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Analysis of regional precipitation patterns in West Africa based on the law of leaks.

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SUMMARY. The description of precipitation patterns over a given region,is usually based on the statistical analysis of point series of rainfall data accumulated over a month or a year. Fitting the best possible statistical model to a given series generally leads to a lack of coherence in the spatial organization of the selected models and to discrepancies between the statistical parameters computed for different periods of accumulation. Furthermore it is difficult to conceive a relationship between these parameters and the underlying climatological events. It is nevertheless believed that such relationships may be found if a suitable statistical model of the rainfall distribution is used. The law of leaks is one among a few such models. The scale parameter of this law characterizes thedistribution of the event point rainfall depths, while the shape parameter is related to the distribution of the time between two consecutive rainfall events For rainfall data collected from events of the same meteorological origin,rainfall depths accumulated over any period are distributed following a law of leaks with a constant scale parameter, while the shape parameter is proportional to the length of the period. Assuming that the law of leaks is a valid model of a point rainfall distribution for a given range of durations, it is then easy to deduce the distribution parameters for other durations from the parameters estimated by fitting it to the total rainfall depths of a given reference duration. This method is illustrated through the study of the monthly rainfall depths measured at 43 stations in Benin over the past 40 years. The results are in good agreement with the expected values,leading us to undertake a similar study over the whole West African subcontinent.

1. Introduction

Regional rainfall frequency analysis is increasingly used to describe climatic patterns, and as a basis for water resource management studies. In contrast to point rainfall frequency analysis, the spatial coherence required by regional analysis leads to put less emphasis on the best possible fit for a given series and requires a distribution model valid for the whole area of study, with parameters varying only to account for the differences between measurement stations. While it may be relatively easy to find such a model by any numerical optimization (e.g. a 3 parameter gamma or beta function), the physical significance of the obtained parameters is often unclear. The problem is further complicated if a model fitted to data for a given period of accumulation (e.g. one month), is to be extended to other periods of accumulation (e.g. one day).This is of special interest in Sahelian countrys where monthly data are generally reliable, which is not the case for daily data. However in terms of climatic knowledge and agricultural needs, it is the frequency analysis of one to fifteen day data which is the most relevant. It is thus a major concern to be able to derive the distribution of rainfall for periods smaller than one month from monthly distributions.

Deeply involved in the study of West African rainfall regimes, ORSTOM scientists were thus led to develop a distribution model allowing extrapolation both in space and in time (period of accumulation) : this distribution is the law of leaks, named after its utilisation in the study of the distribution of gas leakage from gaz pipes by Morlat (cited in Babusiaux, 1969).

2. The law of leaks : a versatile rainfall distribution model.

Presentation of the law of leaks.

The law of leaks originates from the renewal theory (e.g. Cox, 1964), the purpose of which is to study the statistical distribution of the values of chronological events, the duration of the events and the time between two consecutive events being considered as random variables (R.V). To our knowledge it first appeared in the literature in a publication by Einstein(1937) as a special case of Compound Poisson Processes.

Let Y be a R.V. exponentially distributed :

$$f(Y) = 1/s \,.\, e(-Y/s) \qquad (1)$$

where f is the probability density function (pdf) and s is the expectation of Y :

$$E(Y) = s \quad and \quad E(Y-E(Y))^2 = s^2.$$

Let N be the number of occurences of Y over a given period T. N is a R.V. which is assumed Poisson distributed :

$$P(N) = e^{-\lambda} \,.\, \lambda^N / N! \qquad (2)$$

where λ is the mean number of occurences.

