A REVISED MATHEMATICAL MODEL OF MAGNETIC RESONANCE SOUNDING

Anatoly Legchenko

BRGM, Development Planning and Naturals Risks Division, 3, avenue C. Guillemin, BP 6009, 45060, Orléans Cedex 2, France. Currently with IRD: 32, avenue Henri Varagnat, 93143, Bondy Cedex, France.

INTRODUCTION

In standard laboratory magnetic resonance experiments, the artificial static magnetic field is perfectly homogeneous throughout the investigated volume. For magnetic resonance sounding (MRS) measurements, the Earth's magnetic field is taken as the static field. In the subsurface, however, this can be locally modified by the rocks and is generally not homogeneous over a large volume. The geomagnetic field may also vary with time. For a typical MRS setup (square loop of 75x75 m), a cube of 100x100x100 m represents the volume investigated by MRS; the duration of one sounding takes from one to several hours. Under these conditions, the geomagnetic field cannot be considered as constant. In the presence of a non-constant geomagnetic field, phase behavior is more complicated than a simple phase shift caused by electrically conductive rocks. This is not taken into account by currently available models and the associated algorithms can thus only use amplitude for interpreting field measurements. Other important characteristics of the MRS signal are longitudinal relaxation time T_1, transverse relaxation time T_2, and the observed relaxation time T_2^*. In porous media, the relaxation times are proportional to the mean pore size. Because of technical difficulties with measuring T_1 and T_2 in large volumes from the surface, only the MRS T_2^* relaxation time, which can be derived from the envelope of the MRS signal, was used initially. Whilst it is known that T_2^* is proportional to T_2, T_2^* is also sensitive to local inhomogeneities in the geomagnetic field caused by rocks, thus rendering parameter T_2^* less reliable than T_1 or T_2 for pore size estimation.

In this paper, an enhanced mathematical model that improves the accuracy of MRS data interpretation is presented.

IMAGING EQUATION

When using the classical model in the coordinate system rotating with an angular frequency $\omega = -\gamma B_0$, the signal induced in the receiver loop is proportional to the sum of the flux of the transverse components of precessing magnetic moments $M_\perp^2 = M_x^2 + M_y^2$. At time

$t = \tau$ after transmitting an alternating magnetic field $\mathbf{B}_1$, the spin magnetization will have the components:

$$M_x = -M_0 \frac{\omega_1 \Delta\omega}{\omega_{eff}^2}\left(1-\cos(\omega_{eff}\tau)\right), \text{ and } M_y = M_0 \frac{\omega_1}{\omega_{eff}}\sin(\omega_{eff}\tau), \qquad (1)$$

with $\Delta\omega = \omega_0 - \omega$ being a frequency offset between the Larmor frequency and the pulse frequency, $\omega_{eff}^2 = \omega_1^2 + \Delta\omega^2$, and $\omega_1 = 0.5\gamma B_1$. When $\Delta\omega = 0$, the M_x component corresponding to the imaginary part of the MRS signal is zero and hence the signal, which is thus proportional only to M_y, is real. Otherwise it is complex. A phase shift of the transmitted magnetic field $\mathbf{B}_1$ caused by the electrical conductivity of the rocks also creates a phase shift of the MRS response. Assuming a coincident transmitting/receiving loop, we can express the phase of the signal generated by volume dV as:

$$\varphi_0 = \tan^{-1}(M_x / M_y) + 2\tan^{-1}(B_{1x} / B_{1y}) = \varphi_{\Delta\omega} + 2\varphi_\rho, \qquad (2)$$

where $\varphi_{\Delta\omega}$ and φ_ρ are the phase shifts due to, respectively, the frequency offset and the electromagnetic shift caused by the electrical conductivity of the rocks.

The spectrum of the transmitting pulse typical for MRS instruments shows that the amplitude of the second harmonic is about 20% of that of the first one, and that frequency offset is about 35 Hz. In deep water irradiated from the surface, the flip angle of spin magnetization is small even for the first harmonic. This means that water in deep aquifers mostly responds to the first harmonic and effects of higher harmonics on the MRS signal can be neglected. In water close to the surface, however, the flip angle caused by higher harmonics is significant and here must be taken into account. Consequently, in the case of exact resonance ($\Delta\omega = 0$) and non-conductive rocks, the MRS signal generated by deep water may be real; for shallow aquifers, however, more than one harmonic of the pulse that has non-zero frequency offset must be considered and hence the signal is always complex.

