ACCURATE DIFFUSIVE WAVE ROUTING

By Bernard Cappelaere[1]

ABSTRACT: The diffusive wave simplification of the unsteady, open-channel flow equations is a commonly used approach for flood routing applications. Among the many routing methods, except for those based on the De Saint-Venant equations, it may be considered to be the one that most closely complies with the physics of open-channel hydraulics. However, common implementations of the diffusive wave approach (linear diffusion, variable parameter diffusion) involve additional approximations that diminish its physical significance and accuracy. A new formulation for the nonlinear diffusive wave is presented that better respects conservation principles through close consistency with the fundamental flow equations. The accuracy and reliability of the proposed model are shown on test cases consisting both of a hypothetical, regular channel, and of an actual river reach. Simulated discharges are compared to those obtained by a full De Saint-Venant model and by ordinary diffusion methods, as well as to observed hydrographs in the real-world test case. This model, based on hydraulic theory, can be safely applied to a wide range of flow conditions, while complying with the practical constraints of flow routing applications, including the usual nonavailability of adequate channel geometry data.

INTRODUCTION

The diffusive wave approach to flow modeling in open channels has the combined advantage of allowing for:

- The elimination of one of the two state variables (discharge Q or stage Z) from the governing equations
- The aggregation of a great number of basic channel characteristics (describing channel geometry and roughness distribution over space) into more global parameters, namely wave celerity and attenuation. These parameters control the propagation of the remaining variable of concern, generally Q, through a single "advection-diffusion" (ADE) type equation.

This approach is especially suitable for flood routing applications, when available data consist of observed upstream and downstream stage records or hydrographs on a river reach, rather than in the detailed channel description required for full flow modeling. In such cases, variables and wave parameters involved in the diffusion routing model are closer to the observable data and are therefore more meaningful to the user than those describing actual flow conditions (e.g., profile of the free surface, distribution of velocity, wetted surface and perimeter, and roughness along the channel) in a full dynamic model. Hence, a functional representation that uses the fewest possible nonmeasured internal state variables and calibration parameters, while preserving a physically sound basis of the model, is desirable. The underlying simplification of the diffusion method is the neglect of the inertia terms from the momentum conservation part of the De Saint-Venant flow equations. This approximation has been shown to be sufficiently accurate for a wide range of open-channel flow conditions, particularly in rivers and natural streams [e.g., Bocquillon (1978); Ponce et al. (1978); Weinmann and Laurenson (1979)].

More significant departure from the original flow equations is brought about by the additional simplifications that are used in the various common propagation models, such as linear (constant parameter) diffusion, kinematic wave, and reservoir-type conceptualization (momentum equation replaced by some linear storage expression) like the Muskingum method. While these methods do generally satisfy mass conservation, the degradation of the dynamic equation (conservation of momentum) considerably reduces generality and limits the scope of use to quite specific conditions (e.g., for the kinematic wave, small water depths, steep slopes, little lateral inflow), or only to the very situations used for model calibration. Although the solution of these simpler systems is usually easier than that of the flow equations (with or without inertia terms), severe numerical difficulties are not precluded, such as the encounter of kinematic shocks in the kinematic wave approach (Lighthill and Whitham 1955).

The variable parameter diffusion was introduced (Cunge 1969; Price 1973; Bocquillon and Moussa 1988) to account for the strongly nonlinear nature of the momentum equation, while conserving the general form and objectives of the ADE formulation; celerity and attenuation parameters are expressed as functions of the state variable (usually discharge, Q), which makes the equation suitable for numerical resolution without having to solve concurrently for the other variable (Z). This refinement does lend a more realistic representation of propagation dynamics; however, it no longer guarantees strict conservation of fluid mass, as will be shown in this paper. This shortcoming, seldom mentioned in the literature (the ADE equation is usually shown to be derived from the De Saint-Venant equations directly), not only raises a problem from a theoretical standpoint, but also may lead to significant error on predicted discharges, including the peak flow rate, in the very situations where the zero-inertia approximation itself would actually hold true.

Hence, despite this more sophisticated form of the diffusion method, limitations are also imposed to its use, to ensure that mass balance error be kept small; in particular, sufficient channel slopes and slowly varying discharges along time and space are needed. This is due to the fact that accounting for the pressure gradient term of the momentum equation, which is supposed to be precisely the theoretical benefit of the diffusive wave approach over the kinematic propagation theory, is only very partial in common implementations of the diffusion method (hereafter designated as 'ordinary' diffusion method), since it is not reflected in the expressions of the celerity and attenuation coefficients themselves when these are functions of discharge Q only, or constants.

Unlike inertia terms, pressure gradients frequently have significant effects on the propagation process [see, for instance, Morel-Seytoux et al. (1993)]. Their importance increases with lower channel slopes or steeper hydrographs. On the other hand, any attempt at keeping those terms fully in the development of the advection-diffusion equation for discharge Q, makes elimination of the stage variable Z impossible (or vice

[1]Res. Engr., ORSTOM, 911 Ave. Agropolis, B.P. 5045, 34032 Montpellier, France.

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versa). Price (1985) produced a nonlinear Muskingum-like model through approximation of the dynamic flow equation with respect to the pressure gradient term: departing from the diffusive wave formulation, it removes any backwater effect from the discharge routing process.

The purpose of this paper is to provide a derivation of a discharge advection-diffusion type equation [hereafter named 'modified' or high-accuracy nonlinear diffusion method (HAND)] that fully eliminates the stage variable (or the discharge) while closely respecting the influence of pressure gradients on the propagation process. To get the full benefit from this higher precision of the new equation formulation for the diffusive wave, an accurate numerical resolution procedure is implemented to best fit the specific nature of the ADE, that mixes both hyperbolic and parabolic components. Through a fractional-step method, each component is handled separately with a numerical scheme that best suits its particular behavior, instead of globally solving for the whole equation by a single scheme, as is generally done. Special care is brought to the treatment of the convective term, to reduce numerical diffusion effects.

Such refinements, brought both to equation building and solving for the nonlinear diffusive wave approach, make the method appropriate for modeling in a wide range of commonly encountered situations, because it closely follows the fundamental governing equations for flow (including mass conservation) under negligible inertia conditions.

