AGL:Misc/72/14
September 1972

(NOTE: The following is an advance summary of a much more comprehensive paper that will be published in the near future under the sponsorship of ORSTOM (Paris); it should not be quoted therefore without the specific approval of the author).

A MATHEMATICAL MODEL FOR CALCULATION OF AGRICULTURAL PRODUCTIVITY IN TERMS OF PARAMETERS OF SOIL AND CLIMATE

by

J. Riquier [1]

Introduction

When a practical interpretation of scientific data in terms of agricultural productivity is wanted there exists a gap in understanding and communication between pedologists and users of soil and between climatologists and agronomists. The majority of natural phenomena are measurable only with difficulty, and where easily measurable (rainfall, temperature) their actions and interactions on the plant are little known and furthermore require to be integrated over time.

As yet it has not been possible to predict the yield of a crop starting from the production factors (physical and chemical properties of soil, climatic elements). Preliminary experimentation is necessary. A fertilizer trial will show the extra yield that can be obtained under specified conditions of soil, crop and weather but this result can only be extrapolated to precisely the same conditions; that is to say virtually never.

The need, therefore, exists to find an equation, albeit approximate, in which the production factors can be introduced in a quantitative form and which will allow:

1. The identification of the causes of major yield fluctuations
2. A better understanding of the mechanism of yield
3. Evaluation of the economic importance of the factors
4. Quantification of agriculture and natural resources

Methods of evaluation of land

The majority of present evaluation methods are subjective. They group soils which have the same agricultural value according to certain criteria often specific to the region concerned. Land classification is generally based on the most limiting factor. Interactions are not taken into account.

Parametric methods aim at quantifying soil and agriculture, including interactions between the factors studied, to reduce the subjectivity of judgements made in using experimental data. A method proposed previously by the author and others [2] which uses a simple mathematical approach has produced interesting results in the prediction of yields and has indicated the possibility of an evaluation which if not exact is at least an approximation of yields to be expected from a given set of physical and chemical properties of soil.

1/ Director of Research ORSTOM (Paris) and Technical Officer, FAO.

2/ Riquier, J., D.H. Bramao, J.P. Cornet. "A new system of soil appraisal in terms of actual and potential productivity (first approximation)". FAO 1970, AGL:TESR/70/6.

WS/D1927

The Methodology of the present model

Parametric methods have, up to now utilized discontinuous expressions for the intensity of each factor. It is more natural and practical to replace this table of index numbers by a continuous curve and to find the equation for it.

The mathematical model proposed may be regarded as a logical follow-up to the methods proposed by Riquier, Bramao and Cornet (1970)[1/]. It takes the following form:

$$Y = 100 \times C \times Y_p \times Y_s \times Y_v \times Y_o \times Y_t \ldots\ldots\ldots\ldots$$

where Y = Yield in percent of the maximum genetically possible for the variety specified under optimum conditions

C = The growth index of Papadakis (1952)[2/] corrected according to climatic, soil and plant requirements

Y_p, Y_s, Y_v, Y_o, Y_t etc. = Yield in percent as a function of selected diagnostic criteria (e.g.: P = effective depth of soil; S = specific surface of soil; V = base saturation of soil; O = % organic matter content of soil; T = % salt content of the soil etc. ....)

Within this model the following general form has been chosen for each of the yield functions:

$$Q = A\,(1 - e^{-ax})$$

where Q = predicted yield

A = maximum genetic yield

e = Naperian logarithmic base

a = coefficient differing for each kind of plant

This form of the Mitscherlich equation has been chosen because:

(a) the graphic curves to which it gives rise are easily related to the majority of experimental results and to parametric indices

(b) it is easy to calculate (powers of 'e' are available in tabular form)

(c) it contains only one coefficient, 'a'

(d) if Q is expressed as percentage of maximum yield then A = 100.

