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An examination of the Priestley-Taylor equation using a convective boundary layer model

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Abstract. The effect of large-scale parameters on the behavior of the Priestley-Taylor coefficient is addressed by means of a simple analytical model of the convective boundary layer (CBL). In this model, surface and aerodynamic resistances are maintained constant throughout daytime, and the diurnal course of available energy is parameterized in the form of a parabolic curve. To account for entrainment of overlying air, the height of the CBL is assumed to grow as square root of time, and the water vapor saturation deficit in the undisturbed atmosphere above the CBL is represented by a simple linear profile. The Priestley-Taylor coefficient is defined as the ratio of potential evaporation over equilibrium evaporation, and two different ways of defining potential evaporation are considered: (1) as the evaporation of an extensive saturated area (i.e., the whole region influencing the CBL) or (2) as the evaporation of a limited saturated area (small enough that the excess moisture does not modify the characteristics of the CBL). These two ways, called respectively Penman's and Morton's ways, are successively examined. Numerical simulations from the CBL model show that the Priestley-Taylor coefficient (α) does not have a fixed and universal value (1.26) as it has been suggested by these authors. When based on Penman's concept of potential evaporation, α varies as a function of the conditions in the undisturbed atmosphere above the CBL (inversion strength) but also as a function of the characteristics of the surface (aerodynamic resistance). The additional energy implied by a coefficient greater than 1 has to be ascribed only to the entrainment effect. When based on Morton's concept, α depends upon the areal surface resistance and the external conditions above the CBL: The daily mean value of α increases asymptotically with areal surface resistance towards a limit value which grows with inversion strength. In this case the additional energy (implied by $\alpha > 1$) has a double origin: the feedback of areal evaporation on local potential evaporation and the entrainment effect.

1. Introduction

At the land surface–atmosphere interface, strong feedback mechanisms exist between surface fluxes and air characteristics. Evaporation and sensible heat flux affect directly the temperature and humidity of the lowest part of the atmosphere, which in their turn will influence the surface fluxes. In this paper these mechanisms are examined through the Priestley-Taylor equation [*Priestley and Taylor*, 1972], which stipulates that the ratio α between potential evaporation and equilibrium evaporation (i.e., the radiative term of Penman's formula) is constant and equal to 1.26 on average. To do that, we use a simple model which simulates the diurnal surface energy balance in a growing convective boundary layer (CBL), represented by a well-mixed slab of air capped by the free atmosphere. Since the CBL is driven primarily by surface heating during the daytime and vanishes at night, our study will be restricted to fair-weather conditions during the daytime, when the CBL grows. It is during this time that most of the exchange processes occur between the surface and the atmosphere. At night a stable boundary layer forms with the surface cooler than the overlying air [*de Bruin*, 1989]. *Perrier* [1980] and *McNaughton and Jarvis* [1983] were the first who used a closed box model of the CBL to investigate the interactions between surface properties and air characteristics. *De Bruin* [1983] extended this previous model by considering entrainment, assuming that the water vapor flux at the top of the CBL is proportional to the surface evaporation. *McNaughton and Spriggs* [1986] developed a model in which the entrainment term is more physically derived, and they assessed the effects of larger-scale conditions on regional evaporation. *Jacobs and de Bruin* [1992] coupled the big-leaf model to a detailed model of the CBL to study how the feedback affects the sensitivity of transpiration to input variables.

The data generated by the CBL model will serve to test the Priestley-Taylor equation. In this sense this paper has an aim somewhat similar to those of *McNaughton and Spriggs* [1989] and *Culf* [1994]. The former evaluates the Priestley-Taylor equation and the complementary relationship with a CBL model for conditions observed at Cabauw in the Netherlands [*McNaughton and Spriggs*, 1986]. The latter uses a CBL model to investigate the physical basis of the Priestley-Taylor equation. Both conclude that the additional energy, implied by a coefficient α greater than 1, is due to the entrainment of dry air into the mixed layer from above. However, the development which follows differs from these two papers, essentially because the Priestley-Taylor coefficient is not interpreted in the same way. Our interpretation [*Lhomme*, 1997] is based on the concept of saturated area (completely wet surface) and on two

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possible definitions of potential evaporation leading to two different calculations of the coefficient α. Section 2 describes the CBL model. Section 3 deals with the definition and expression of the Priestley-Taylor coefficient, explaining how our definition differs from that of *McNaughton and Spriggs* [1989]. And section 4 is devoted to numerical results illustrating the behavior of this coefficient.

2. Modeling the Convective Boundary Layer

The CBL represents the turbulent layer of the atmosphere which develops during the daytime from the ground upwards because of the convective motions generated by the sensible heat flux released at the surface. Essentially, the CBL comprises a relatively thin surface layer, where the gradients of temperature and humidity may be significant, and a well-mixed layer where the potential saturation deficit D is assumed to be constant with height [*McNaughton*, 1989]. Above the capping inversion of the well-mixed layer is the undisturbed atmosphere, whose properties are determined by synoptic-scale processes.

The areal latent heat flux at the surface is governed by the Penman-Monteith equation:

$$\lambda E = \frac{\varepsilon A + \rho\lambda D/\langle r_a\rangle}{\varepsilon + 1 + \langle r_s\rangle/\langle r_a\rangle} \tag{1}$$

where A is the available energy ($A = R_n - G$, with R_n being net radiation and G being soil heat flux), $\langle r_a\rangle$ is the bulk aerodynamic resistance to heat and water vapor transfer through the surface layer, $\langle r_s\rangle$ is the bulk surface resistance to water vapor transfer (the resistances are denoted with the areal averaging operator $\langle\ \rangle$ to specify they are defined for the whole area influencing the CBL), ρ is the air density, λ is the latent heat of vaporisation, and ε is the dimensionless slope of the saturation specific humidity. Sensible heat flux is given by a similar equation obtained from $H = A - \lambda E$.