DIFFUSIVE WAVE THEORY

Written in terms of discharge Q and water flow depth h (rather than stage Z) as state variables, the general system of equations for one-dimensional, unsteady, open-channel flow known as the De Saint-Venant equations, is

$$B(h)\frac{\partial h}{\partial t} + \frac{\partial Q}{\partial x} = 0 \quad \text{(continuity equation)} \tag{1}$$

$$\frac{1}{g}\frac{\partial u}{\partial t} + \frac{u}{g}\frac{\partial u}{\partial x} + \frac{\partial h}{\partial x} - S_o + S_f = 0 \quad \text{(dynamic equation)} \tag{2}$$

where t = time, x = longitudinal coordinate (abscissa along channel reach); u = flow velocity [$u = Q/A(h)$, A being the cross-sectional area of flow]; B = channel width at water surface; S_o = channel bottom slope; and S_f = friction slope (slope of energy line).

Neglecting the inertia terms [first two terms of (2), that is, accelerations over time and space], and estimating S_f as $S_f = Q^2/D^2(h)$, where $D(h)$ is the section conveyance for flow depth h, then (2) can be rewritten as

$$Q^2/D^2(h) + \partial h/\partial x = S_o \tag{3}$$

Eq. (3) yields the discharge Q as a function of h and $\partial h/\partial x$

$$Q = \bar{Q}(h)\cdot \text{COR} \tag{4}$$

where $\bar{Q}(h) = D(h)\sqrt{S_o}$ and $\text{COR} = \sqrt{1 - S_o^{-1}\partial h/\partial x}$.

$\bar{Q}(h)$ is the so-called "normal" discharge for depth h, defined as the discharge value under zero pressure-gradient conditions ($\partial h/\partial x = 0$, COR = 1), that is, when the free water surface is parallel to the channel bottom. COR is a dimensionless correcting factor, accounting for the effect of the pressure gradient $\partial h/\partial x$, relative to the channel slope S_o. An expression for $D(h)$ is provided, for instance, by Manning's formula: $D(h) = K\cdot A(h)\cdot R^{2/3}(h)$ with R and K being the hydraulic radius and Strickler's roughness coefficient, respectively. The system of (1) and (3) forms the basis for the diffusive wave approach.

Classically, to obtain a discharge propagation equation, the derivatives of h are eliminated by combining (1) and (3) once differentiated with respect to x and t, respectively, eliminating the remaining $\partial h/\partial t$ using (1), expanding the differentiation of products, and dividing by $-2BQ/D^2$. Writing dD/dh as D' and dB/dh as B', the general diffusive wave equation, based only on the zero-inertia approximation, is

$$\frac{\partial Q}{\partial t} + \frac{\partial Q}{\partial x}\left(\frac{Q}{B}\frac{D'}{D} + \frac{D^2}{2B^2Q}\frac{\partial B}{\partial x}\right) = \frac{D^2}{2BQ}\frac{\partial^2 Q}{\partial x^2} \tag{5}$$

that amounts to the advection-diffusion equation for discharge Q

$$\frac{\partial Q}{\partial t} + C_Q\frac{\partial Q}{\partial x} = \sigma_Q\frac{\partial^2 Q}{\partial x^2} \tag{6}$$

$$\text{where } C_Q = \frac{Q}{B}\frac{D'}{D} + \frac{\sigma_Q}{B}\left[\left(\frac{\partial B}{\partial x}\right)_h + B'\frac{\partial h}{\partial x}\right] \tag{7}$$

$$\text{and } \sigma_Q = D^2/(2BQ) \tag{8}$$

C_Q and σ_Q are the celerity and attenuation parameters for the diffusive wave routing process. Because (4) expresses Q as a function of h and $\partial h/\partial x$, it is seen that C_Q and σ_Q also are functions of h and $\partial h/\partial x$. To complete the full elimination of variable h from the discharge propagation equation [(6)–(8)], C_Q and σ_Q should be functions of Q and its derivatives only, which is not the case. At most, only one of the two unwanted variables h or $\partial h/\partial x$ could be further eliminated [through use of (3)] but not both, thus precluding a resolution based solely on this diffusive wave equation. Further approximation(s) beyond the zero-inertia hypothesis must therefore be made to produce a solvable propagation equation.

Before going deeper into these approximations, a few remarks may be made.

1. With lateral inflow (or outflow), it can be shown that the diffusive wave equation [(6)] becomes

$$\frac{\partial Q}{\partial t} + C_Q\frac{\partial Q}{\partial x} = \sigma_Q\frac{\partial^2 Q}{\partial x^2} + C_Q\cdot q_e \tag{9}$$

   where $q_e = q - (\sigma_Q/C_Q)\cdot\partial q/\partial x$, q being the algebraic lateral flow rate per unit channel length (positive for inflow). If variations of q along the channel abscissa x can be considered small, then $q_e \approx q$.
2. Expressing C_Q and σ_Q without direct dependence on h implies that the conveyance $D(h)$, and consequently the "normal" discharge $\bar{Q}(h)$, are invertible functions of h, so that h may be eliminated as inv$\bar{Q}$(Q/COR) [see (4)], where inv$\bar{Q}$( ) designates the inverse function of $\bar{Q}(h)$. This simply means that there exists a single-valued, monotonic relationship between h and $\bar{Q}$. This excludes, for instance, the case of a closed pipe where some normal discharge values can be obtained for two distinct flow depths.
3. If the diffusive wave equation had been derived for the variable h [eliminating the Q instead of the h derivatives from (1) and (3)], then the celerity and attenuation parameters C_h and σ_h for flow depth would have been $C_h = QD'/BD$ [first term of C_Q, see (7)], and $\sigma_h = \sigma_Q = D^2/(2BQ)$. Hence, C_h and σ_h also are functions of h and $\partial h/\partial x$ only, and elimination of the variable Q from the equation for diffusive propagation of flow depth h is complete [thanks to (4)], with no need for any further approximation

$$C_h = \bar{C}(h)\cdot\text{COR} \quad \text{and} \quad \sigma_h = \bar{\sigma}(h)/\text{COR} \tag{10}$$

$$\text{where } \bar{C}(h) = B^{-1}d\bar{Q}(h)/dh \quad \text{and} \quad \bar{\sigma}(h) = \bar{Q}(h)/(2B\cdot S_o) \tag{11}$$

Expression (11) is well-known formulas for the propagation coefficients; they yield the correct values for flow depth propagation when the flow conditions are "normal". Equation (10) gives the stage propagation coefficients as products of these zero pressure-gradient values (function of h only) and of the correction factor COR due to the pressure gradient $\partial h/\partial x$. Also note that the attenuation coefficients for discharge and flow depth are identical, and that the celerities are also equal in the case of constant width (uniform rectangular channel).