We are now interested in studying the distribution of Y_T , which is obtained by accumulating all the realizations of Y over the period T. For a given number of realizations, n , the scaled variate :

$$U = 1/s \,.\, Y_T$$

is Pearson III distributed, with pdf :

$$f(U) = e^{-U} \,.\, U^{n-1} / [s \,.\, (n-1)!]$$

The marginal distribution of U, letting n vary from one to infinity is then :

$$f(U) = \sum_{n=1}^{\infty} e^{-\lambda} \,.\, \lambda^n / [(n!).\,(n-1)!] \,.\, u^{n-1} \,.\, e^{-U}$$

or

$$f(U) = e^{-\lambda} \,.\, \lambda \,.\, e^{-U} \,.\, \sum_{n=0}^{\infty} [\,\lambda u]^n / [n!\,(n+1)!]$$

let

$$J = \sum_{n=0}^{\infty} [\,\lambda u]^n / [n!\,(n+1)!]$$

then J may be written :

$$J = I_1(2\sqrt{\lambda U}) / [\sqrt{\lambda U}]$$

where $I_1\,(2\sqrt{\lambda u})$ is the modified first order Bessel function. The distribution of Y_T is thus given by :

$$f(U) = e^{-\lambda} . \lambda . e^{-U} . I_1(2\sqrt{\lambda U}) / [\sqrt{\lambda U}] \quad \text{for } U>0 \qquad (3)$$

$$\text{and} \quad F(o) = F0 = e^{-\lambda}$$

$$\text{with} \quad U = Y_T / s.$$

As can be seen in figure 1, this pdf is J shaped for <1, and tends towards a Gaussian pdf as λ approaches infinity.

Using the characterizing function :

$$\phi(t) = Fo . e^{it0} + \int_0^\infty e^{itx} . f(x)\,dx$$

Babusiaux(1969) has shown that the first three moments of the law of leaks are :

$$K_1 = E(Y) = \lambda s$$

$$K_2 = E[(Y - E(Y))^2] = 2\lambda s^2$$

$$K_3 = E[(Y - E(Y))^3] = 6\lambda s^3$$

and thus the coefficient of variation (CV) and the skewness coefficient :

$$CV = [K_2]^{1/2} / K_1 = [2/\lambda]^{1/2}$$

$$\gamma_1 = K_3 / [K_2]^{3/2} = 3/[2\lambda]^{1/2}$$

It is easy to see that for a given CV, the leak distribution is more symetric($\gamma_1 / CV = 3/2$) than the Pearson III distribution ($\gamma_1 / CV = 2$).

Several methods were developed by Babusiaux(1969) to fit the leak distribution. The maximum likelihood method was derived later by Ribstein (1983) and proved more effective.

Application to rainfall distributions.

It is widely acknowledged that the distribution of individual events is approximately exponential (Lebel and Guillot,1983), the difficulty being to identify what an individual rainfall event is. Denoting this elementary rainfall event as Y, its pdf is given by equation (1). Further assuming that for large durations of accumulation ($T =$ one month for instance), the total rainfall depth results from the sum of N independent realizations of Y, and that N is a Poisson distributed R.V., the total rainfall has a leak distribution, its scale parameter being the average amount of rainfall produced by an elementary event and its shape parameter N being the average number of elementary events over the period . It is then theoretically easy to deduce the distribution of rainfall over a period T' from the leak distribution fitted to the period data : the scale parameter remains constant while the shape parameter may be computed as :

$$N_{T'} = N_T . T'/T \qquad (4)$$

This may prove useful when daily data are unfit for processing due to missing observations which are subsequently accumulated over a longer period. Furthermore the fitting of a leak distribution to the time series available ove a whole region gives physical significance to the spatial variations of the parameters : an increase (or decrease) in average total precipitation can be explained either by an increase (or decrease) in the average number of events or by an increase (or decrease) of the average rainfall depth delivered by an elementary event.

