rd_n and P_n are the runoff depth and the rainfall amount, respectively, both expressed in mm. P_{0n} (mm) is the rainfall below which there is no runoff. K_n (in mm$^{-1/2}$) is a parameter depending on the soil surface hydraulic conductivity, on the catchment area and on the proportion of the catchment contributing to runoff.

Because it has been observed that all the measured values of $\sqrt{\mathrm{Rd}_n}$ and P_n were included between two straight lines (see for instance Fig. 3 for the El Cura data), K_n can be expressed as:

$$K_n = K_{\min} + \left[(K_{\max} - K_{\min})/\left(P_{0n}^{\max} - P_{0n}^{\min}\right)\right] \times \left(P_{0n}^{\max} - P_{0n}\right) \tag{2}$$

where $K_{\max}$, $K_{\min}$, $P_{0n}^{\max}$ and $P_{0n}^{\min}$ correspond to the maximum and minimum values, respectively, of K and P_{0n} for either the plots or the catchments.

(ii) By assimilating the soil to a reservoir (Fig. 4), P_{0n} can be expressed as:

$$P_{0n} = C(H_{\max} - \mathrm{API}_n) \qquad \text{with } \mathrm{API}_n \leq H_{\max} \tag{3}$$

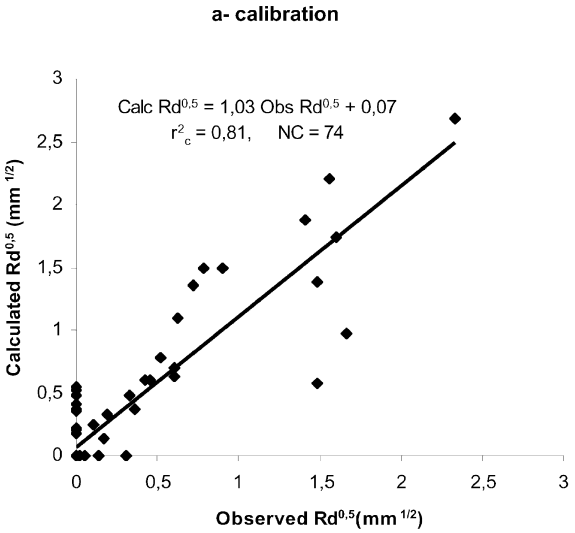
Table 4

Parameters of calibration of the NAZASM model (Eq. (5)) and comparison of the results with the empirical model (Eq. (1)) (r^2 Eq. (1) and r^2 Eq. (5) are the coefficient of determination of the linear regressions between $\sqrt{\mathrm{Rd}_n}$ and P_n based on Eq. (1) [$\sqrt{\mathrm{Rd}_n} = K_n(P_n - P_{0n})$] and Eq. (5) (NAZASM model), respectively. r_c^2 and r_v^2 are the coefficients of determination of the regression between calculated and observed runoff yield for the calibration (r_c^2) and the validation (r_v^2) of NAZASM. N is the number of events considered. All the other parameters are defined in the text. Plots 11–19 are located on conglomerates lying in a pasture area; plots 21–27 are on ignimbrites, in a pasture zone; plots 30–32 are on ignimbrites, in a forest environment)