The multifactoral equation thus resembles that due to Baule but is enlarged by factors other than the fertilizer elements. It is of the multiplicative type:

$$Q = A\,(1 - e^{-ax})\,(1 - e^{-by})\,(1 - e^{-cz})$$

For any factor that becomes limiting through excess, a modified Mitscherlich equation is used:

$$Q = A\,(1 - e^{-cx})\,(e^{-kx^2})$$

As described in the following paragraphs, the functions used to determine each of the yield factors in the model are adapted from these general forms through consideration of basic principles and study of experimental data. Although the individual functions

---

1/ Riquier, J., D.H. Bramao, J.P. Cornet. "A new system of soil appraisal in terms of actual and potential productivity (first approximation)". FAO 1970 AGL TESR/70/6.

2/ Agricultural Geography of the World by J. Papadakis, Buenos Aires, 1952, P.43, edited by the author, Nahuel Huapi 4984 B.A. Argentine.

may be complex, each gives rise to a family of graphic curves that reflect the influence of the factor in question (other factors considered to be optimum) on the yield of individual kinds of plant, or on groups of related plant species or varieties. Once the curve for a given variety is established empirically, the curve can be used to estimate a value for the factor in question in the overall model for areas in which only basic physical data is available. For example, once a curve relating effective soil depth to the yield of a given plant variety has been established (other factors being considered optimum) a value for Y_p in the model can be read off the appropriate graph for all other areas where the effective soil depth is known.

Index of the climatic influence on growth

This index (shown as C in the model) is based on an empirical equation proposed by Papadakis in 1952 (loc. cit):

$$C = \frac{h}{12}\left(\frac{8H^2\,T_1^{\,2}}{(1+4H^2)\,(20^2+T_1^{\,2})+10^{-6}(1+2H)\,T_1^{\,6}}\right)$$

where
- h = day length in hours
- T_1 = mean temperature in C^o (corrected in relation to the thermal needs of the plant concerned).
- H = an index of humidity calculated from the hydraulic balance sheet of the soil by a method analogous to that of Thornthwaite using current climatic data = $\frac{\text{available soil water (AVW)}}{\text{real maximum evapotranspiration (ETRM)}}$ 1/

By taking H = 1 and h = 12 a graph for growth as a function of temperature is obtained having a form fully comparable with those obtained experimentally. With T_1 = constant and h = 12 the variation in growth index as a function of humidity is obtained which also compares favourably with the experimental evidence except in that part of the curve which corresponds to an excess of water. The depressive effect of excess water is taken into consideration in relation to soil texture rather than climate in the model as a whole.

An additional factor of light intensity is taken into account when special circumstances make this important - for crops grown under shade, for example, or where light conditions are generally poor as on the northern aspect of mountains in the northern hemisphere. In these circumstances the factor C is multiplied by:

$$1 - e^{-Lr}$$

where r = radiation in cal/cm^2/day
and L = a coefficient varying from 1 for trees and shade plants to 0.25 for wheat, sugarcane and other light-loving species.

A correction is made to the temperature factor in the Papadakis' equation to reflect the climatic requirements of individual kinds of plants. For each plant variety there is a critical minimum temperature (θ_m) below which growth does not take place, an optimum growth temperature (θ_o) and a critical maximum temperature above which the plant dies (θ_M)2/.

1/ ETRM = C_m x E
where E = potential evapotranspiration (Penman, Turc, Papadakis, etc.)
C_m = coefficient varying from month to month depending on the phenological development of the plant.

2/ For wheat : θ_m = 2°C; θ_o = 20°C; M = 34°C;
For cotton : θ_m = 15°C; θ_o = 25°C; M = 38°C.

T_1 is obtained by a linear correction to T (the mean temperature for the period in question) according to its displacement from these critical temperatures using the following equations:

If T less than θ_o, $T_1 = T \frac{(T - \theta_m)}{(\theta_o - \theta_m)}$

If T more than θ_o, $T_1 = T \cdot \frac{(\theta M - T)}{(\theta M - \theta_o)}$

A further modification of Papadakis' original formulae required for the model is the expression of all indices of C as a percentage of the maximum obtainable index for the given plant. The latter can be calculated for substituting optimum values of T_1 and H in the equation (θ_o and 3 respectively).

The data required to calculate this climatic growth index are monthly figures for rainfall and for mean maximum and minimum temperature, the latitude, the soil texture and the critical temperatures (thermal growth limits) of the plant in question. The actual process of calculation, can be greatly simplified using tables of the kind prepared by Papadakis.