The inversion cap of the well-mixed layer, whose height h grows during the daytime, is not impermeable. The incorporation of a thin layer of air of thickness dh and saturation deficit D_+ into the mixed-layer with saturation deficit D leads to the following conservation equation [*McNaughton and Spriggs*, 1986; *Raupach*, 1991; *McNaughton and Raupach*, 1996]:

$$\frac{dD}{dt} = \frac{D_e - D}{T_e} + \left(\frac{D_+ - D}{h}\right)\frac{dh}{dt} \tag{2}$$

with

$$D_e = \frac{\varepsilon A}{\varepsilon + 1}\frac{\langle r_s\rangle}{\rho\lambda} \qquad T_e = h\left(\langle r_a\rangle + \frac{\langle r_s\rangle}{\varepsilon + 1}\right) \tag{3}$$

T_e is the time constant in a closed-box model (when $dh/dt = 0$), that is, the time needed to reach 63% of the equilibrium value (D_e) in the case of a steady forcing (when A is assumed to be constant with time). The upper limit conditions are taken from *Raupach* [1991], who assumes that the vertical profile of potential saturation deficit D_+ in the undisturbed atmosphere is linear $D_+ = \gamma_D z$, where z is the altitude and γ_D is a positive parameter with units of kg kg^{-1} m^{-1}; γ_D is written as $\gamma_D = (\varepsilon c_p/\lambda)\gamma_\theta - \gamma_q$, where γ_θ and γ_q are respectively the slopes of potential temperature and specific humidity just above the CBL. The value of γ_D can be adjusted as a function of the dryness of the air above the capping inversion. The height of the CBL (h) is assumed to grow as square root of time $h(t) = (Kt)^{1/2}$, where K is a growth-rate parameter with the dimension of diffusivity (m^2 s^{-1}). K has been parameterized as a function of surface resistance $\langle r_s\rangle$ (in the form of an increasing function) since all other conditions being equal, the greater the surface resistance, the greater the sensible heat flux and the faster the growth of the CBL (see the appendix).

Available energy $A(t) = R_n(t) - G(t)$ is assumed to vary as a parabolic curve, which intends to simulate its diurnal behavior over the day length δ: $A(t) = 0$ at the initial time $t = t_0$ and at the time $t = t_0 + \delta$, and $A(t) = A_x$ (a maximum value) at the time $t = t_0 + \delta/2$. A parabolic curve was chosen instead of a sine wave because it leads to a much simpler analytical solution for the differential equation. Under these conditions $A(t)$ can be written as

$$A(t) = A_x F(t) \qquad F(t) = -4[t^2 - (\delta + 2t_0)t + t_0(t_0 + \delta)]/\delta^2 \tag{4}$$

If at the initial time t_0 the height of the CBL is assumed to be h_0, these two parameters are linked by $t_0 = h_0^2/K$. Although the surface resistance ($\langle r_s\rangle$) shows a significant diurnal variation (approximately constant in the morning with an increase in the afternoon), it is kept constant in the model. It seems that for practical calculations, the CBL model is relatively insensitive to this effect [*de Bruin*, 1989]. The bulk aerodynamic resistance through the surface layer ($\langle r_a\rangle$) is also assumed to be constant during the daytime.

Putting $D_x = \varepsilon\langle r_s\rangle A_x/[(\varepsilon + 1)\rho\lambda]$, $\tau = Kr_e^2$ with $r_e = T_e/h = \langle r_a\rangle + \langle r_s\rangle/(1 + \varepsilon)$, equation (2) can be rewritten as

$$\frac{dD}{dt} + \left[\frac{1}{(\tau t)^{1/2}} + \frac{1}{2t}\right]D = \frac{D_x F(t) + \Delta}{(\tau t)^{1/2}} \tag{5}$$

where the parameter Δ is defined as $\Delta = \gamma_D K r_e/2$ (with the dimension of D). The slope of the saturation specific humidity ε is calculated at the mean diurnal value (T_a) of the potential temperature in the well-mixed layer, and T_a constitutes an input to the model. Equation (5) is a linear, first-order differential equation in $D(t)$ with nonconstant coefficients. The solution is given by

$$D(t) = \Delta\left[1 - \frac{1}{2}\left(\frac{\tau}{t}\right)^{1/2}\right] + \frac{1}{t^{1/2}}\left\{D_x L(t) + \Psi(t_0)\exp\left[-\frac{2}{\tau^{1/2}}(t^{1/2} - t_0^{1/2})\right]\right\} \tag{6}$$

where

$$\Psi(t_0) = (D_0 - \Delta)t_0^{1/2} + \Delta\tau^{1/2}/2 - D_x L(t_0) \tag{7}$$

D_0 being the saturation deficit at $t = t_0$ (in our simulations D_0 is logically taken to be equal to $D_+(h_0) = \gamma_D h_0$), and

$$L(t) = at^{5/2} + bt^2 + ct^{3/2} + dt + et^{1/2} + f \tag{8}$$

with

$$\begin{aligned} a &= -4/\delta^2 \\ b &= 10\tau^{1/2}/\delta^2 \\ c &= 4[(\delta + 2t_0) - 5\tau]/\delta^2 \\ d &= -3c\tau^{1/2}/2 \\ e &= [-4t_0(\delta + t_0) + 6(\delta + 2t_0)\tau - 30\tau^2]/\delta^2 \\ f &= -e\tau^{1/2}/2 \end{aligned} \tag{9}$$

When the available energy A is assumed to be constant (i.e., $F(t) = 1$), $L(t) = t^{1/2} - \tau^{1/2}/2$ and the solution of the differential equation simplifies into

$$D(t) = D_\infty\left[1 - \frac{1}{2}\left(\frac{\tau}{t}\right)^{1/2}\right] + \frac{1}{\sqrt{t}}\left[D_0 t_0^{1/2} + D_\infty(\tau^{1/2}/2 - t_0^{1/2})\right] \cdot \exp\left[-\frac{2}{\tau^{1/2}}(t^{1/2} - t_0^{1/2})\right] \quad (10)$$

where $D_\infty = D_x + \Delta = D_x + \gamma_D K r_e/2$ is the steady limiting value of saturation deficit at large time [*Raupach*, 1991]. For a closed-box model (i.e., without considering entrainment) the limiting value of D is simply D_x.