	Area	Year	r^2 Eq. (1)	r^2 Eq. (5)	α (day^{-1})	C	$P_{0n}^{\max}$ (mm)	$K_{\max}$ (mm$^{-1/2}$)	$K_{\min}$ (mm$^{-1/2}$)	$H_{\max}$ (mm)	r_c^2	NC	r_v^2	NV
Catchments														
Pilitas	51 km^2	1995	0.06	0.18	0.20	0.09	8.9	0.10	0	100	0.52	6	0.66	5
		1996	0.54	0.66	0.09	0.20	2.9	0.17	0	161	0.83	34	0.81	33
		1997	0.71	0.95	0.095	0.08	12	0.035	0.035	79	0.74	19	0.79	19
		95–97	0.37	0.46	0.054	0.13	23	0.08	0.004	120	0.76	59	0.81	58
Cura	21.8 km^2	1995	0.82	0.99	0.12	0.05	13	0.11	0.07	100	0.86	17	0.79	16
		1996	0.67	0.73	0.03	0.015	4	0.12	0	260	0.91	36	0.78	36
		1997	0.67	0.79	0.09	0.02	4.5	0.18	0.18	230	0.91	13	0.67	13
		1998	0.74	0.94	0.15	0.07	2.5	0.11	0.05	100	0.80	9	0.62	9
		95–98	0.61	0.75	0.03	0.08	14	0.09	0.06	223	0.81	74	0.64	73
Posta	8.61 km^2	1997	0.69	0.94	0.02	0.04	15	0.06	0.06	100	0.76	11	0.75	11
Manga	3.1 km^2	1995	0.49	0.67	0.05	0.1	3.8	0.1	0	44	0.74	17	0.68	16
		1996	0.37	0.67	0.02	0.03	8	0.12	0	250	0.77	35	0.93	34
		1997	0.31	0.77	0.02	0.013	17	0.03	0	59	0.39	16	0.46	15
		1998	0.66	0.67	0.1	0.2	18	0.035	0	80	0.84	12	0.84	11
		95–98	0.61	0.75	0.01	0.06	17	0.13	0	250	0.75	79	0.76	78
Esmeralda	1.28 km^2	1995	0.91	0.91	0.34	0.4	10	0.21	0.08	40	≈ 1	8	≈ 1	7
		1996	0.77	0.89	0.11	0.05	8	0.17	0	140	0.86	35	0.90	34
		1997	0.40	0.68	0.04	0.22	10	0.08	0.06	45	0.60	9	0.93	9
		1998	0.75	0.98	0.1	0.05	20	0.065	0	150	0.74	17	0.89	17
		95–98	0.59	0.81	0.10	0.02	4.8	0.08	0	129	0.66	68	0.72	68
Plots														
Manga site														
11	50 m^2	1996	0.34	0.60	0.18	0.06	4	0.08	0	48	0.57	16	0.82	15
		1997	0.8	0.98	0.10	0.10	7	0.18	0.02	35	0.79	17	0.54	16
12	50 m^2	1996	0.33	0.76	0.08	0.16	25	0.06	0.06	90	0.50	16	0.88	15
		1997	0.84	0.98	0.06	0.08	7	0.14	0.02	35	0.82	17	0.77	16
13	50 m^2	1996	0.43	0.70	0.07	0.17	24	0.12	0.12	100	0.68	16	0.90	15
		1997	0.77	0.96	0.06	0.12	7	0.16	0.02	35	0.74	17	0.77	16
		1998	0.89	0.93	0.08	0.25	10	0.11	0.11	41	0.93	13	0.87	13
		96–98	0.54	0.68	0.05	0.14	24	0.14	0.03	131	0.72	45	0.77	45
14	50 m^2	1996	0.45	0.69	0.07	0.17	24	0.10	0.10	100	0.69	16	0.92	15

Table 4 (*continued*)