ORDINARY DIFFUSIVE WAVE METHODS

To solve the diffusive wave equation (9), two alternative approaches are generally used, based on simplifying assumptions on the C_Q and σ_Q coefficients; (1) C_Q and σ_Q are taken as prescribed functions of Q only {the so-called variable parameter diffusion method, noted as VPD [see for example, Price (1975)]} or (2) C_Q and σ_Q are considered to be constant (linear diffusion, first introduced by Hayami 1951), for which case an analytical solution is readily obtained. The latter simplification can be viewed as a particular case of the more general first approximation, which is the only one to be discussed further.

Considering C_Q and σ_Q as functions of Q only, implies that in expressions (7) and (8) of C_Q and σ_Q, the influence of $\partial h/\partial x$ on C_Q and σ_Q is neglected, and therefore that Q is simplified to $\bar{Q}$, h to $\bar{h}$ = inv$\bar{Q}(Q)$ (the "normal" flow depth for discharge Q), C_Q and σ_Q (as well as C_h and σ_h) to $\bar{C}$ and $\bar{\sigma}$.

With the correcting factor COR being thus implicitly ignored, this first-order approximation is valid only if $\partial h/\partial x$ is small compared to the channel slope S_o [see (4)]. Otherwise, the diffusion equation (6) no longer complies with (5), and therefore departs from its founding equations (1) and (3). Hence, not only is distortion brought into the conservation of momentum, but more importantly mass conservation is no longer guaranteed. The annoying point is that the diffusive wave approach was developed precisely for the case where $\partial h/\partial x$ cannot be neglected in (3). [Note that when it can be neglected altogether, the system of (1) and (3) yields the much simpler, attenuation-free, kinematic wave approximation: $\partial Q/\partial t + \bar{C}\cdot\partial Q/\partial x = 0$, where the kinematic wave speed $\bar{C} = B^{-1}dQ/dh$ can be obtained as a function of Q only].

Hence, even if this simple method for nonlinear diffusion can provide a better solution to propagation than the kinematic wave approach when $\partial h/\partial x$ may not be neglected altogether, this solution contains a bias that can be strong and detrimental, since the basic equations are not satisfied. Mass conservation not being guaranteed is a major concern, from a theoretical as well as practical standpoint, when handling any flow propagation problem. Volumes would be preserved only if some appropriate constraint could be put on the two variable parameter functions, binding very strictly $C_Q(Q)$ and $\sigma_Q(Q)$ to each other. One such condition that would ensure mass conservation is σ_Q/C_Q = constant, for all Q.

However this property of (6) is of little practical interest, since C_Q and σ_Q seldom follow a proportionality relationship in real situations. Note that Hayami's linear diffusion model is only a particular case of this condition being fulfilled. In fact, the method that will be proposed in the next section, imposes some link between the C_Q and σ_Q values in (9) through the expressions by which they are derived from the state variable Q and its derivatives.

Example

To illustrate the behavior of the ordinary nonlinear diffusion approach, a regular channel reach with known geometry and roughness is used, the celerity and attenuation parameters C_Q and σ_Q being taken as the $\bar{C}(Q) = B^{-1}dQ/dh$ and $\bar{\sigma}(Q) = Q/(2BS_o)$ functions, precalculated for these channel characteristics (as already mentioned, this method assimilates Q to $\bar{Q}$ as far as the propagation coefficients are concerned). Using celerity and attenuation functions derived from geometry rather than calibrated from given hydrographs does not alter the generality of the example presented here. Indeed, since the method itself does not impose any specific constraint on the shape of the prescribed functions for these parameters, the values calculated under the hypothesis of "normal" flow conditions are as good as any, if not better, to represent such functions.

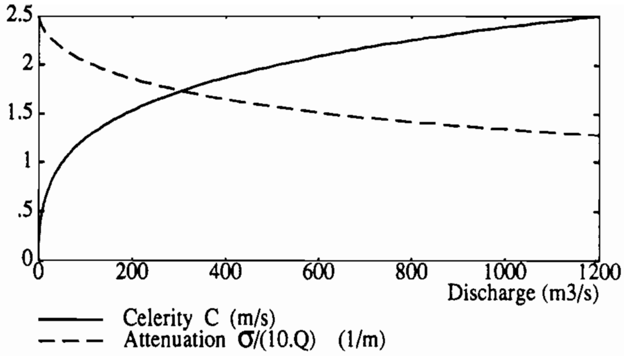

FIG. 1. Celerity and Attenuation versus Discharge for Hypothetical, Trapezoidal Channel [Attenuation Is Plotted as Ratio σ/(10.Q)]

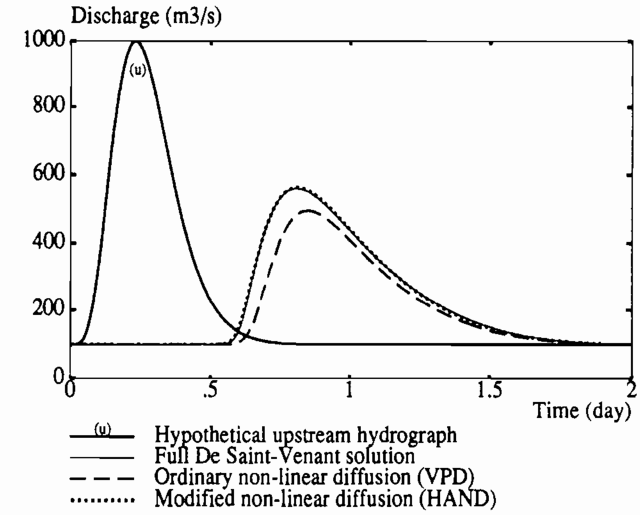

FIG. 2. "Ordinary" (VPD) versus Modified (HAND) Nonlinear Diffusion in Hypothetical, Trapezoidal Channel

A 100-km long trapezoidal channel, 12 m high, with lower and upper widths equal to 40 and 80 m, respectively, a 5.10^{-4} slope, and a roughness value K = 20 (SI units), is used for illustrative purposes. Functions $C_Q(Q)$ and $\sigma_Q(Q)$ are tabulated for the values of normal discharge corresponding to a 0.1-m water depth increment (Fig. 1), and are linearly interpolated in between.