The main role of the model is to simulate the diurnal variation of the potential saturation deficit in the mixed-layer and, consequently, to allow the calculation of the corresponding evaporation rates. The inputs to the model are two resistances (the surface resistance $\langle r_s \rangle$ and the aerodynamic resistance $\langle r_a \rangle$), and three climatic data: the maximum available energy A_x, the parameter γ_D giving the profile of potential saturation deficit above the CBL and the mean air temperature at the ground T_a from which ε is calculated (over the range of temperature 10°C–40°C, ε increases roughly from 1.3 to 6.0). The day length d has been considered as fixed (12 hours). Nevertheless, an additional parameter is needed for the model to work. It is the CBL height h_0 at the initial time which is used to define t_0. In fact, the influence of this initial CBL height on the daily course of saturation deficit is very weak: A change in h_0 from 1 to 100 m leads to an increase in D of only 0.0002 kg kg^{-1} at the beginning of the day and of less than 0.0001 kg kg^{-1} at the end. In all the simulations performed a fixed value of 10 m has been adopted for h_0.

3. Defining and Expressing the Coefficient α

3.1. Definition of the Priestley-Taylor Coefficient

The coefficient α of *Priestley and Taylor* [1972] is defined as the ratio of potential evaporation (E_p) over equilibrium evaporation (E_{eq}): $\alpha = E_p/E_{eq}$, where E_{eq} is given by $\lambda E_{eq} = \varepsilon A/(\varepsilon + 1)$. For these authors E_p represents the evaporation from a "horizontally uniform saturated surface (land and water)," sufficiently extended to obviate any significant advection of energy from outside. Nevertheless, to the authors the precise physical significance of E_p and the notion of saturated surface are not as clear as it seems at first glance. These authors recognize that a surface is saturated when water vapor of the air in contact with this surface is saturated, but they consider irrigated pots carrying pasture to be saturated, in the same way as open water. For them, potential evaporation, which generally refers to the evaporation rate of open water site and moist land sites (when all exchange surfaces are wet, like just after rainfall or dew deposit, for instance), represents the same concept as potential evapotranspiration, which commonly refers to the maximum rate of transpiration from an area completely and uniformly covered by a vegetation with an adequate supply of water [*Thornthwaite*, 1948; *Penman*, 1956; *Brutsaert*, 1982]. Now, we know that a well-watered crop, completely covering the ground, has a surface resistance different from zero (around 70 s m^{-1}) and from a strict physical point of view cannot be considered as saturated like open water (for which surface resistance is zero).

This inaccuracy in the definition and representation of potential evaporation has led to some misinterpretations that will be examined hereafter. First, actual evaporation has often been substituted for potential evaporation or potential evapotranspiration in the definition of α. For instance, variations of α as a function of the areal surface resistance $\langle r_s \rangle$ are given by *de Bruin* [1983] and *McNaughton and Spriggs* [1989] as outputs of their CBL model, and these variations encompass conditions far beyond the potential case. Second, when potential evaporation is retained in the correct sense of moist surface, the controversy about the size of the saturated surface generates some additional ambiguity [*Nash*, 1989; *Granger*, 1989]. It is generally accepted that the area at potential rate must be extensive enough to avoid considering oasis situations, where advection from upwind surfaces can enhance potential evaporation [*Penman*, 1963; *Brutsaert*, 1982]. Nevertheless, for *Morton* [1969, 1983] potential evaporation represents the evaporation that would occur from a moist surface with an area small enough that the effects of the evaporation on the overpassing air would be negligible. Both definitions correspond to quantities which can be measured in the real world and can be simulated by the CBL model described above. The latter definition, recommended by *Nash* [1989], was the one used in another paper [*Lhomme*, 1997] in which the Priestley-Taylor coefficient was examined in light of a closed-box model of the CBL. We are not sure in fact which type of potential evaporation the original Priestley-Taylor coefficient refers to. A priori it refers to an extensive area, but as stipulated by *Lhomme* [1997], one can wonder whether, in trying to obtain the evaporation from a large saturated area, these two authors did not estimate a potential evaporation close to the one proposed by Morton, since "the only observations available as a basis [for the derivation of the value of α] are those from individual sites, subject in some cases to quite apparent small-scale nonuniformity and advection" [*Priestley and Taylor*, 1972, p. 82].

3.2. Expression of the Priestley-Taylor Coefficient

In this study, potential evaporation (E_p) will systematically correspond to the evaporation rate of saturated land site (or open water sites), when there is no significant surface or physiological control on the evaporation, that is, when the surface resistance in the Penman-Monteith model (equation (1)) is zero. In each of the two cases mentioned above E_p is expressed in the form of a Penman-type equation obtained from (1):

$$\lambda E_p = \frac{\varepsilon A + \rho\lambda D/r_a}{\varepsilon + 1} \quad (11)$$

and the Priestley-Taylor coefficient is written as a function of time as

$$\alpha(t) = 1 + \frac{\rho\lambda}{\varepsilon r_a}\frac{D(t)}{A(t)} \quad (12)$$

where $D(t)$ and $A(t)$ are respectively given by (4) and (6).