	Area	Year	r^2 Eq. (1)	r^2 Eq. (5)	α (day^{-1})	C	$P_{0n}^{\max}$ (mm)	$K_{\max}$ (mm$^{-1/2}$)	$K_{\min}$ (mm$^{-1/2}$)	$H_{\max}$ (mm)	r_c^2	NC	r_v^2	NV
		1997	0.83	0.97	0.05	0.09	5	0.13	0.01	35	0.82	17	0.76	16
15	50 m^2	1996	0.31	0.97	0.06	0.03	22	0.05	0	120	0.66	16	0.95	15
		1997	0.88	0.99	0.05	0.04	5	0.15	0	35	0.85	17	0.65	16
16	1 m^2	1997	0.56	0.99	0.001	0.03	12	0.13	0.01	30	0.62	17	0.64	16
		1998	0.74	0.97	0.07	0.01	3	0.25	0	180	0.75	13	0.92	12
17	10 m^2	1997	0.90	0.99	0.1	0.10	5	0.13	0.1	12	0.90	10	0.80	9
18	10 m^2	1997	0.84	0.88	0.01	0.25	2.5	0.28	0	10	0.84	10	0.56	9
19	10 m^2	1997	0.82	0.64	0.08	0.20	5	0.30	0	28	0.93	10	0.62	9
Aguaje site														
21	50 m^2	1996	0.76	0.96	0.046	0.02	7	0.1	0.06	221	0.80	24	0.86	24
		1997	0.87	0.99	0.08	0.05	6	0.08	0.08	60	0.85	18	0.84	18
22	50 m^2	1996	0.85	≈1	0.05	0.01	5	0.17	0.17	220	0.85	24	0.81	24
		1997	0.79	0.99	0.10	0.01	9	0.21	0.21	60	0.79	18	0.80	18
		1998	0.65	0.83	0.07	0.04	6	0.13	0	99	0.81	14	0.89	13
		96–98	0.59	0.99	0.1	0.02	17	0.12	0	100	0.63	56	0.73	56
23	50 m^2	1996	0.88	≈1	0.04	0.01	5	0.12	0	229	0.88	24	0.75	24
		1997	0.81	≈1	0.12	0.01	5	0.08	0.08	30	0.80	18	0.78	18
24	50 m^2	1996	0.63	0.84	0.04	0.01	5	0.11	0	232	0.79	24	0.81	24
		1997	0.87	≈1	0.08	0.01	5	0.07	0.07	40	0.86	18	0.82	18
25	50 m^2	1996	0.78	0.94	0.04	0.01	6.7	0.11	0	230	0.83	24	0.82	24
		1997	0.80	≈1	0.01	0.01	5	0.08	0.08	60	0.80	18	0.85	18
26	1 m^2	1997	0.70	0.70	0.013	0.03	7	0.18	0	110	0.73	13	0.67	13
		1998	0.87	0.98	0.01	0.10	7	0.19	0.19	57	0.90	14	0.92	13
27	1 m^2	1998	0.86	0.97	0.1	0.23	7	0.1	0.1	30	0.91	14	0.88	13
Rosilla site														
30	50 m^2	1995	0.76	0.83	0.1	0.10	5	0.18	0.01	50	0.79	14	0.79	13
		1996	0.75	0.80	0.035	0.02	5	0.19	0	168	0.91	19	0.75	19
31	50 m^2	1995	0.78	0.97	0.06	0.08	4	0.06	0.02	45	0.81	14	0.73	13
		1996	0.74	0.84	0.04	0.13	34	0.16	0.11	184	0.88	19	0.80	19
32	50 m^2	1995	0.52	0.89	0.01	0.07	8	0.04	0.01	80	0.55	14	0.54	13
		1996	0.21	0.42	0.03	0.30	20	0.1	0.05	160	0.56	19	0.51	19
Mean values			0.660.84							0.78		0.78		


a- calibration

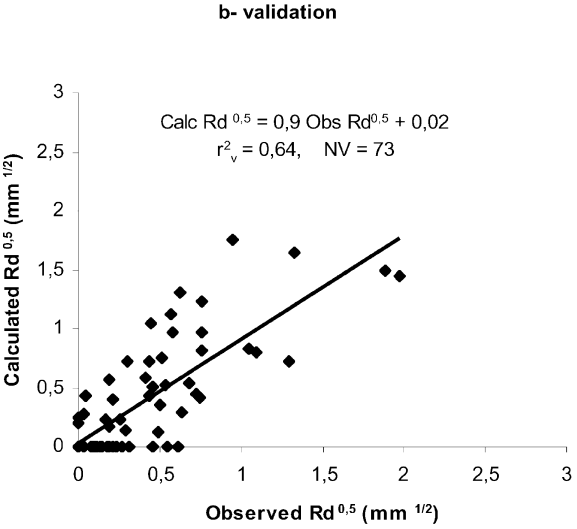

b- validation

Fig. 5. Calibration (a) and validation (b) of NAZASM for the Cura catchment (1995–1998): comparison between calculated and observed values of runoff (square root).

where C is a parameter taking into account most likely rainfall intensity and indirectly the catchment heterogeneity, the water storage of the soil surface (including vegetation and litter) and the mechanical effect of raindrops on the soil. $H_{\max}$ is the maximum water storage of the reservoir (mm) and API_n (mm) is its actual level at a given time (Fig. 4).

(iii) Following the definition of the API (Kohler and Linsley, 1951; Chevallier, 1983), API_n is calculated as:

$$\mathrm{API}_n = (\mathrm{API}_{n-1} + P_{n-1})\exp(-\alpha\Delta t) \qquad (4)$$

where $\Delta t = t_n - t_{n-1}$ is the time (day and/or fraction of day) elapsed between the end of the previous rain event P_{n-1} and the beginning of the current one (P_n).

The parameter α (day^{-1}) is the inverse of the characteristic time of soil moisture depletion.

Introducing Eq. (4) into Eq. (3) and then into Eq. (1), gives:

$$\sqrt{\mathrm{Rd}_n} = K_n\{P_n - C[H_{\max} - (\mathrm{API}_{n-1} + P_{n-1}) \times \exp - \alpha(t_n - t_{n-1})]\} \qquad (5)$$

The model (Eq. (5)) has seven parameters ($C, H_{\max}, \alpha, K_{\max}, K_{\min}, P_{0n}^{\max}, P_{0n}^{\min}$) to be determined. This was achieved by splitting the time series of observed (P_n, Rd_n) values in two parts (one event out of two): one half being used for the calibration of the parameters by best fitting between calculated and measured values of runoff depths; and the other one for the validation, the values of the parameters being kept unchanged.