3. Rainfall related characteristics of the West African climate.

In the early days of the science of meteorology tropical meteorology was the most advanced branch. The first general circulation model designed by Hadley in 1735 was anchored in a description of the trade winds regime. During the first half of our century, on the initial impetus of the Norwegian school, the meteorology (and thus its associated science, climatology) of temperate regions saw remarkable progress, while the meteorology of tropical regions stagnated. Insufficient knowledge in these regions has been since recognized as a major impediment to further improvement of General Circulation Models and climatology in general (e.g. Newell and Kidson, 1979), leading the Global Atmospheric Research Programme (GARP) of the WMO to instigate several experiments in tropical countries (GATE and WAMEX among others). These experiments have provided the scientific community with a better understanding of the synoptic, mesoscale and local meteorological processes associated with the wet season precipitations in West Africa(June to September). It was first confirmed (e.g. Barnes and Sieckman, 1984) that the most significant phenomenon associated withw rainfall production was the squall line. Squall lines are westward travelling disturbances with very active convection. They appear as large and dark cloud fronts to ground observers and can remain organized for more than 3000 km and travel at an average speed of 40 to 70 km/h (ASECNA, 1976). They display a discontinuous activity, the cells of maximum rainfall being separated by zones of very low or even zero rainfall. It seems that the travelling speed of the squall lines must be related to the maximum rainfall intensities rather than to the total amount of rainfall. Even though there is not yet any satisfactory description of their interaction with the general circulation, the PREGATE-ASECNA experiment of 1973, followed by numerous studies of cloud clusters using satellite imagery allowed Desbois et al.(1988) to give a fairly "objective" definition of a squall line as viewed by satellite :

"- sharp edge on the west side
- persistence during at least 12 hours
- independence from the cloudy environment (clear area ahead of the line)."

Ground measurements show that at a given station the rainfall lasts a few hours The most significant rainfalls over West Africa, north of the 1200 mm yearly rainfall isoline thus display a marked intermittent character at the daily scale, and thus fall in the category of random process described in section 2.Nowhere is a good statistical modelling of rainfall distributions more needed than in Sahelian Africa. This region is highly dependent on surface water for its agricultural development but a large number of streams are not perennial. The crops are thus grown during the rainy season and are very sensitive to any variation of the climate, whether in space or in time.

4. The Benin case study.

As a first test of the applicability of the leak distribution as a model of monthly rainfall distribution, the rainfall of Benin was chosen. This country is not representative of the region, since the south is under the influence of the ocean and a mountain range is present in the center. The variations of the yearly rainfall are thus not as meridionally organized as the are further north (for more details see Le Barbe and Ale, 1989).

Forty three stations are available over the period 1950-1985. The calendar months were chosen to define twelve different populations, each represented by a sample of 36 values. The lengths of a few series are much longer (up to 60 years for Cotonou). The study of these series showed no major differences in the average distribution between the period 1950-1985 and the longer periods This may be related to the fact that the fifty's were wetter than normal, while since 1968, the drought has been severe.

The leak distribution generally fits the experimental distribution well, especially in the middle of the rainy season (July, August and September), as can be seen from figure 2. The parameters of the leak distribution were therefore mapped. The maps shown in figure 3, display a spatial organisation of these parameters, in agreement with prior knowledge. In particular the specific behavior of the stations bordering the Gulf of Guinea is striking. Over the rest of the country, the shape parameter isolines are roughly organised along latitudes. The organisation of the scale parameter isolines reveals two regions : 1) the south where its value decreases northward, which corresponds to the decreasing influence of the ocean and, 2) the north where no organisation is apparent, which seems to mean that the average magnitude of an elementary event is roughly constant. In addition the variations of the mean monthly rainfall from March to October are correlated with the variations of the shape parameter (number of events), while the scale parameter remains roughly constant.

For a few stations the parameters of the five and ten day rainfall distribution were computed from the monthly parameters, the scale parameter remaining constant ($s_5 = s_{10} = s_{30}$) and the shape parameter being obtained using equation 4 with $T = 30$ and $T' = 5$ or 10. The leak distributions so obtained were plotted along with the corresponding experimental distributions and figure 4 shows that using this procedure, one is able to compute a theoretical distribution that fits the experimental distribution relatively well.