When potential evaporation is defined for an extensive area (we will refer to it as Penman's definition), all the region influencing the CBL is assumed to be saturated and to evaporate at potential rate: In (12) the aerodynamic resistance r_a is the areal resistance $\langle r_a \rangle$, A is the areal available energy $\langle A \rangle$, and D is the saturation deficit calculated by the CBL model with $\langle r_s \rangle = 0$. When potential evaporation refers to a small area (Morton's definition), it is assumed that this small saturated area is surrounded by a surface with a resistance $\langle r_s \rangle \neq 0$. In this case A is the available energy of the small area and

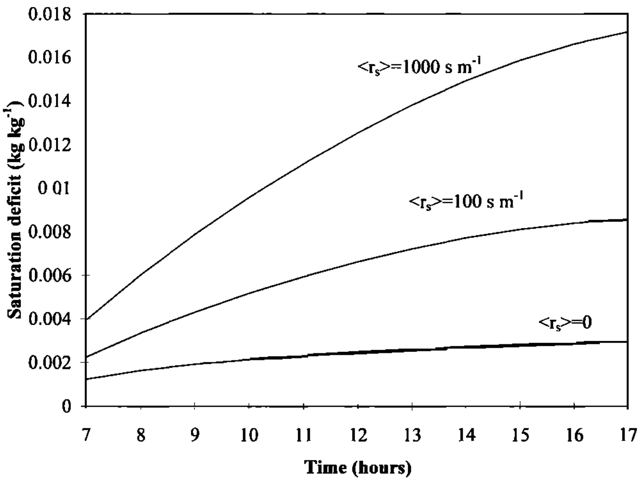

Figure 1. Diurnal variation of saturation deficit into the well-mixed layer for three different values of surface resistance ($\gamma_D = 10^{-5}$ kg kg^{-1} m^{-1}, $T_a = 30$ °C, $A_x = 500$ W m^{-2}, and $\langle r_a \rangle = 50$ s m^{-1}).

the aerodynamic resistance r_a of the saturated patch represents the resistance to scalar transfer, from this patch to the well-mixed layer, horizontally integrated over the whole patch [*Raupach*, 1991, p. 115]. (In the simulations, for the sake of convenience, we will assume that $r_a \approx \langle r_a \rangle$ and $A \approx \langle A \rangle$, but the results would not change substantially if r_a and A are taken different from $\langle r_a \rangle$ and $\langle A \rangle$). D in this last case is the saturation deficit calculated by the CBL model with a surface resistance $\langle r_s \rangle \neq 0$ representative of the area surrounding the small saturated patch. The calculation of this potential evaporation is as easy as the former, but its physical significance needs some additional explanation. For (11) to be valid from an experimental viewpoint, the surface maintained at potential rate must be small enough that the excess moisture flux does not alter the characteristics of the CBL in equilibrium with the areal actual evaporation. But at the same time it must be large enough that the height of the internal boundary layer can reach the height of the areal surface layer. Such conditions have been examined by *Lhomme* [1997]. They can be met only if the small saturated area has a minimum size of 500–1000 m (depending on its roughness). This means that the minimum size of the region influencing the CBL ranges from 10 to 20 km (if the small saturated area is assumed to represent no more than 5% of the total area).

3.3. The Priestley-Taylor Coefficient at Equilibrium

When the input of available energy A is maintained constant (at its maximum value A_x), the solution of the differential equation (5) is given by (10) and the equilibrium value of D (for a very large time) is $D_\infty = D_x + \Delta$. It leads to the following equilibrium value for α, obtained by substituting D_∞ for D in (12) [*Lhomme*, 1997]:

$$\alpha_{eq} = \alpha_{0,eq}(1 + \omega) \qquad \alpha_{0,eq} = 1 + \frac{1}{1+\varepsilon}\frac{\langle r_s \rangle}{r_a}$$
$$\omega = \frac{\rho \lambda K \gamma_D}{2\varepsilon A_x} \tag{13}$$

In a closed-box model of the CBL, when entrainment is not taken into account (i.e., $K = 0$), ω is equal to 0 and the equilibrium value of α reduces to $\alpha_{0,eq}$. When the whole region influencing the CBL evaporates at potential rate, $\langle r_s \rangle$ is equal to 0 (i.e., $\alpha_{0,eq} = 1$), the equilibrium coefficient α_{eq} reduces to $1 + \omega$, and the additional energy implied by a coefficient greater than 1 is due only to the entrainment effect. In the case of a small saturated area surrounded by a dry area, $\langle r_s \rangle$ is different from 0 and $\alpha_{0,eq} > 1$, which means that the additional energy has to be ascribed both to the feedback of regional evaporation ($\alpha_{0,eq}$) and to the entrainment effect ($1 + \omega$). Equation (13) predicts then that the drier a region ($\langle r_s \rangle$ high), the greater the local potential evaporation (E_p).