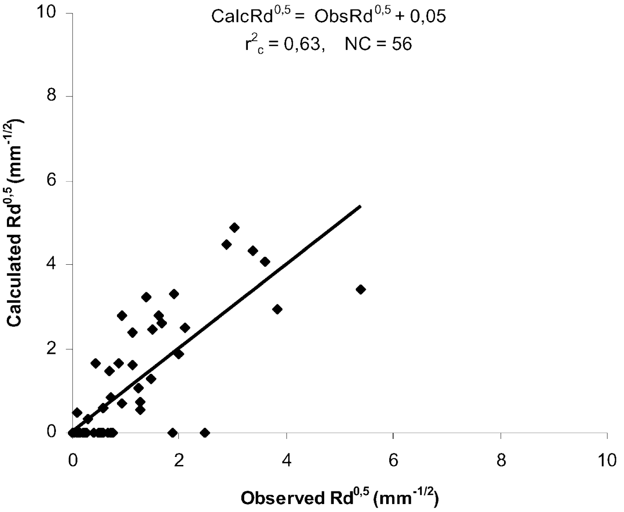
The model is initialised at the beginning of the rainy season, where API_0 is assumed to be zero.

5. Results

The NAZASM model has been calibrated and tested for all the catchments and plots, which were surveyed in the high Nazas river basin during four consecutive years (1995–1998). It has also been evaluated by regrouping several years: 1995–1997 for the Pilitas catchment, 1995–1998 for other three and 1996–1998 for two plots of 50 m^2 located at the La Manga and El Aguaje sites. The total represents 58 tests and all the results including the model fitted parameters are summarised in Table 4.

5.1. Parameter estimates

Table 4 gives for each test the values of coefficient of determination of the regression between calculated and observed runoff values for the calibration (r_c^2) and a- calibration



b- validation

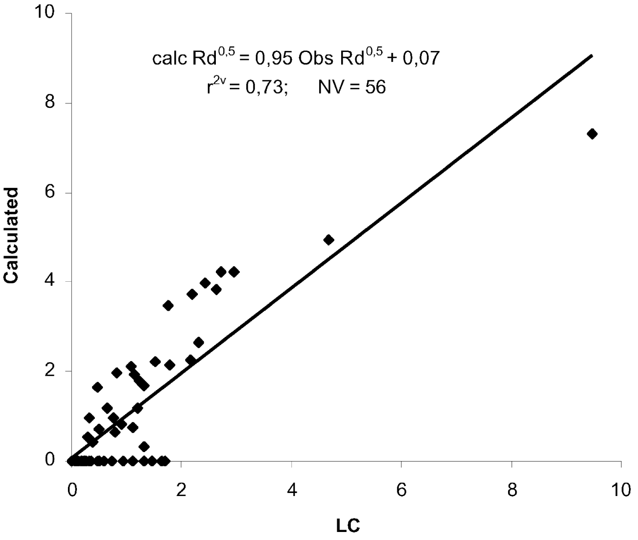


Fig. 6. Calibration (a) and validation (b) of NAZASM for the plot 22 (area = 50 m^2; El Aguaje site, 1996–1998): comparison between calculated and observed values of runoff (square root).