A calculated growth index approaching zero indicates that the moisture and temperature requirements of the plant will not be satisfied. Indeed, months which yield a value less than 20 must be considered unsuitable for plant growth. This permits the growing season to be calculated if it was previously unknown.

The mean value of monthly indices for the growing period provides an approximate measure of the maximum possible yield under the prevailing climate given optimum soil conditions. The index provides a useful measure of vegetative production from forests and pastures. It is less reliable for plants such as cereals in which yield can be significantly influenced by a dessicating wind or excessive rain during the short but critical flowering period.

It is emphasized that, as yet, the method does not take account of photo periodism or diurnal temperature change. Yield is assumed to be proportional to the state of plant development and to the amount of vegetative growth. This is not always the case.

It is proposed to weight the monthly indices by a correction for the phenological development of the plant instead of using a simple mean of monthly indices during the growing period. It is quite practical to calculate the index for weekly or ten day periods.

Yield as a function of soil depth

The function takes the following form:

$$Y_p = 1 - e^{-aP}$$

where Y_p = yield as a percent of the genetic maximum;
a^p = rooting coefficient for the given kind of plant;
P = effective depth of the soil.

The curves obtained with this function are considered to represent variations in yield in relation to effective depth of the soil (other growth factors being optimum). In reality they are "rooting curves" showing the percentage/weight distribution of roots according to depth. It is assumed that yield is lowered by a percentage proportional to the amount by which the roots cannot reach their normal distribution of development with depth due to the presence of an impenetrable layer. The shape of the "rooting curves" and thus the value of the coefficient 'a' is determined by observation of the normal rooting distribution of the plant in question. Since these will vary slightly according to soil humidity and texture it is clearly preferable, where possible, to determine these values *in situ*.

Extreme values are: a = 0.30 for prairie grasses growing over a high water table.
a = 0.075 for Araucaria, Oaks, etc.

Making the further assumption that each horizon of a soil assists productivity to an extent proportionate to its share of plant roots, the "rooting curves" can be very useful for integrating a complex profile with varying horizons. For example, consider the significance of base saturation on the growth of barley in a soil having the following characteristics:

Horizon	0– 30 cm	Base saturation		V_{0-30}	= 80%
"	30– 50 cm	"	"	V_{30-50}	= 65%
"	50–100 cm	"	"	V_{50-100}	= 50%

Barley is found to have a rooting coefficient 'a' of 0.03 and from the corresponding rooting curve the expected root distribution in these horizons may be determined:

Horizon	0– 30 cm	proportion of total barley roots	= 58%
"	30– 50 cm	" " " " "	= 18%
"	50–100 cm	" " " " "	= 18%

A weighted value for base saturation in the top metre of this soil with respect to barley can then be calculated:

$$V_{0-100} = \frac{80 \times 58 + 65 \times 18 + 50 \times 18}{100} = 69.4\%$$

This value for base saturation would then be used for calculating the value Y_v (yield percentage as a function of base saturation, see later).

Where soil depth is limited by a water table[1/] the capillary fringe must be taken into consideration. For purposes of calculation the measured depth to the water table (P_1) must be decreased by 20 cm for sands, 30 cm for clays and 80 cm for silts (i.e. $P = P_1 - 20$ in a sandy soil with high water table).

Yield as a function of specific surface of the soil

The function takes the following form:

$$Y_s = (1 - e^{-bS}) (e^{-c(S/1\,000)^2}).$$

where Y_s = yield in percent of the genetic maximum
b = coefficient of response of a given plant to soil texture
S = specific surface of the soil in M^2/gram.
c = coefficient complex (a function relating soil structural stability, climate and plant requirements).

Soil texture is known to influence production through its influence on water retention, on the capacity for retaining and exchanging nutrients, the aeration of the root system and in other ways. Much of this influence relates to differences in the specific surface of the soil which can be measured or calculated:

1/ If a water table is present close to the soil surface, the water supply may be regarded as constant and the value H = 3.0 used in calculating the climatic growth index.

$$S = \text{clay \%}\ (0.10\ (\text{exchange capacity of the clay in m.eq/100 grams}) + 0.5) + \text{silt \%} \times 0.21 + \text{fine sand \%} \times 0.021 + \text{coarse sand \%} \times 0.002.$$

(NOTE: in this equation clay, silt, fine sand and coarse sand are expressed as percentages of the total soil not of the fine earth fraction - account thus being taken of the coarser particles, stones, rocks, etc. in the soil).