4. Numerical Results

4.1. Performance of the CBL Model

An important parameter in the modelling process is the parameter γ_D, which represents the slope of the profile of saturation deficit in the atmosphere just above the CBL. Its range of variation has been determined in the following way. According to *McNaughton and Spriggs* [1989], during the 9 days of data recorded at the site of Cabauw, in the Netherlands, the inversion strength on potential temperature (γ_θ) varied approximately from 10^{-3} to 2.10^{-2} K m^{-1}. Assuming that $\gamma_q \approx 0$, γ_D ($= (\varepsilon c_p/\lambda)\gamma_\theta - \gamma_q$) varies roughly from 10^{-6} to 2.10^{-5} kg kg^{-1} m^{-1} (taking $\varepsilon c_p/\lambda \approx 10^{-3}$). Consequently, these two values (10^{-6} and 2 10^{-5}) have been systematically chosen to characterise the range of variation of γ_D. Figure 1 shows the diurnal variation of water vapor saturation deficit (D) for different values of surface resistance ($\langle r_s \rangle$) and an inversion strength $\gamma_D = 10^{-5}$. Very similar results were obtained by *McNaughton and Spriggs* [1989, Figure 6], with a model where entrainment is parameterized in a different way. In Figure 2 the diurnal variation of saturation deficit is plotted for different inversion strengths (γ_D) at the top of the CBL, all other conditions being equal. Figures 3 and 4 show the diurnal variation of actual evaporation and potential evaporation (Morton's definition) for two different values of areal surface resistance ($\langle r_s \rangle = 100$ and $\langle r_s \rangle = 1000$ s m^{-1}): A greater surface resistance damps actual evaporation and enhances potential

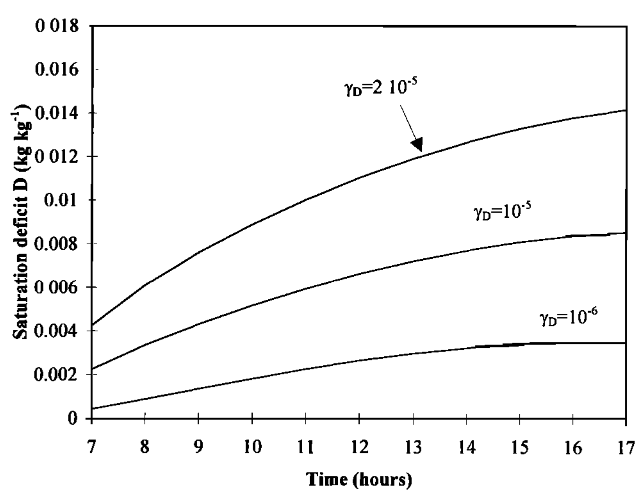

Figure 2. Diurnal variation of saturation deficit into the well-mixed layer for three different values of the inversion strength γ_D (kg kg^{-1} m^{-1}) with $T_a = 30$ °C, $A_x = 500$ W m^{-2}, and $\langle r_a \rangle = \langle r_s \rangle = 50$ s m^{-1}.

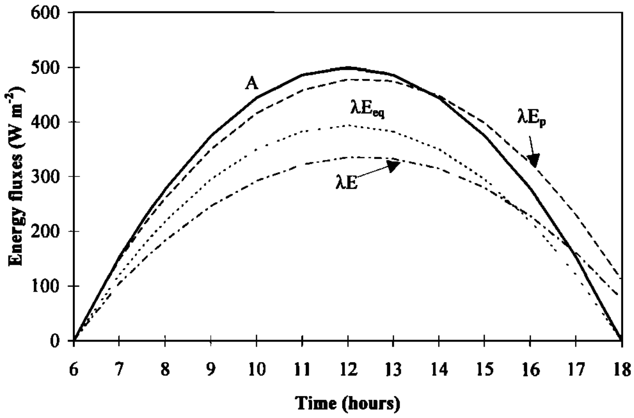

Figure 3. Diurnal variation of available energy (A), equilibrium evaporation (λE_{eq}), actual evaporation (λE), and potential evaporation (λE_p), in the sense of Morton, for $\langle r_s \rangle$ = 100 s m^{-1}. The other parameters are $\gamma_D = 10^{-5}$ kg kg^{-1} m^{-1}, T_a = 30 °C, A_x = 500 W m^{-2}, and $\langle r_a \rangle$ = 50 s m^{-1}.

evaporation (through a greater saturation deficit within the CBL).

4.2. Behavior of the Priestley-Taylor Coefficient

Since there exist two different ways of defining and calculating potential evaporation (as the evaporation from a small saturated area, or as the evaporation from an extensive saturated area), these two ways will be successively examined. Potential evaporation defined as the evaporation from "an extended saturated surface" (Penman's definition) will be denoted by E_p^a, and the corresponding Priestley-Taylor coefficient will be denoted by $\alpha^a = E_p^a/E_{eq}$. And the Priestley-Taylor coefficient calculated with Morton's definition of E_p will be denoted simply by α without subscript or superscript.

Table 1 shows the variation of the equilibrium value of α^a (denoted by α_{eq}^a and given by (13) with $\langle r_s \rangle = 0$) as a function of inversion strength (γ_D) for a surface resistance $\langle r_a \rangle$ = 50 s m^{-1}. When there is no entrainment α_{eq}^a is equal to 1. And when entrainment is accounted for, α_{eq}^a increases linearly with γ_D up to a value of 1.37 for $\gamma_D = 2\ 10^{-5}$. In Figure 5 the

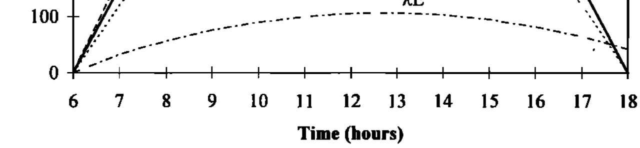

Figure 4. Diurnal variation of available energy (A), equilibrium evaporation (λE_{eq}), actual evaporation (λE), and potential evaporation (λE_p), in the sense of Morton, for $\langle r_s \rangle$ = 1000 s m^{-1}. The other parameters have the same values as those in Figure 3.