A hydrograph with equal values for initial and final steady-state discharges is fed into the channel upstream end, to enable the direct comparison of upstream and downstream hydrograph volumes (there is no difference in water stored in the channel between the initial and final steady states). The nonlinear partial differential equation (P.D.E) [(6)] for discharge propagation is solved by the numerical method presented in the later section entitled "Numerical Method."

The propagated hydrograph resulting at the downstream end of the channel is presented in Fig. 2 (VPD curve). There is nearly a 20% difference in volumes above base-flow, obtained between upstream and downstream hydrographs (20.5 and 16.4 hm^3, respectively). This discrepancy can be attributed to the model formulation itself [(6) with C_Q and σ_Q dependent on Q only], not to numerical resolution error. The same numerical method applied to (6) with constant C_Q and σ_Q, or with C_Q and σ_Q respecting the very peculiar condition mentioned earlier (σ_Q/C_Q = constant), yields no such mass loss. If (6) were now slightly transformed to take the form of a true ADE with variable parameters (comparable for instance to a solute transport equation, where Q would represent some solute concentration)

$$\frac{\partial Q}{\partial t} + \frac{\partial (C_Q Q)}{\partial x} = \frac{\partial}{\partial x}\left(\sigma_Q \frac{\partial Q}{\partial x}\right)$$

while keeping the $C_Q(Q)$ and $\sigma_Q(Q)$ functions of Fig. 1 (or as a matter of fact, any prescribed $C_Q(Q)$ and $\sigma_Q(Q)$ functions), the solution obtained for Q with exactly the same numerical method, is found to conserve the mass of Q (here a solute mass), the flux through a section now being $C_Q Q - \sigma_Q \cdot \partial Q/\partial x$, instead of Q itself. These tests show that the numerical procedure used in the model can be used reliably to highlight the deviation from fundamental fluid movement principles produced by the simple, variable parameter diffusion approach.

To further qualify the solution obtained by this approach, the downstream hydrograph resulting from simulation by a full De Saint-Venant model [solving the basic set of (1) and (2) with an implicit finite-difference scheme (Fread 1985), and of course abiding totally by the principles of mass and momentum conservation] is plotted on Fig. 2. Comparison of hydrographs shows the damping effect on discharge values, especially on peak discharge, produced by the simple nonlinear diffusion method. Suppressing inertia terms in the De Saint-Venant solution leads to no visible difference with the full model, showing that the zero-inertia approximation inherent to diffusion theory is perfectly acceptable.

A conclusion from this section is that the ordinary VPD diffusion method can safely be employed only in situations remaining close to the domain of validity of either the linear diffusion, or the kinematic wave approximation. Much of the interest lying in such nonlinear method is thus lost. Ordinary diffusive wave methods, with constant or variable parameters, must be viewed as conceptual models that do not abide by strict hydraulic principles. The following section develops the equations that make nonlinear diffusion applicable to the general case of negligible inertia.

MODIFIED METHOD FOR VARIABLE-PARAMETER DIFFUSION (HAND METHOD)

The full expressions for the coefficients C_Q and σ_Q are [see (7) and (8)]

$$C_Q = C_h + \frac{\sigma_Q}{B}\left[\left(\frac{\partial B}{\partial x}\right)_h + B'\frac{\partial h}{\partial x}\right] \quad \text{where} \quad C_h = \frac{Q}{B}\frac{D'}{D} \qquad (12)$$

$$\sigma_Q = \sigma_h = D^2/(2BQ) \qquad (13)$$

The purpose of this modified method is to show that when Q is used in conjunction with its derivative $\partial Q/\partial x$ to control the expression of C_Q and σ_Q, an approximation of (5) can be obtained with second-order accuracy. Both mass and momentum conservation are then closely satisfied. Conversely, if C_Q and σ_Q are functions of Q and $\partial Q/\partial x$ respecting the expressions proposed hereafter, conservation principles can be expected to be satisfied. Let us see in (12) and (13) how C_Q and σ_Q, functions of h and $\partial h/\partial x$, can be approximated as functions of Q and $\partial Q/\partial x$.

The term $(\partial B/\partial x)_h$ which expresses the variability of channel geometry along the x axis, will hereafter be omitted. This is not to say that this term is always negligible in reality; however, propagation models usually schematize the actual channel into a series of prismatic reaches, each being characterized by a set of space-lumped parameter values, and therefore implicitly, by some equivalent, fictitious, uniform channel geometry (see section "Validation by full hydraulic modeling of the equivalent channel" for a discussion of this geometry). It is to this equivalent channel that the theory developed hereafter applies. Effects of cross-section variations on hydrograph routing have been discussed in the literature [see for instance Price (1975)]. The suitability of the concept of equivalent, uniform channel is presented by Price (1985).

First, $\partial h/\partial x$ can be eliminated as $S_o(1\text{-COR}^2)$ using (4). Equation (10) gives C_h and σ_h (i.e. σ_Q) as functions of $\bar{C}$ and $\bar{\sigma}$ (zero pressure-gradient celerity and attenuation for "normal" flow conditions in the section; functions of h or $\bar{Q}(h)$, indifferently), and COR. Finally, B'/B can be eliminated by writing

$$\frac{dB}{dh}\frac{1}{B} = \frac{dB}{d\bar{Q}}\frac{d\bar{Q}}{dh}\frac{1}{B} = \frac{dB}{d\bar{Q}}\bar{C} = \frac{\bar{C}}{2\bar{\sigma}S_o}\left(1 - \frac{\bar{Q}}{\bar{\sigma}}\frac{d\bar{\sigma}}{d\bar{Q}}\right)$$

the expression for $dB/d\bar{Q}$ being obtained through derivation of $B = \bar{Q}/(2\bar{\sigma}S_o)$, given by (11). Equations (12) and (13) then become

$$C_Q = \frac{\bar{C}}{2}\left[\text{COR}\left(1 + \frac{\bar{Q}}{\bar{\sigma}}\frac{d\bar{\sigma}}{d\bar{Q}}\right) + \frac{1}{\text{COR}}\left(1 - \frac{\bar{Q}}{\bar{\sigma}}\frac{d\bar{\sigma}}{d\bar{Q}}\right)\right]; \sigma_Q = \bar{\sigma}/\text{COR} \qquad (14a,b)$$

where $\bar{Q} = Q/\text{COR}$. These are the general expressions for C_Q and σ_Q, for the assumptions of negligible inertia and channel section variability but none on pressure gradients. Only the correction factor COR, given by (4), is still h-dependent: the $\partial h/\partial x$ term can be eliminated through approximation of the equation for flow depth propagation $\partial h/\partial t + C_h \partial h/\partial x = O(\partial^2 h/\partial x^2)$. Dropping the right-member term and eliminating $\partial h/\partial t$ with the continuity equation [(1)], C_h with (10) and (4), and finally using (11) to clean out all undesirable variables (S_o, B, and $\bar{Q}$), COR can be approximated as

$$\text{COR} = \sqrt{1 - \frac{2\bar{\sigma}}{\bar{C}\cdot Q}\frac{\partial Q}{\partial x}} \qquad (15)$$

This relationship still holds true when there is some lateral inflow, q. The approximation relative to the pressure gradient term is selective in the sense that it bears only on the estimation of the corrective factor for the equation parameters [(15)], not on the formulation of the propagation equation itself [(14)].