Inclusion of exchange capacity in these equations provides a correction for differing kinds of clay.

The coefficient 'b' is derived empirically and reflects the superior performance of certain plants on soils having certain ranges of texture. Thus, in order of preference for increasing clay content:

rye, asparagus, groundnuts, potatoes	b = 0.1
the majority of plants	b = 0.05
rice, cotton, pasture	b = 0.02

The coefficient complex 'c' is included to provide corrections for poor aeration and other adverse effects relating to soil structure and soil moisture relationships.

$$c = 10\ kt\ (\log_{10} 10\ Is)$$

where k = a coefficient of plant susceptibility to lack of root aeration ($k = 0$ for paddy rice; $k = 1$ for the majority of plants; $k = 2$ for very susceptible plants such as cotton, bananas, tobacco and tomatoes.)

t = a climatic coefficient related to excess soil water. (t is equal to the maximum value during the growing season of the index of humidity H_{max}, used previously, if the latter is greater than unity. Otherwise $t = 2 - H_{max}$).

$\log_{10} 10Is$ = the logarithm to base ten of ten times the value of Henin's index of soil structural instability - a measure of the strength or rather the weakness of soil structural aggregation when the soil is wetted.

A graphic relationship has been very tentatively established which permits substitution of permeability measurements for the Henin instability index where the latter has not been measured.

Yield as a function of base saturation in the soil

This function which takes account of the chemical fertility of the soil takes the following form:

$$Y_v = 1 - e^{-dV}$$

where Y_v = yield in percent of the genetic maximum

d = plant coefficient reflecting response to levels of base saturation and pH range.

V = percentage base saturation = $\dfrac{\text{Content of exchangeable bases}}{\text{Base exchange capacity}} \times 100$

The function gives rise to a family of curves relating base saturation to percentage of maximum yield, the exact shape of each curve depending on the coefficient 'd' which is determined empirically. Broadly speaking, for cultivated crops $d = 0.03$, for pastures $d = 0.05$ and for forest $d = 0.10$. It is better, however, to obtain values specific to the crop in question. Heather, for example, grows in infertile acid soils - $d = 0.10$ and the same value can be used for oil palm, tea and coffee. For groundnuts, a more demanding plant $d = 0.05$ and for tobacco and maize $d = 0.03$.

Yield as a function of organic matter content of the soil

This function takes the form

$$Y_o = 1 - e^{-fO}$$

where Y_o = yield in percent of the genetic maximum
O = percentage organic matter content in the soil
f = coefficient of efficacy of organic matter.

The influence of humus and organic matter on plant growth is complex and varies according to the type of organic matter and its degree of decomposition as reflected by the carbon/nitrogen ratio (C/N). Of special significance to productivity is the magnitude of the nitrogen supply provided by O.M. decomposition. It is this factor which is mainly reflected in the coefficient 'f'.

$$f = n \times \frac{15}{C/N}$$

where n = a coefficient relating to the nitrogen requirement of a particular kind of plant.

For nodulated legumes a value of n = 1 fits the observed facts; n = 0.70 for most plants including vines, rice, wheat and cotton but, for plants with a high nitrogen requirement such as cereals, forage plants, cabbages, tomato, cocoa and sugarcane n = 0.50.

Yield as a function of soil salinity

This function takes the form:

$$Y_t = e^{-(gT - 0.2)}$$

where Y_t = yield in percent of the genetic maximum
T = percentage salt content of the soil
g = coefficient of plant susceptibility to salt toxicity and soil texture.