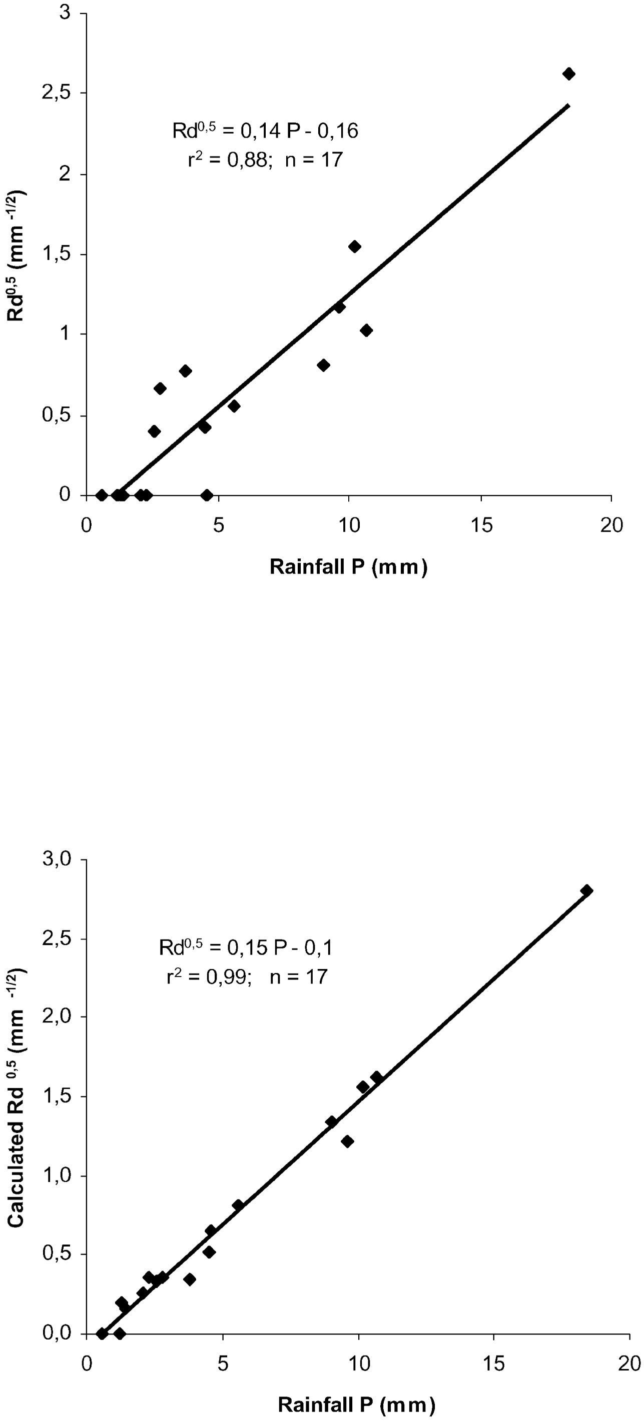


Fig. 7. Comparison between rainfall and runoff relationship given by the experimental relationship $\sqrt{\mathrm{Rd}_n} = K_n(P_n - P_{0n})$ (Eq. (1)) (a) and by NAZASM (b) (Eq. (5)) for the plot 15 (1997).

Table 5
Variation of α calculated by API $-$ Rd_{n-1} instead of API and taking into account PU10 instead of P (API $-$ Rd_{n-1} is calculated by subtracting from API runoff depths (Rd_{n-1}) induced by previous rainfall events. API (PU10) is the API calculated with an 'effective rainfall' (corresponding here to intensity higher than 10 mm/h, PU10) instead of the rainfall P)

API-type introduced in Nazas model	α (day^{-1})	r^2
API	0.071	0.84
API $-$ Rd_{n-1}	0.0699	0.845
API (PU10)	0.068	0.835

the validation (r_v^2) of the model, as well as the number of rain events considered, respectively (NC and NV). As an example, the relations between calculated and observed $\sqrt{\mathrm{Rd}_n}$ is shown in Fig. 5 for the catchment Cura (1995–1998) and in Fig. 6 for the plot no. 22 (1996–1998), for both calibration (Figs. 5(a) and 6(a)) and validation (Figs. 5(b) and 6(b)) of NAZASM.

It can be noticed that:

r_c^2 and r_v^2 are quite similar (about 0.78 in average) whatever the plots and catchments;

for five cases, both calibration and validation r^2 are quite low, due to a high scattering in soil response to the rain: Pilitas catchment in 1995; Manga catchment in 1997; plot 16 in 1997 and plot 32 in 1995 and 1996. In the last two cases, only 3 and 5 events have produced runoff, due to the high thickness of the litter;

in some other tests, only one of the r^2 coefficient is satisfying, while the other is not: Cura catchment in 1998, Esmeralda catchment in 1997; plots 11 and 12 in 1996; plots 15 and 18 in 1997);

in all the other cases (47 of 58 tests), both calibration and validation coefficients of determination are satisfying whatever the size, the lithology and the vegetation of the catchment are.

5.2. *Empirical versus API-type model*

As it can be seen in Table 4 (columns 4 and 5) the use of NAZASM (Eq. (5)) instead of the empirical approach (Eq. (1)) significantly improves the estimation of runoff yield: the mean r^2 increases from 0.66 to 0.84. The rise is from 0.70 to 0.88 for the plots (38 tests) and from 0.59 to 0.76 for the catchments (20 tests). As an example, Fig. 7 shows for plot the no. 15 in 1997 the rainfall runoff relationship obtained by the empirical relation (Eq.(1); Fig. 7(a)) and calculated by NAZASM (Fig. 7(b)).