Table 1. Variation of the Equilibrium Value of α^a, α_{eq}^a, as a Function of Inversion Strength γ_D (kg kg^{-1} m^{-1}) With T_a = 30 °C, A_x = 500 W m^{-2}, and $\langle r_a \rangle$ = 50 s m^{-1}

	α_{eq}^a
No entrainment	1.00
$\gamma_D = 10^{-6}$	1.02
$\gamma_D = 10^{-5}$	1.19
$\gamma_D = 2 \times 10^{-5}$	1.37

equilibrium value α_{eq} of the coefficient (based upon Morton's definition) is plotted as a function of surface resistance for different values of inversion strength; α_{eq} increases linearly with surface resistance and inversion strength. When $\langle r_s \rangle$ is large, these equilibrium values are too great to be plausible, but they can never be achieved in the span of a day (as we will see below) because the corresponding time constants are too large.

The actual values of α^a and α have been explored hereafter by means of the CBL model. The ratio α^a was calculated for different scenarios represented by different values of the aerodynamic resistance $\langle r_a \rangle$ and of the slope of the potential saturation deficit just above the capping inversion (specified by the parameter γ_D). The results are given for a daily basis in Table 2. They show that α^a is not a constant, but depends upon synoptic-scale conditions and surface aerodynamic characteristics. It varies roughly in the range 1–1.3 when γ_D varies from 10^{-6} to 2 10^{-5} and $\langle r_a \rangle$ from 20 to 200 (in SI units): It increases when γ_D increases and decreases when $\langle r_a \rangle$ increases. The additional energy implied by a coefficient α^a greater than 1 is due to the entrainment of dry air downwards within the CBL: The drier the air above the CBL, the greater the coefficient α^a. For a given saturation deficit above the CBL, α^a increases with surface roughness; but for a small external saturation deficit ($\gamma_D < 10^{-6}$), the value of α^a is nearly constant and very close to 1 whatever the value of $\langle r_a \rangle$.

Figure 6 shows the diurnal course of α (based upon Mor-

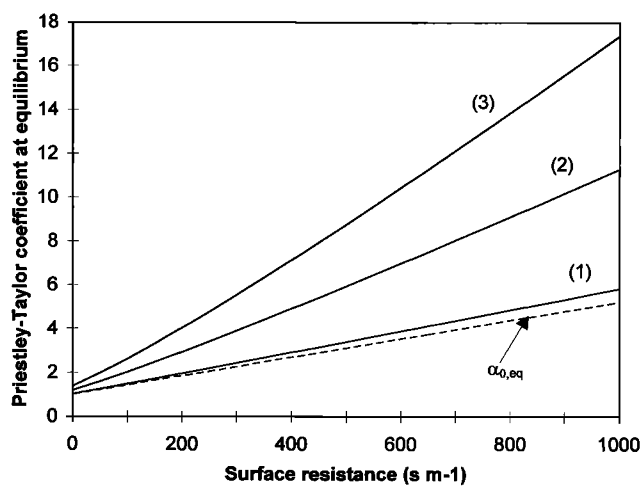

Figure 5. Variation of the theoretical coefficient α_{eq} at equilibrium, given by equation (13), as a function of surface resistance $\langle r_s \rangle$ for three different values of inversion strength: (1) $\gamma_D = 10^{-6}$, (2) $\gamma_D = 10^{-5}$, and (3) $\gamma_D = 2.\ 10^{-5}$ (units in kg kg^{-1} m^{-1}) (T_a = 30 °C, A_x = 500 W m^{-2}, and $\langle r_a \rangle$ = 50 s m^{-1}).

Table 2. Variation of Coefficient $\alpha^a = E_p^a/E_0$ Calculated on a Diurnal Basis as a Function of $\langle r_a \rangle$ (s m^{-1}) and γ_D (kg kg^{-1} m^{-1}) with T_a = 30 °C and A_x = 500 W m^{-2}

	$\gamma_D = 10^{-6}$	$\gamma_D = 10^{-5}$	$\gamma_D = 2\ 10^{-5}$
$\langle r_a \rangle$ = 20 (forest)	1.02	1.16	1.32
$\langle r_a \rangle$ = 50 (grass)	1.01	1.10	1.20
$\langle r_a \rangle$ = 200 (water)	1.00	1.03	1.06

The value given is the mean value between 0800 and 1600.

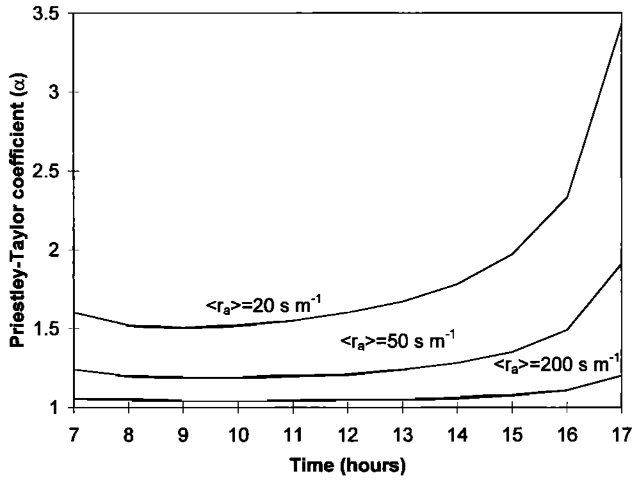

Figure 7. Diurnal variation of α ($=E_p/E_{eq}$) for three different values of aerodynamic resistance $\langle r_a \rangle$, with $\gamma_D = 10^{-5}$ kg kg^{-1} m^{-1}, T_a = 30 °C, A_x = 500 W m^{-2}, and $\langle r_s \rangle$ = 100 s m^{-1}.