5. Conclusion

Regional rainfall frequency analysis is a cornerstone of water resource management in West Africa, and especially in the assessment of its agricultural potentatial. The leak distribution, one parameter of which (the scale parameter s) is the average rainfall depth of the elementary rainfall event and the other (the shape parameter N) is the average number of events over a given period, provides a good fit for the 43 experimental point monthly distributions available over the period 1950-1985 in Benin. The physical significance of the parameters of the leak distribution allows an interpretation of their spatial variations from a climatological point of view. The main point resulting from this analysis are the following :

- the average number of events increases regularly from the south to the north, for all four months of the rainy season.

- the spatial organisation of the scale parameter allows to divide the area into two regions : 1) the south, under the influence of the Gulf of Guinea with a progressive decrease of the scale parameter from the south to the north; 2) the north, where the scale parameter varies more or less randomly, showing that the meridional rainfall gradient is primarily caused by a variation of the number of events rather than that of the average depth of the rainfall events.

- the end of the rainy season in the north is related to a sharp decrease of the number of rainfall events in October, the average depth of the rainfall events remaining almost constant from March to October.

- the leak distribution allows direct computation of the parameters of the five and ten day rainfall distributions, using the parameters of the leak distribution fitted to monthly data. This is of particular interest when monthly data are available and data over smaller durations are of questionable reliability.

This work is the first step of a larger study of the regional precipitation patterns over West Africa. The suitability of the leak distribution for such a purpose has been established for Benin and will have to be confirmed on a broader scale. This second phase is in progress and has already produced some interesting results.

References

ASECNA (B.E), 1976. Analyse de lignes de grains dans la boucle du Niger. La meteorologie, VIeme serie, 6, 183-189.

Babusiaux C., 1969. Etude statistique de la loi des fuites. Thesis prepared at the Paris University of Science.

Barnes G.M., and K. Siekman, 1984. The environment of fast and slow moving tropical mesoscale convective cloud lines. Monthly Weather Review, 104,1029-103

Cox D.R., 1964. Renewal theory. Ed. Methuen. French edition : Dunod. Paris

Desbois M, T. Kayiranga, B. Gnamien, S. Guessous, and L. Picon, 1988. Characterization of some elements of the sahelian climate and their interannual variations for July 83, 84 and 85 from the analysis of Meteostat ISCCP data. To appear in Journal of Applied Meteorology.

Einstein H.A., 1937. Der geschiebetrieb als wahrscheinlichkeitsproblem, mitteilung der verschsanstalt fur wasserbau,an der Eidgenossische Technische Hochschule in Zurich, Verlag Rascher & Co., 110 p. Translation into english by W. W. Sayre, Bed load transport as a probability problem, Sedimentation, H.W. Shen ed.. Fort Collins, Colorado. 1972.

Le Barbe L., and G. Ale, 1989. Monographie des ressources en eau de surface du Benin. ORSTOM-Direction de l'Hydraulique du Benin. Ed. ORSTOM. Paris.

Lebel T., and P. Guillot, 1983. Statistical analysis of point and areal hourly rainfalls. Vth International Conference on Hydrometeorology. October 1983. Tulsa. OK. AMS proceedings pp 17-25.

Newell R.E., and J.W. Kidson, 1979. The tropospheric circulation over Africa an its relations to the global tropospheric circulation. Saharan Dust, C. Morales, Ed., Wiley and sons, 133-169.

Ribstein P., 1983. Loi des fuites. Cahiers ORSTOM, serie hydrologique, 20(2), 117-145.