Additional comments can be made:

(i) coefficients of determination of the rainfall–runoff relationship given by Eq. (5) are satisfying in all cases (58 tests) except three (Pilitas catchment in 1995 and 1995–1997; plot 32 in 1996) (Table 4, column 5);

(ii) similar results have been recently obtained for several 5000 km^2 watersheds located in the same area (Viramontes, 2000). Because of the longer response times of these basins, monthly values of rainfall and runoff have been considered in the analysis, instead of event values.

(iii) incidentally, subtracting from the API runoff depths (Rd_{n-1}) induced by previous rainfall events, as suggested by Chevallier (1983) did not improve the results (see Table 5). Furthermore, considering the effective rainfall as defined for instance by Peugeot et al. (1997), Descroix et al. (2001a) instead of using the raw ones in Eq. (3) did not significantly increase the explained variances as it is shown in Table 5. In that case, effective rainfall corresponds to intensity higher than 10 mm/h (PU10).

6. Discussion on the parameters of the API-type model

In the following, we discuss the influence of different environmental factors (rainfall, size of the studied areas, lithology and vegetation type cover) on the NAZASM parameters. Table 6 summarises the corresponding results expressed in terms of mean (and its significance), CV (coefficient of variation) and median values calculated for each parameter. The last column gives the number (N) of plots or catchments included in each category, which have been used to calculate the statistics.

It should be mentioned that all the CV values are quite important (generally higher than 0.5, and

Table 6

Statistics of the NAZASM parameters for different environmental conditions. N in the number of plots or catchments considered in the statistical analysis (M is the mean. CV is the coefficient of variation. m is the median. (1) Sigf. shows if averages are significantly different according to the Student test. * * * different at the 99% confidence level. * * different at the 85% confidence level. * different at the 80% confidence level. NA not applicable)

	α				C				$P_0^{\max}$				$K_{\max}$				$K_{\min}$				$H_{\max}$				N
	M (day^{-1})	Sigf. (1)	CV	m (day^{-1})	M	Sigf.	CV	m	M (mm)	Sigf.	CV	m (mm)	M (mm$^{-1/2}$)	Sigf.	CV	m (mm$^{-1/2}$)	M (mm$^{-1/2}$)	Sigf.	CV	m (mm$^{-1/2}$)	M (mm)	Sigf.	CV	m (mm)	
1996 ($P = 600$ mm)	0.06	NA	0.64	0.05	0.08	NA	1.08	0.03	12.3	*	0.81	7	0.12	NA	0.33	0.12	0.04	NA	1.4	0	171	* * *	0.37	168	17
1997 ($P = 350$ mm)	0.06		0.63	0.06	0.08		0.97	0.05	7.5		0.50	6.5	0.13		0.55	0.13	0.05		1.15	0.03	56		0.86	37.5	20
Plots	0.06	*	0.59	0.06	0.09	NA	0.94	0.07	9.7	NA	0.80	6.9	0.13	* *	0.45	0.13 0.10	0.05	NA	1.20	0.02	93	* *	0.75	60	38
Catchments	0.09		0.87	0.09	0.10		0.35	0.07	10.8		0.57	10	0.10		0.48		0.03		1.53	0	133		0.55	110	20
Rhyolite and ignimbrite	0.07	NA	0.86	0.06	0.07	*	1.29	0.04	8.9	NA	0.73	6.9	0.12	NA	0.40	0.11	0.06	NA	1.09	0.06	124	* *	0.60	100	32
Conglomerates and tuffs	0.07		0.66	0.07	0.11		0.64	0.10	11.6		0.69	8.5	0.13		0.54	0.13	0.02		1.59	0.002	85		0.78	69	26
Savanna	0.06	NA	0.62	0.06	0.08	* *	0.96	0.05	9.4	NA	0.71	7	0.13	NA	0.45	0.12	0.05	NA	1.25	0.02	106	NA	0.76	85	42
Forest	0.09		0.92	0.08	0.14		0.82	0.09	12.3		0.76	9.5	0.12		0.55	0.09	0.03		1.15	0.01	107		0.48	110	16

Table 7
Statistics of the soil porosity and the gravimetric water content according to vegetation cover (*N* is the number of replications)

	Porosity			Water content		
	Mean	CV	*N*	Mean	CV	*N*
Forest	0.374[a]	0.20	28	0.18[b]	0.24	95
Savanna	0.355[a]	0.22	41	0.155[b]	0.23	156

[a] Statistically different at the 90% confidence level.
[b] Statistically different at the 99% confidence level.

frequently superior to 1) indicating that the spatial variability of the soil characteristics and the vegetation cover is important.