ton's definition) for different values of the areal surface resistance $\langle r_s \rangle$. When surface resistance increases, the diurnal curve of α rises, which is logical since saturation deficit and E_p increase with $\langle r_s \rangle$. The value of α is fairly constant until midafternoon and grows later. A similar form for the diurnal curve (concave up) has been found by *de Bruin* [1983], although his definition of α is different (E_p being replaced by areal actual evaporation). In Figure 7 the diurnal course of α is plotted for different values of the areal aerodynamic resistance $\langle r_a \rangle$: α is a decreasing function of $\langle r_a \rangle$, that is, an increasing function of roughness length. Figure 8 shows the diurnal course of α for different values of inversion strength. As for α^a, the diurnal curve of α rises when inversion strength increases. In Figure 9 the daily mean value of α is plotted against the surface resistance for different values of inversion strength. It appears that the theoretical limit (α_{eq}) mentioned above is obviously never reached on a daily basis. The value of α varies in a relatively restricted range which includes the value of 1.26 experimentally established by *Priestley and Taylor* [1972]. All the curves tend to a sort of asymptotic value when $\langle r_s \rangle$ increases, this asymptotic value rising with γ_D. For a typical value of γ_D of 10^{-5} and $\langle r_a \rangle$ = 50 s m^{-1} (typical value of aerodynamic resistance for grass), α increases with surface resistance from 1.1 (for $\langle r_s \rangle$ = 0) to an asymptotic value of about 1.5 (for $\langle r_s \rangle$ tending to infinity). Consequently, the experimental value of 1.26 proposed by *Priestley and Taylor* [1972] appears to be better accounted for by the coefficient α than by the coefficient α^a, which predicts a range of variation from 1.01 to 1.20 for the same value of the aerodynamic resistance (see Table 2).

4.3. A Look at the Complementary Relationship

The complementary relationship is based upon an idea formulated by *Bouchet* [1963] and reworked in a series of papers by *Morton* [1969, 1975, 1983]. This relationship treats potential evaporation (E_p), defined at local scale in the sense of Morton, and actual evaporation (E^a) at regional scale as complementary quantities. It states that when external conditions do not change and in the absence of large-scale advection, the decrease in actual evaporation generates an equal but opposite change in potential evaporation, implying a constant sum. This statement results in the following equation:

$$E^a + E_p = 2E_p^a \tag{14}$$

where E_p^a is the areal potential evaporation (Penman's definition), obtained when the environment is completely wet (in this case $E^a = E_p$). Although this relationship has been widely

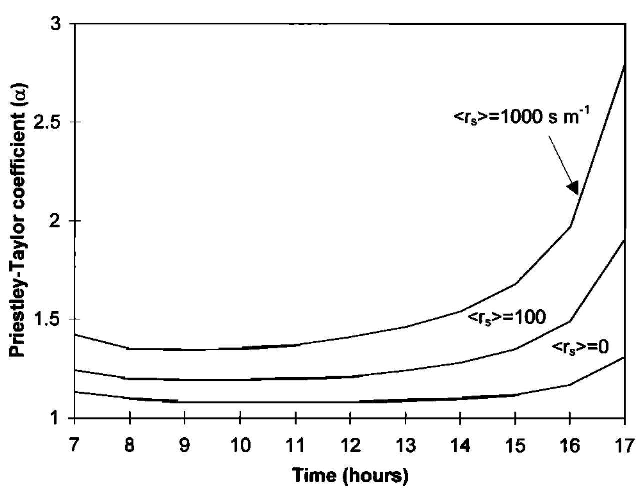

Figure 6. Diurnal variation of α ($=E_p/E_{eq}$) for three different values of surface resistance $\langle r_s \rangle$, with $\gamma_D = 10^{-5}$ kg kg^{-1} m^{-1}, T_a = 30 °C, A_x = 500 W m^{-2}, and $\langle r_a \rangle$ = 50 s m^{-1}.

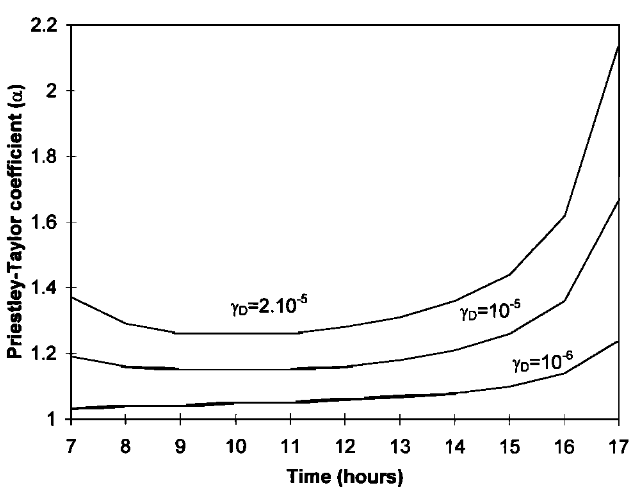

Figure 8. Diurnal variation of α ($=E_p/E_{eq}$) for three different values of inversion strength γ_D (kg kg^{-1} m^{-1}), with T_a = 30 °C, A_x = 500 W m^{-2}, and $\langle r_a \rangle = \langle r_s \rangle$ = 50 s m^{-1}.

used and commented upon, it has never been derived completely from physical principles, and doubts remain about its reliability. Each quantity in (14) can be easily calculated from the CBL model, and the validity of (14) can be examined. The areal actual evaporation E^a is inferred from (1). The local potential evaporation E_p and the areal potential evaporation E_p^a are calculated from (11) in the way described above. To test the validity of the complementary relationship, the coefficient η was calculated on a daily basis as $\eta = (E^a + E_p)/E_p^a$. If the complementary relationship is true, η must be constant and equal to 2, whatever the areal surface resistance. Table 3 shows that η is not constant but decreases when surface resistance increases. Consequently, one has to admit that the complementary relationship, as written by *Morton* [1969], is not verified by the numerical results generated by the CBL model.