Equation (14), with (15), provides the expressions for celerity and attenuation coefficients, with only Q and $\partial Q/\partial x$ as variables, and prescribed functions $\bar{C}(\bar{Q})$ and $\bar{\sigma}(\bar{Q})$ as variable parameters. Together with the propagation equation [(9)] itself, this makes a fully determined P.D.E. system (five equations for five unknowns: Q, $\bar{Q}$, COR, C_Q and σ_Q), that can be solved numerically for the main unknown Q, given the $\bar{C}(\bar{Q})$ and $\bar{\sigma}(\bar{Q})$ functions. COR, C_Q and σ_Q are explicit intermediate variables used only for convenient writing; they can be eliminated directly if desired, leaving only two remaining equations with the unknowns Q and $\bar{Q}$.

The very interesting feature of this set of equations is that indeed all basic, h-dependent physical data, describing channel geometry and roughness, have disappeared [including slope S_o, which was not the case in (10) for stage propagation, where S_o appears in the expression of COR], the two global wave parameters $\bar{C}$ and $\bar{\sigma}$ sufficing to describe discharge propagation along the channel. Hence, topography is not necessary, when the model can be calibrated from recorded upstream and downstream hydrographs. Calibration of the modified nonlinear diffusion model (HAND) is not significantly different from ordinary variable-parameter (VPD) calibration. For an example of a calibration method, see Price (1975). Note that when

$\bar{\sigma}$ may be neglected, the kinematic wave model is indeed obtained, for which $C_Q = \bar{C}$ and $\bar{Q} = Q$; of course $\partial Q/\partial x = 0$ (COR = 1) yields $C_Q = \bar{C}$, $\sigma_Q = \bar{\sigma}$ and $\bar{Q} = Q$. The problem of the radicand in (15) becoming negative or nil should not theoretically arise (aside from numerical divergence of the model), since hydrograph routing cannot be expected to produce $Q \leq 0$ conditions due to a horizontal or counter-slope water surface.

NUMERICAL METHOD

Given the distinct natures of the propagation equation's convective and diffusive components, an operator-splitting approach is used to handle the two terms separately. This approach allows giving each term a specific numerical treatment, that best suits its particular behavior, instead of globally solving for the whole equation by a single scheme, as is most often done.

At each time step Δt, the propagation equation [(9)] is replaced by a set of two partial differential equations that are solved successively, with a step fractioning technique [see, for instance, Usseglio and Chenin (1988)]

$$\frac{\partial Q}{\partial t} + C_Q \frac{\partial Q}{\partial x} = C_Q \cdot q_e \quad \text{a pure convection equation} \qquad (16)$$

$$\frac{\partial Q}{\partial t} = \sigma_Q \frac{\partial^2 Q}{\partial x^2} \quad \text{a pure diffusion equation} \qquad (17)$$

The advection equation [(16)] is solved first, over the time step Δt, starting with known discharge values $Q(x, t)$ at the beginning of the time step. A method of characteristics with third-order interpolation is used that minimizes numerical diffusion error. The solution obtained from this first equation, $Q^*(x)$, is then fed as initial values for the time-step to the parabolic, diffusion equation [(17)]; the latter is solved through a six-point, Crank-Nicholson implicit finite-difference scheme, over the time step Δt, to produce the solution $Q(x, t + \Delta t)$ at the end of the time step.

The solution $Q(x, t + \Delta t)$ thus obtained through this operator-splitting, fractional-step technique, is at least a second-order accuracy approximation to the original propagation equation. This method overcomes the strict condition that must be observed on time and space steps ($\Delta x/\Delta t = n \cdot C_Q$, where n is an integer), for pure finite-difference schemes to avoid numerical diffusion. Respecting this condition is not possible when C_Q varies over time and space, as is the case for the nonlinear diffusive wave. Finite-difference methods thus produce significant error on the convective part of the propagation equation that is eliminated by the highly accurate, characteristics approach used with the HAND method.

Initial conditions are taken as steady-state discharges. Inflow hydrographs are forced into the model as the upstream boundary condition and lateral inflow. For the diffusion step, the downstream boundary condition simulates the effect of a semi-infinite uniform channel prolongating to infinity the last internal point's diffusion equation.

IMPLEMENTATION ON TEST CASES

The modified nonlinear diffusive wave method was first implemented on regular channels. Results for the hypothetical test case used as an example for the ordinary variable parameter method will be presented hereafter.

The same input data (tables for zero pressure-gradient propagation parameters $\bar{C}$ and $\bar{\sigma}$ as functions of Q; channel length; upstream hydrograph), as well as time and space steps are used. The numerical method is basically identical (the COR variable just has to be set to 1, meaning that the effect of $\partial Q/\partial x$ is neglected, to come back to the ordinary model).

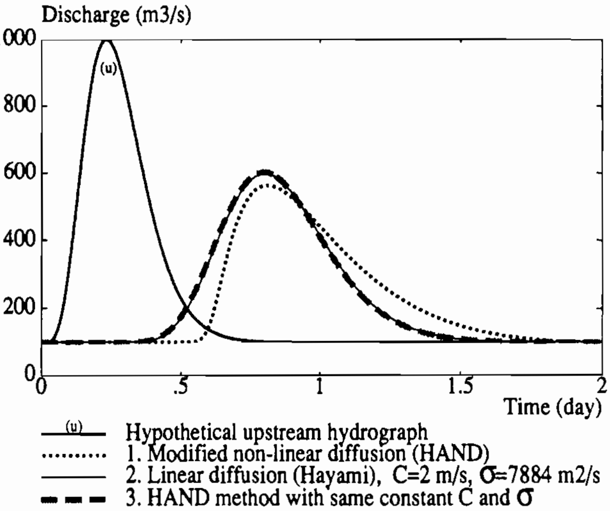

FIG. 3. Linear Versus Nonlinear Diffusion with Modified (HAND) Routing Method in Hypothetical, Trapezoidal Channel

The hydrograph routed by the HAND model is plotted on Fig. 2, together with those produced by the De Saint-Venant and ordinary diffusion (VPD) methods. It can be seen that the output of this new, accurate model closely follows the De Saint-Venant solution, with only a 0.2% volume discrepancy, compared to the 20% loss incurred with the ordinary model.