The parameter α represents the soil moisture decay, i.e. the capacity of the soil to retain infiltrating water after a rainfall event.

A literature survey shows that this parameter is often estimated at a value of 0.5 day^{-1} in the sahelian regions (Chevallier, 1983; Seguis, 1987; Albergel, 1988) and in France (Grésillon, 1994), but only at 0.11–0.16 for central and eastern United States (Chevallier, 1983).

As it can be seen in Table 4, most of the optimised α values fall within the range of values reported by Chevallier (1983) for the inter-tropical zone (i.e. between 0.01 and 0.2 day^{-1}). Although the mean values may appear weakly variable, it is worthwhile to note that they vary according to the context. As a matter of fact, Table 6 shows that α is under the influence of some key factors, namely:

(i) the size of the studied area: the α mean value is 0.06 for plots and 0.09 for catchments. It is the only significant difference. However, taking into account values obtained by Viramontes (2000) for catchments of around 5000 km^2 (α ranging from 0.02 to 0.06) would suggest that there is no linear relation between α and area size.
(ii) types of vegetation cover: considered areas in savanna have a mean value of α parameter of 0.06, instead of 0.09 for forested zones. But, this difference is not significant.

Table 8
Statistics of the hydraulic conductivity according to lithology (values were obtained by tension infiltrometer tests performed at an applied water pressure head of −10 mm. *N* is the number of replications)

Hydraulic conductivity	Ignimbrites and rhyolites	Tuffs and conglomerates
Mean (m s^{-1})	5.5×10^{-6}[a]	8.4×10^{-6}[a]
CV	0.71	0.43
N	46	42

The mean C value is significantly smaller in savanna ($C = 0.08$) than in forested areas ($C = 0.14$), the role of soil and vegetation reservoir being bigger in the latter environment. Porosity and gravimetric soil water content are higher in forest than in savanna as it is shown in Table 7. That is mainly due to the presence of both litter and forest making easier soil surface water storage. For the same reasons, C values are higher in conglomerates and tuffs (0.11) than in rhyolite (0.07), the latter one being less permeable, as it can be seen in Table 8, which gives values of soil surface hydraulic conductivities obtained by tension disc infiltrometer (see for instance Vandervaere et al. (2000)).

The $P_{0n}^{\max}$ values are around 5–20 mm of water. Here, the only noticeable difference is between 1996 and 1997. In the wet year 1996, vegetation growth induces higher infiltration rates, so the minimum rainfall value necessary to produce runoff increases. In other respects, $P_{0n}^{\max}$ is higher in forest (12.3 mm) than in savanna (9.4 mm); $P_{0n}^{\max}$ increases logically with the area size and is higher for permeable fields (conglomerates and tuffs) than for rhyolite and ignimbrites ones. That is classical but not statistically significant here. In the other hand, $P_{0n}^{\min}$ appeared to be always equal to 0.

$K_{\max}$ presents no significantly different means except between plots on the one hand, catchments on the other (probably due to the different area sizes). This is a general behaviour that increasing the catchment leads to a decrease in runoff coefficient. $K_{\min}$ is mostly close to 0 and too scattered to be easily explainable.

The maximum capacity of the soil, $H_{\max}$, is higher in a rainy year, because the whole soil profile is functioning as a reservoir. There is also a significant difference between plots (mean $H_{\max} = 93$ mm) and catchments ($H_{\max} = 133$ mm) because of the general

Table 9
Comparison between monthly values of measured and calculated runoff depths by NAZASM (Cura catchment, 1995–1998)

Month	Calculated runoff depth (mm)	Observed runoff depth (mm)
June 1995	0.61	0.84
July 1995	2.79	2.72
August 1995	0.79	0.28
September 1995	0.57	0.00
Year 1995	4.76	3.84
June 1996	0.87	0.72
July 1996	1.03	1.21
August 1996	13.14	15.19
September 1996	12.27	14.16
Year 1996	27.31	31.28
June 1997	0.14	0.22
July 1997	2.80	3.06
August 1997	2.04	4.43
September 1997	0.11	0.00
Year 1997	5.08	7.71
June 1998	0.00	0.00
July 1998	0.00	0.00
August 1998	10.87	7.26
September 1998	0.71	0.45
Year 1998	11.58	7.71

higher roughness of greater areas and the presence of storage areas (talwegs, vegetalised stripes, etc.).