Table 3. Variation of Coefficient $\eta = (E^a + E_p)/E_p^a$ as a Function of Areal Surface Resistance $\langle r_s \rangle$ (s m^{-1}) With $\langle r_a \rangle = 50$ s m^{-1}, $\gamma_D = 10^{-5}$ kg kg^{-1} m^{-1}, $T_a = 30$ °C, and $A_x = 500$ W m^{-2}

$\langle r_s \rangle$	η
0	2.00
50	1.98
100	1.93
200	1.85
500	1.71
1000	1.60
5000	1.47

The value given here is the mean value calculated between 0800 and 1600. If the complementary relationship were true, η would be constant and equal to 2.

5. Conclusion

A simple CBL model with entrainment, built on the one originally devised by *Raupach* [1991] and leading to an analytical solution, has been used to assess the Priestley-Taylor coefficient, defined as the ratio between potential evaporation (E_p) and equilibrium evaporation (E_{eq}). Since there are two different ways of defining potential evaporation, as the evaporation of a completely wet environment (Penman's definition) or of a small saturated surface (Morton's way), there are two possible ways of calculating the Priestley-Taylor coefficient. These two ways have been successively examined.

In no case does the Priestley-Taylor coefficient appear to have a constant and universal value. When this coefficient is defined for an extended saturated area (denoted α^a), it depends upon the conditions in the undisturbed atmosphere above the CBL (inversion strength) and also upon the characteristics of the surface (roughness length). The additional energy implied by a coefficient greater than 1 is due to the entrainment effect. When defined in Morton's way, the coefficient (denoted α) does not appear constant either, and the additional energy originates both from the feedback of areal evaporation on potential evaporation and from entrainment effect. There is a strong dependence of α on areal surface resistance and on the inversion strength above the capping inversion, which tends to rise the saturation deficit within the CBL. For given external conditions (γ_D), when areal surface resistance increases, the daily value of α has an asymptotic behavior towards a limit value which rises with γ_D. The numerical results obtained from the model tend to prove than the experimental value of 1.26 proposed by *Priestley and Taylor* [1972] is better accounted for by a coefficient α defined for a small saturated area (Morton's way) than for an extended saturated area (Penman's way).

The Bouchet-Morton complementary relationship has also been examined at the light of the CBL model but finds no support in the numerical results obtained. The sum of areal evaporation (E^a) and potential evaporation (E_p) is not constant and generally not equal to twice the areal potential evaporation (E_p^a).

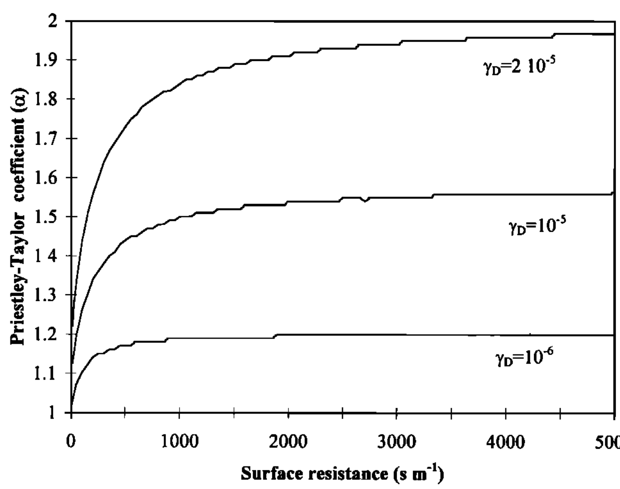

Figure 9. Variation of coefficient α ($=E_p/E_{eq}$) as a function of surface resistance $\langle r_s \rangle$, for three different values of the inversion strength γ_D (kg kg^{-1} m^{-1}); α is calculated as the mean value of 9 hourly values (from 0800 to 1600). $T_a = 30$ °C, $A_x = 500$ W m^{-2}, and $\langle r_a \rangle = 50$ s m^{-1}.

Appendix: Parameterization of the Growth-Rate Parameter *K*

In the model the height of the CBL is assumed to grow with time according to the equation $h(t) = (Kt)^{1/2}$. Since K is an increasing function of the areal surface resistance $\langle r_s \rangle$ (the greater $\langle r_s \rangle$, the greater the sensible heat flux and the faster the growth of the CBL), a hyperbolic model has been chosen of the form

$$K = K_x \frac{\langle r_s \rangle + \mu}{\langle r_s \rangle + \nu} \qquad \text{(A1)}$$

where K_x is the maximum value of K, obtained when $\langle r_s \rangle$ tends to infinity, and μ and ν are two constants empirically determined by writing that the minimum value of the parameter (K_n) is obtained for $\langle r_s \rangle = 0$, and that an intermediate value (K_i) is obtained for $\langle r_s \rangle = r_i$. In this case we have

$$\mu = r_i \frac{K_n}{K_x} \frac{K_x - K_i}{K_i - K_n} \qquad \nu = r_i \frac{K_x - K_i}{K_i - K_n} \qquad \text{(A2)}$$

There exists a simple relationship between the parameter K and the height of the CBL (h_f) at the end of the diurnal cycle (d). This relationship is obtained from the equation which governs the growth rate of the CBL and reads as $h_f^2 = K(t_0 + d)$, t_0 being the initial time obtained from the input parameter h_0. So it is possible to relate K_n, K_i, and K_x directly to the corresponding values of h_f (h_{fn}, h_{fi}, and h_{fx}). In our simulations we used $d = 12$ hours, $h_{fn} = 1000$ m for $\langle r_s \rangle = 0$, $h_{fx} = 3000$ m for $\langle r_s \rangle$ very large, and $h_{fi} = 1500$ m for $\langle r_s \rangle = r_i = 100$ s m^{-1}.

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