The correction for salinity takes account of varying plant susceptibility in interaction with soil texture and the presence of sodium carbonate (Na_2CO_3). Values for the coefficients 'g' derived empirically can be set out as follows:

Sodium Carbonate negligible	Texture	g	Sodium Carbonate present
Plants very resistant to salinity:	Clay	g = 0.8	
	Silt	g = 1.0	
	Sand	g = 1.5	
Ordinary plants:	Clay	g = 1.5	Very resistant plants
	Silt	g = 2.0	
	Sand	g = 3.0	
Plants very susceptible to salinity:	Clay	g = 3.0	Ordinary plants
	Silt	g = 4.0	
	Sand	g = 5.0	

Other factors affecting productivity

A number of other factors may need to be considered in local circumstances. These include sulphur toxicity; excess of lime or gypsum; deficiency of micro-nutrient elements; aluminium toxicity and even parasites and predators. Each may require to be evaluated on the basis of the percentage reduction in yield to which it will give rise multiplied together with the other partial yield factors.

1. Fertilizers

Calculation of the influence of fertilizer applications on productivity presents special problems since this is strongly affected by short term weather effects and does not conform, therefore, to the time scale in years applicable to other aspects of the general hypothesis.

If a factor reflecting fertilizer application is indispensable to the general equation and, in particular, if it is desired to show the relation between fertilizer and other properties of soil and climate, an attempt must be made to evaluate it although it will be very difficult.

Although open to criticism, the Baule equation is presently used for this purpose:

$$Y_{NPK} = Y_{max} \; (1 - e^{aN}) \; (1 - e^{bP}) \; (1 - e^{cK})$$

In the equation, the values of N, P and K are the Kg/ha of the nutrients applied as fertilizer added to the content of these nutrients already present in available form in the soil. By suitable adjustment of the scale of the abscissa a single curve can suffice to relate all three elements to the percentage of maximum yield.

2. Erosion

The soil loss factor has not been studied for two reasons:

a) it occurs more in connection with management than as part of soil productivity. In fact when a loss starts certain remedial measures can be taken to offset the effect.
b) the loss involved in erosion also involves loss of OM, of exchange capacity, of a top horizon rich in bases, and the destruction of the structure. All these alterations to the soil properties must be taken into account in making a calculation of productivity even at the time of measurement.

On the other hand if the loss of soil is a long continuing one the estimation of soil potential is difficult since there is an acceleration year by year in the loss of (beneficial) properties.

3. Slope

Slope also affects the water balance sheet and consequently the growth index C for it involves surface flow (run off and wash) and less water penetrating the soil. The coefficient of run off is a function of the soil cover and of the soil permeability. Its evaluation is possible but the biggest problem to be resolved is the cumulative effect of the surface flow which noticeably modifies the water balance sheet at the bottom of slopes or in depressions. It cannot be evaluated without knowing the receiving surface, the area affected by wash, the length of the slope etc. This problem is most important in the semi-arid areas.

4. Management

The proposed mathematical model permits the calculation of a potential productivity following significant managerial improvements (irrigation, drainage, etc.). It is only necessary to recompute using the improved factors (modified by the improvements). For example, irrigation raises the humidity index from x to 3 (the optimum value) in the calculation of the climatic index, drainage increases the depth P, green manure produces a 0.5% increase in the organic matter factor O and so on.

Conclusions

This method of calculating productivity is simple for it is sufficient to carry out multiplication of percentages, all derived from graphs (apart from that for climate which takes a little longer). Electronic computation saves time and allows plant coefficients to be deduced from statistical calculations.

The principle aims of the approach may be summarized as:

1. To compare different soils for a specific crop in the same climate;
2. To establish the capacity of a soil for different crops. The crop showing the highest yield is obviously the most desirable if economic factors are also favourable;
3. To study the influence of a single factor, e.g.: fertilizer. The action of other factors having been established through calculation, the factor being studied can be better understood;
4. To study the productivity of the same soil in different latitudes and to eliminate the climatic factor;
5. To interpret pedological and climatic observations in terms of productivity of a crop.

Knowledge of yield is indispensable for the economic evaluation of land. It constitutes the profit from agriculture the expenses arising from cultivation, management, improvement practices. The possibilities of profit making and the costs of management are dependent upon the physical and chemical properties of soil and upon climate and thus can be evaluated by a parametric method but this will be dealt with subsequently.