To compare with the linear diffusion method, a set of constant celerity and attenuation coefficients is arbitrarily taken as the $\bar{C}$ and $\bar{\sigma}$ values obtained for a discharge value denoted Q_{ref} equal to half the peak upstream discharge. Fig. 3 shows the hydrographs obtained with, respectively,

1. The modified nonlinear diffusion (HAND) method with the full, variable parameter tables
2. The analytical (Hayami) solution to the linear diffusion equation for the $\bar{C}(Q_{ref})$ and $\bar{\sigma}(Q_{ref})$ constants (2 m/s and 7,884 m^2/s)
3. The modified nonlinear diffusion method with the same $\bar{C}(Q_{ref})$ and $\bar{\sigma}(Q_{ref})$ constants

This shows that the proposed method

- Allows for good modeling of the nonlinearities of channel routing (compare curves 1 and 2, together with the De Saint-Venant hydrograph of Fig. 2)
- Is able to handle varied parameter configurations, including constant $\bar{C}$ and $\bar{\sigma}$ values (compare methods 2 and 3; in this case, agreement would be good between the two variable-parameter modeling methods, because nonconservation of mass no longer occurs when the ordinary diffusion equation is linear)

The method is now tested on the real case of a natural stream, drawn from the data provided by Price (1975) for the Erwood-Belmont reach of the river Wye in Great Britain. This 69.75 km-long reach is reported to have a large flood plain and small lateral inflow, making it of significant interest for flood routing tests. Curves for celerity and attenuation versus discharge, as established by Price (1975), are shown in Fig. 4, and were used 'as is'; that is, with no additional calibration (the calibration criteria used by Price were the value and time of peak).

Results obtained for two major flood events, those of January 1948 (upstream peak discharge of about 800 m^3/s) and December 1960 (peak around 1,200 m^3/s), are presented in Figs. 5 and 6, respectively. In the plot for the 1960 flood (Fig. 6), an 8.5% reduction is applied to all values of the down-

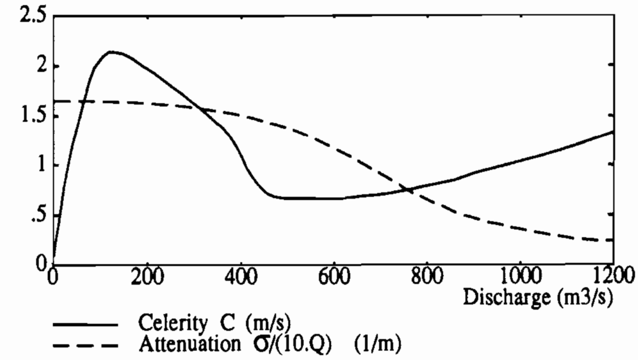

FIG. 4. Celerity and Attenuation Versus Discharge, for Erwood-Belmont Reach of River Wye [Attenuation Is Plotted as Ratio $\sigma/(10.Q)$]

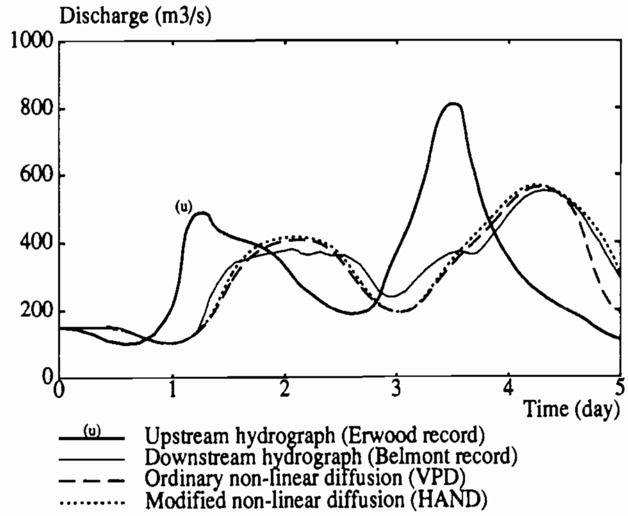

FIG. 5. Routing of January 1948 Flood, Erwood-Belmont Reach of River Wye

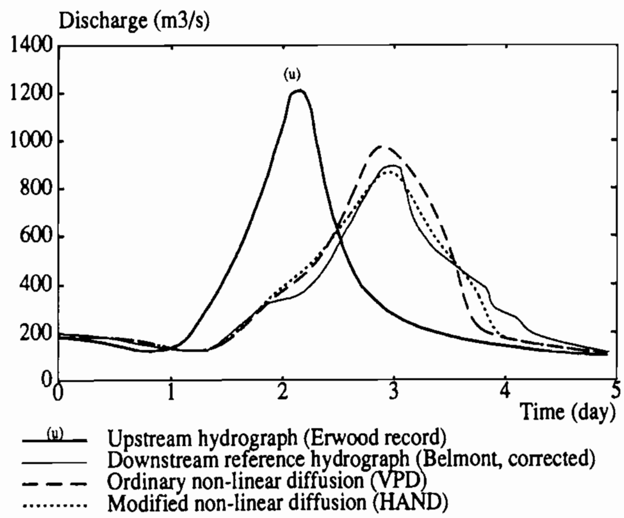

FIG. 6. Routing of December 1960 Flood, Erwood-Belmont Reach of River Wye

stream hydrograph recorded at Belmont (leading to what is called the 'reference' hydrograph, on Fig. 6), to account for the 8.5% volume discrepancy between the reported upstream and downstream hydrographs. The source of this discrepancy is not known (lateral inflow, or rating error, or both), nor is a better correction procedure to allow for proper observed-to-predicted comparisons. The affine hydrograph reduction applied is tantamount to consider that when there is lateral inflow, its variations closely follow those of the main discharge, and that the relative error on discharge estimation is constant. Figs. 5 and 6 show that the overall shape of the downstream hydrograph is better replicated with the modified diffusion method than with the ordinary method, especially in the recession limbs. The same parameter and computational increments were used for both methods. In other words, no attempt was made whatsoever to improve agreement with records.