The results have shown that taking into account the previous soil moisture improves significantly the rainfall runoff relationships. However, the western Sierra Madre seems to be located on a hydrologic boundary. As a matter of fact, during the observation time, its functioning has had clearly a hortonian pattern, except in 1996 (year with a slight rainfall surplus) and in some other short periods of rainfall events, as it occurs when tropical depressions cross the whole Sierra Madre, and during very abundant rainy seasons. In these cases, surface saturated areas can appear and enhance strongly the runoff coefficient.

Nevertheless, an analysis performed on the long-term regional rainfall and discharge data (50 and 30 years, respectively) shows that the 1994–2000 period was very dry. Except the year 1996 (see Table 2) all other years had a strong rainfall deficit in this region: −40% of the mean annual rainfall in 1994, −20% in 1995 and in 1997, −35% in 1998 and −30% in 1999 and 2000. This clearly appears in the base flows of the two basins of Rio Ramos (7130 km^2) and Rio Sextin (4660 km^2), which were very low on these years. Therefore, the regular regime is probably closer to the 1996's one. Under more normal conditions, groundwater plays an important role by allowing a permanent river flow during several weeks and even months all along the rainy season and sometimes after.

Comparing rainfall amount and hillslopes behaviour in space (Estrada, 1999) and in time (Viramontes, 2000) in northern Mexico allows to assume that during the rainy season there is a rainfall threshold of about 500 mm (depending on the temporal distribution of events) up to which runoff is mainly hortonian, and not only hortonian above. Viramontes (2000) determined that α parameter was increasing from the seventies decade to the nineties one in two catchments of the upper Rio Nazas basin, due to the degradation of pasture and to the deforestation.

As long as the considered period includes wet and dry years, the parameters are more significantly fitted. Furthermore, NAZASM is applicable at plot scale (1–50 m^2), as well as at basin scale (from 1 to 10,000 km^2), making it a robust lumped deterministic model for semi-arid mountains and for all scales. It can be fairly applied yearly of monthly, as it is shown in Table 9, which presents the observed and calculated runoff depth for the Cura basin.

However, the use of the API-type model for runoff prediction could be hazardous if the calibration sample does not include events of dry and wet years. As an example, for the Esmeralda catchment, the use of the model parameters fitted on the dry 1996 year would lead to calculated runoff values equal to 32, 45 and 51% of the observed ones in 1995, 1997 and 1998, respectively. This is due to the big difference between processes observed in wet and dry years.

7. Conclusions

In a subhumid mountainous region with a tropical climate, it has been shown that hortonian runoff is mostly widespread. Consequently, the initial soil moisture content prior to a rainfall event plays a key role, especially during a year characterised by rainfalls close to the climatic average.

For dry years (as in 1995, 1997 and 1998 in study area) in catchments with rhyolites and ignimbrites in pasture land areas, it appeared that the soil water reservoir remains very small. Indeed, rainfall is so infrequent and light that it prevents the formation of water storage, which could reduce infiltration. However, even during dry years, soil moisture content plays a major role at least in two contexts:

In a forest environment, where the existence of a soil-vegetation reservoir is preserved. This represents 15–50% of catchment areas at altitudes between 2200 and 2600 m, but almost 100% above. These areas are important, because they receive the highest rainfall amount (up to 900 mm per year). However, it should be noted that certain areas are suffering from very rapid deforestation, which could threaten the sustainability of this water storage;

In conglomerate areas found only in grabens, where soil moisture is trapped into topographic depressions (and thus without any major influence on downstream flows) which receive less amount of rain.

All the results have shown that a simple model based on the concept of the API contributes to better explain the runoff formation and significantly improves the classical empirical rainfall–runoff relationships. This improvement has been verified for plots of different sizes (1, 10 and 50 m^2), as well as for catchments ranging from 1 to 50 km^2 and up to 7000 km^2 according to other studies performed in the same area.

Acknowledgements

The authors thank Jose Luis Montoya and Avelina Carrillo from CONAGUA (Comisión Nacional del Agua, the Mexican Water Resources Administration) for making available to them the climatologic data.

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