We assume the spin system to be linear, which makes it possible to calculate the MRS response using the first few harmonics generated by the pulse. Using the reciprocity theorem and taking into account the resistivity distribution and frequency offset, the induced voltage thus becomes (Legchenko et al., 2003):

$$e_0(q) = \omega_0 \int_V \sum_k \left(\frac{B_{1k}(\mathbf{r}) e^{j\varphi_{0k}(\mathbf{r})} M_{\perp k}(\mathbf{r})}{I_{0k}} \right) w(\mathbf{r}) dV(\mathbf{r}), \qquad (3)$$

where $B_{1k}(\mathbf{r})$ is transmitted by the k^{th} harmonic magnetic-field component perpendicular to the geomagnetic field, $0 \le w(\mathbf{r}) \le 1$ is the water content, $q = I_0 \tau$ is the pulse parameter (I_0 and τ are the amplitude and duration of the pulse respectively) and $\mathbf{r} = r(x, y, z)$ is the coordinate vector.

Numerical modeling shows that if the existing simplified model is used, then under certain conditions the neglect of the complex nature of the MRS signal could lead to interpretation errors, such as incorrectly created or incorrectly eliminated aquifers. Complex signals can only be inverted using the enhanced model. Field measurements confirm these results and reveal a good correlation between theoretical signals obtained using the enhanced model and borehole data.

RELAXATION TIME T_1

It was experimentally shown (Legchenko et al., 2002) that in limestone (magnetization $\approx 10^{-4}$ A/m), the signals from both free and capillary-bound water were relatively long ($T_2^* > 70 \div 80$ ms), considering the threshold of the MRS instruments (30 ms). On the contrary, in basaltic gravel (magnetization $\approx 10^{-1}$ A/m), even the signal from free water was very short ($T_2^* \approx 10$ ms) and, therefore, could not be measured with a standard MRS instrument. Thus, the sensitivity to magnetization of the rocks limits the reliability of T_2^* as a parameter for estimating hydraulic conductivity and encourages the application of T_1 or T_2.

The saturation recovery method (Dunn et al., 2002) was adapted to MRS measurements. Two pulses are applied to the investigated volume and, after the first pulse, the spin magnetization **M** of the sample dV is turned off at the angle θ. During the delay τ_p, it builds up towards equilibrium along the geomagnetic field with the time constant T_1. Assuming the spin system to be linear, and neglecting relaxation during the pulse, ($\tau << T_2^*, T_2, T_1$), the perpendicular to the Earth's magnetic field component of the spin magnetization after the second pulse can be described by the equation:

$$M_{\perp}(\tau_p) = M_0 \exp(-\tau_p / T_1) \sin(\theta + \theta_2) + M_0 \left(1 - \exp(-\tau_p / T_1)\right) \sin(\theta_2), \qquad (4)$$

where θ_2 is the flip angle caused by the second pulse. If both pulses are set to be equal ($q_1 = q_2 = q$) and the phase shift between the current of the second pulse is equal to π relative to the current of the first pulse, then $\theta_2 = -\theta$ and Equation 4 can be simplified to:

$$M_{\perp}(\tau_p) = -M_0 \left(1 - \exp(-\tau_p / T_1)\right) \sin(\theta). \qquad (5)$$

For calculating the amplitude of a MRS signal measured after the second pulse, Equation 3, which describes the amplitude after the first pulse will be replaced by:

$$e_{02}(q,\tau_p) = \omega_0 \int_V \sum_k \left(\frac{B_{1k}(\mathbf{r}) e^{j(\varphi_{0k}(\mathbf{r})+\pi)} M_{\perp k}(\mathbf{r},\tau_p)}{I_{0k}} \right) w(\mathbf{r}) x(\mathbf{r}) dV(\mathbf{r}), \tag{6}$$

where $x(\mathbf{r}) = 1 - \exp\left(-\tau_p / T_1(\mathbf{r})\right)$. Resolution of Equation 3 can provide the water content $w(\mathbf{r})$ and then, $T_1(\mathbf{r}) = -\tau_p / \log(1 - x(\mathbf{r}))$ can be derived from Equation 6.

For demonstration, normalized amplitudes measured at two sites versus the delay τ_p are shown in Figure 1. As expected, a longer T_1 was observed at the site where the aquifer has a higher yield.

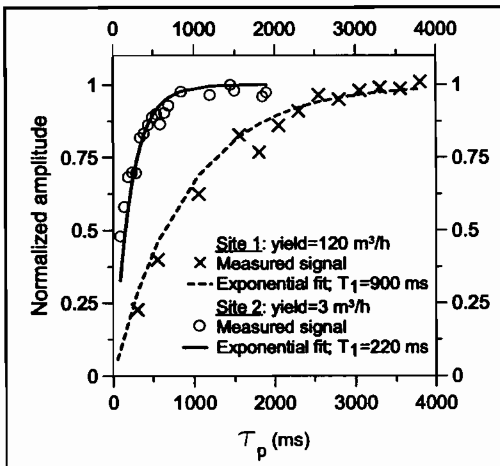

Fig. 1. Example of T_1 measurements.

CONCLUSIONS

Thus, the accuracy and reliability of MRS results can be improved by taking into account the frequency offset between the pulse frequency and the Larmor frequency and the few first harmonics of the pulse for modeling the MRS response, and by replacing measurements of T_2^* by T_1 for estimating hydraulic conductivity.

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