Results for the ordinary diffusion method (VPD curves) are consistent with those reported by Price (1975). Note that to improve the agreement between observations and predictions in the recession limb, some artificial, empirical control over the solution was introduced by Price (1975) that consisted of maintaining the value of $\bar{C}$ constant when the computed discharge at any point fell below a certain value, namely 400 m^3/s, provided it had previously exceeded some value greater than the bankfull discharge in the natural river.

Volume conservation is better satisfied with our modified method than with the ordinary method; differences with upstream volumes are −0.1 against −7.6% for the 1948 flood, and 0.7 against 5.7% for the 1960 event, respectively. The values of Nash's efficiency criterion (Nash and Sutcliffe 1970) were computed for all four solutions; results are 88.7% in 1948 and 86.8% in 1960 for the ordinary diffusion method, against 93.1 and 95.7%, respectively, for the proposed modified method. This quantifies the better agreement with observations obtained for the modified method, given that maximum theoretical efficiency would be 100% for an ideal model.

VALIDATION BY FULL HYDRAULIC MODELING OF THE EQUIVALENT CHANNEL (MEC METHOD)

Given two $\bar{C}(\bar{Q})$ and $\bar{\sigma}(\bar{Q})$ functions (as tabulated data, or as analytical expressions, for instance), it is possible to derive a synthetic geometry for a fictitious equivalent channel that behaves similarly to the actual reach (the one described by the two celerity and attenuation input functions) with respect to hydrograph propagation. To find this equivalent channel, one has to solve for the channel characteristics that satisfy the expressions [(11)] of $\bar{C}$ and $\bar{\sigma}$ for a given channel.

In actual fact, the solution to this inverse problem does not need to be provided as the full geometrical description of an equivalent channel; determining the variations of width $B(h)$ and conveyance $D(h) = \bar{Q}/\sqrt{S_o}$ with water depth h, is sufficient to fully describe the hydraulic behavior of the equivalent cross section. Besides, the solution of the inverse problem is not unique; there is even an infinity of them, one for a given value of slope S_o

$\bar{C}(\bar{Q})$, $\bar{\sigma}(\bar{Q})$ and S_o being provided, one can compute

$$B(\bar{Q}) = \frac{\bar{Q}}{2S_o \cdot \bar{\sigma}(\bar{Q})} = f_1(\bar{Q}), \quad \text{and} \quad \frac{d\bar{Q}}{dh} = B \cdot \bar{C} = \frac{\bar{Q}}{2S_o}\frac{\bar{c}(\bar{Q})}{\bar{\sigma}(\bar{Q})} = f_2(\bar{Q})$$

$$\text{Hence } \frac{d\bar{Q}}{f_2(\bar{Q})} = dh \quad \text{yielding} \quad h(\bar{Q}) = \int_0^{\bar{Q}} \frac{dQ}{f_2(Q)}$$

Therefore, integration of $1/f_2(Q)$ over Q (performed numerically, with the trapezoidal rule), from $Q = 0$ to $Q = \bar{Q}$, provides the water depth h in the cross section for normal discharge $\bar{Q}$; the corresponding conveyance $D(h)$ and width $B(h)$ are obtained as $\bar{Q}/\sqrt{S_o}$ and $f_1(\bar{Q})$, respectively. This makes $h(\bar{Q})$ a monotonic, increasing function, thereby ensuring the physical viability of the synthetic channel configuration. This way, routing parameters, supplied as $[Q, \bar{C}(Q), \bar{\sigma}(Q)]$ input tables, can be transformed into tabulated hydraulic characteristics $[h, B(h), D(h)]$, enabling full flow modeling in the equivalent, prismatic channel. To ensure true equivalence of the two sets of tables, it might be necessary to intensify the discretization of the $(Q, \bar{C}, \bar{\sigma})$ input tables before they are processed into (h, B, D) tables, since the transformation (from routing character-

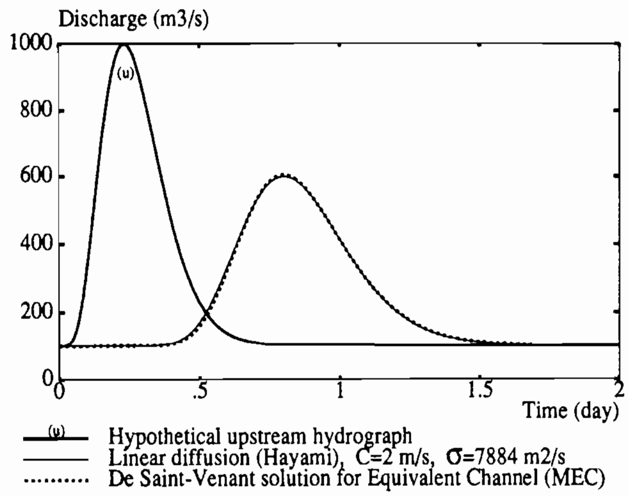

FIG. 7. Equivalent Channel (MEC) Method, Linear (Constant Coefficients) Case

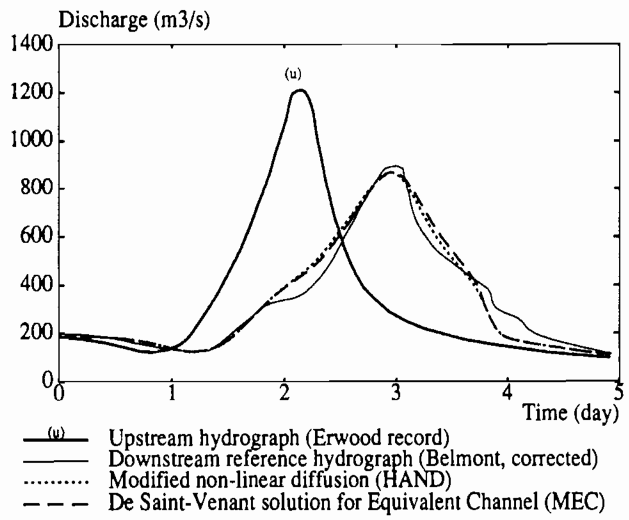

FIG. 8. Routing of December 1960 Flood by Equivalent Channel (MEC) Method, Erwood-Belmont Reach of River Wye

istics to hydraulic characteristics) is highly nonlinear. Routing of a given hydrograph in the equivalent channel can then be modeled by solving the De Saint-Venant equations for these uniform cross-sectional characteristics, over the length of the actual reach, with the given slope S_o (set as the real average slope if known, or as any arbitrary value otherwise).