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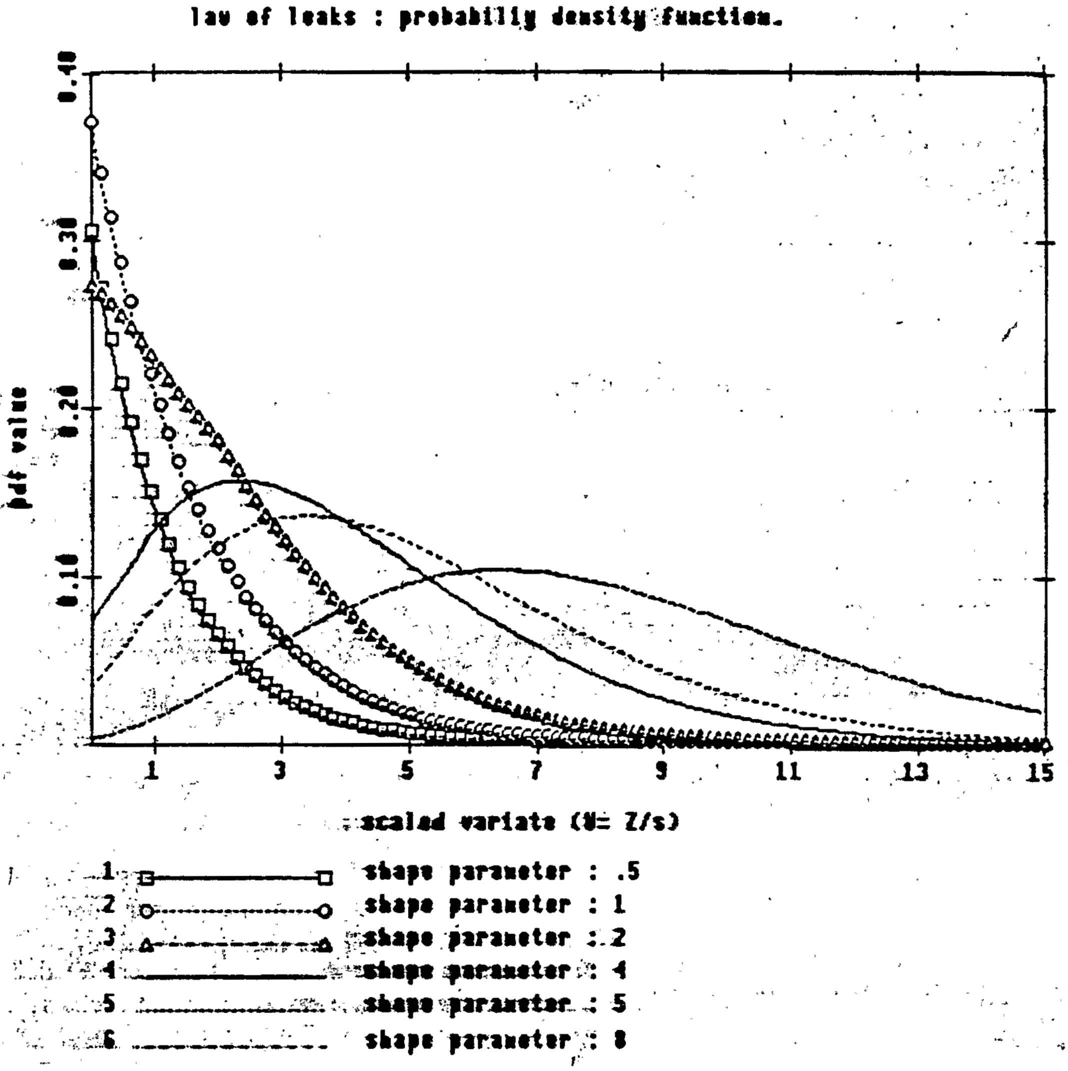
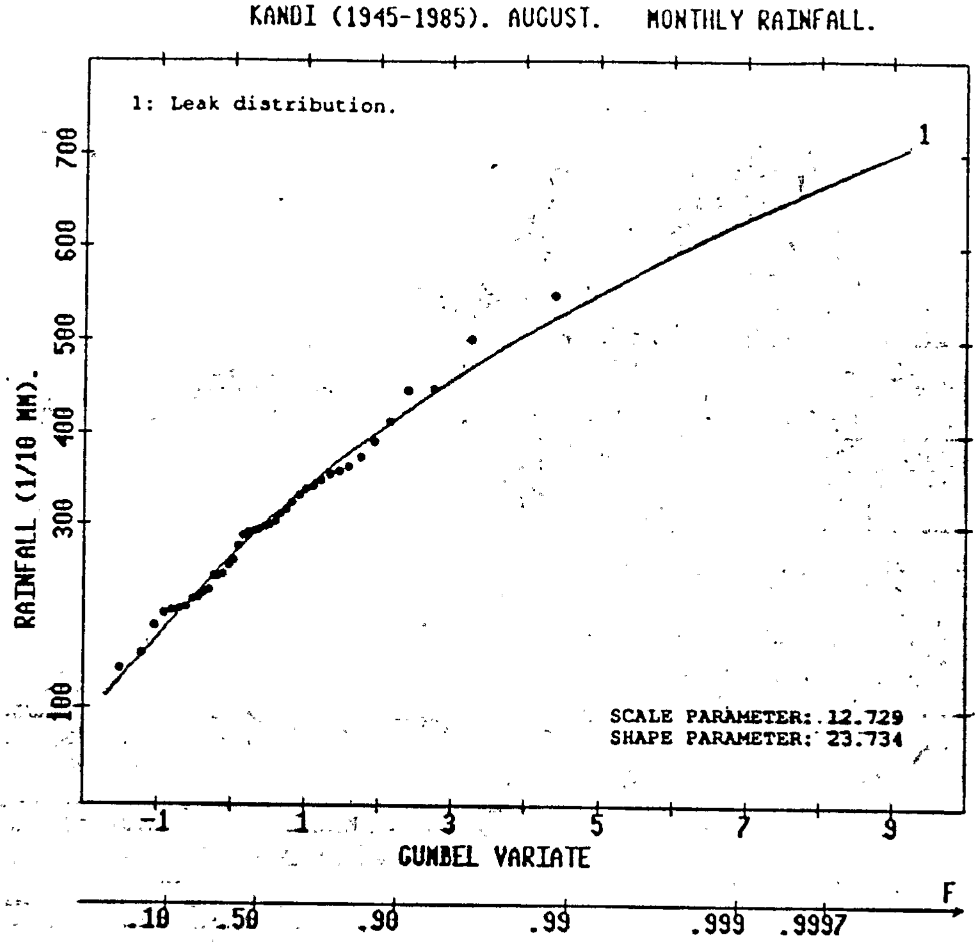