When applied, for instance to the $\tilde{C}(\bar{Q})$ and $\tilde{\sigma}(\bar{Q})$ input tables corresponding to the regular channel reach already used earlier, this method of hydraulic routing along a synthetic, equivalent channel, for various arbitrary slope values (i.e., equal or not to the true slope) does produce the same routed hydrograph as the original De Saint-Venant solution for the actual slope, geometry, and roughness. The "synthesized" geometry is found to be identical to the actual geometry when and only when the original slope value is used for the equivalent channel reach.

If the MEC method is now applied with constant $\tilde{C}$ and $\tilde{\sigma}$ values for all discharges Q, excellent agreement is found with Hayami's analytical solution to the linear diffusion [see Fig. 7 for the set of values $\tilde{C}(Q_{ref})$ and $\tilde{\sigma}(Q_{ref})$ defined in the previous section; slope S_o is given the arbitrary value 0.001]. Incidentally, this means that some channel geometry can theoretically be found to satisfy any combination of $\tilde{C}(\bar{Q})$ and $\tilde{\sigma}(\bar{Q})$ input functions, including constant $\tilde{C}$ and $\tilde{\sigma}$ coefficients. These results show the value of this MEC method as an alternate solution for nonlinear diffusion modeling. The advantage of this method is that satisfaction of the physical laws of mass and momentum conservation is guaranteed, as they are explicitly formulated by the equations being solved. A drawback, aside from the larger volume of computations to be performed (the system to be solved is twice as big, the numbers of unknowns and differential equations being doubled; small space and time steps may be required, due to rugged shapes of the inferred channel characteristics), is the totally fictitious hydraulic description that is produced, bearing no significance with respect to actual flow in the channel.

Since for a given set of $\tilde{C}(\bar{Q})$ and $\tilde{\sigma}(\bar{Q})$ input functions, the HAND method can be viewed as providing an approximate solution to the more accurate MEC formulation, the validity of the HAND solution can be checked against the MEC solution. This was done for the case of the Wye river described in the previous section. Fig. 8 compares the HAND and MEC solutions (together with the reference hydrograph), for the December 1960 flood. It can be seen that agreement between the two solutions is very good, leading to the conclusion that the HAND method can be confidently used to correctly solve the nonlinear diffusion problem for flood routing. It is probable that the imperfect agreement of both solutions with the reference hydrograph is due to insufficient calibration of the celerity and attenuation functions and to the absence of lateral inflow estimation (these adjustment steps were not considered in the present work), rather than to the actual model formulation and solution method.

CONCLUSION

A new method was devised for modeling flood routing in the very common situation where the diffusive wave analogy, that is, the zero-inertia approximation, is suitable. This method, named HAND, improves the accuracy of the general nonlinear diffusive wave approach, through a modification of the variable-parameter diffusion equation for flow routing that better ensures accounting for the effect of the pressure gradient term on discharge propagation. Increasing model compliance with the fundamental De Saint-Venant equations guarantees that the basic principles of momentum and mass conservation are better satisfied, while still solving for discharge as the only state variable, like with the ordinary diffusive wave models. Hence, the advantages of (1) A potentially large applicability for this method; (2) a simpler model; and (3) reduced data-needs compared to a full De Saint-Venant model, are combined. To keep up with the higher accuracy of this new equation formulation, a precise numerical solution procedure, based on a fractional-step, process-splitting method, was implemented, that is well suited to the form of this nonlinear diffusion-type equation.

The modeling capability of the HAND method was demonstrated both for a hypothetical, regular channel, against a full De Saint-Venant solution for this completely defined hydraulic system, and on an existing natural river reach, with recorded hydrographs but unknown geometry: the Wye river between Erwood and Belmont (U.K.). It was also tested against an alternative, high-accuracy approach, the MEC method, consisting of full hydraulic modeling, by the De Saint-Venant equations, using a fictitious synthetic channel; this channel is designed to be equivalent to the actual river reach as far as the laws of variation of the propagation coefficients with discharge are concerned. In all cases, agreement of the results between the HAND method and the full De Saint-Venant solutions was found to be very good. Improvement over the ordinary variable-parameter approach (VPD) can be very significant, both in terms of replication of reference hydrograph (observed or computed), and of satisfaction of the basic physical principles, especially mass conservation.

It can thus be concluded that the HAND method can be used more reliably for a wider range of propagation problems than other diffusive or kinematic wave methods: close accordance with the physics of open-channel flow provides both a higher capability for modeling propagation dynamics than the linear diffusion and kinematic wave approaches, and safer and more accurate results than the ordinary VPD method, the latter being reasonably applicable under quasi-linear or quasi-kinematic conditions only.

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APPENDIX II. NOTATION

The following symbols are used in this paper:

A = cross-sectional area of flow;
B = channel width at water surface;
B' = dB/dh (derivative of B with respect to h);
C_h = celerity of water depth wave;
C_Q = discharge wave celerity;
$\tilde{C}$ = C_h and C_Q under zero pressure-gradient conditions (kinematic wave celerity);
COR = dimensionless correcting factor, accounting for effect of the pressure gradient;
D = section conveyance;
D' = dD/dh (derivative of D with respect to h);
h = water flow depth;
inv$\tilde{Q}$( ) = inverse function of $\tilde{Q}(h)$ (yields the "normal" water depth for a given discharge value);
K = Strickler's roughness coefficient;
Q = actual discharge;
$\tilde{Q}(h)$ = "normal" discharge for depth h (i.e., under no pressure gradient);
q = algebraic lateral flow rate per unit channel length (positive for inflow);
q_e = $q - (\sigma_Q/C_Q)\partial q/\partial x$ (effective discharge gradient due to lateral flow introduced in the convective term of the diffusive wave equation);
R = hydraulic radius;
S_o = channel bottom slope;
S_f = friction slope (slope of energy line);
t = time;
u = flow velocity;
x = longitudinal coordinate (abscissa along channel reach);
Z = water stage;
σ_h = attenuation parameter for the water depth diffusive wave;
σ_o = attenuation parameter of the diffusive discharge wave; and
$\tilde{\sigma}$ = σ_h and σ_Q simplified to zero pressure-gradient conditions.