Figure 1. Probability density function of the leak distribution for various shape paramaters.



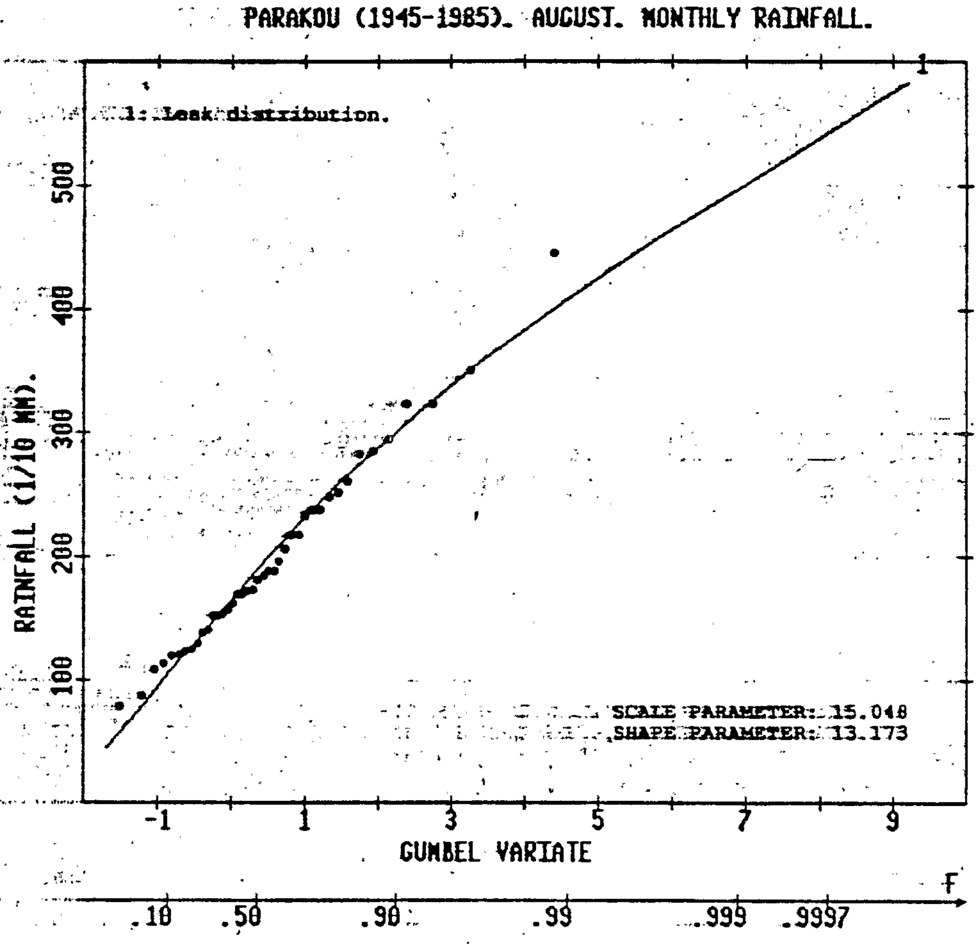
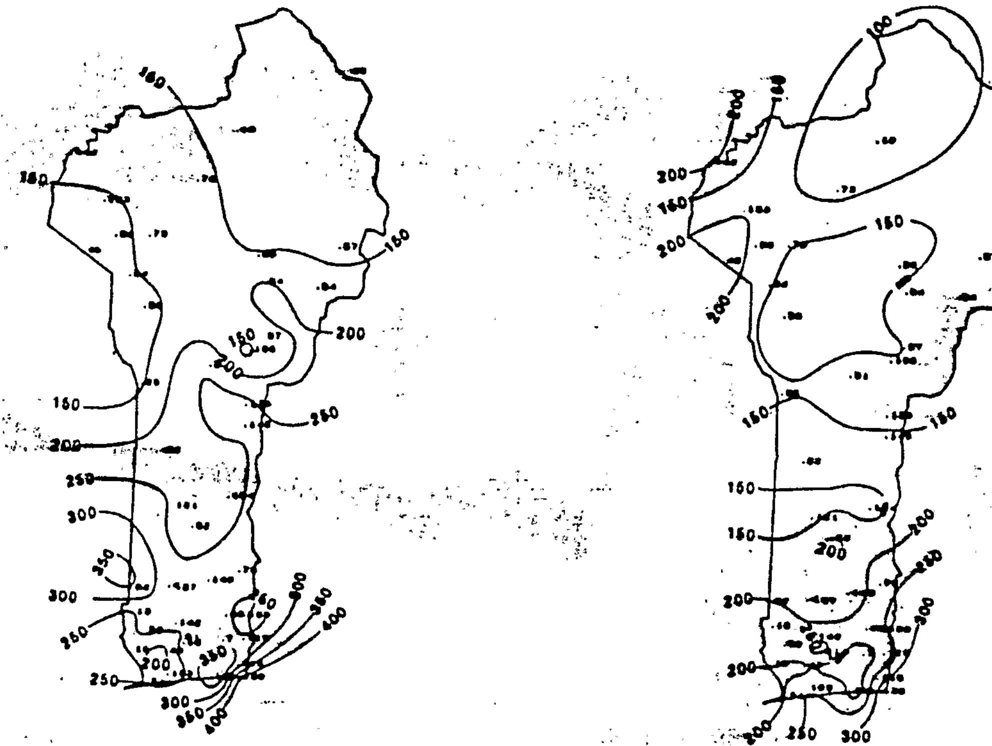


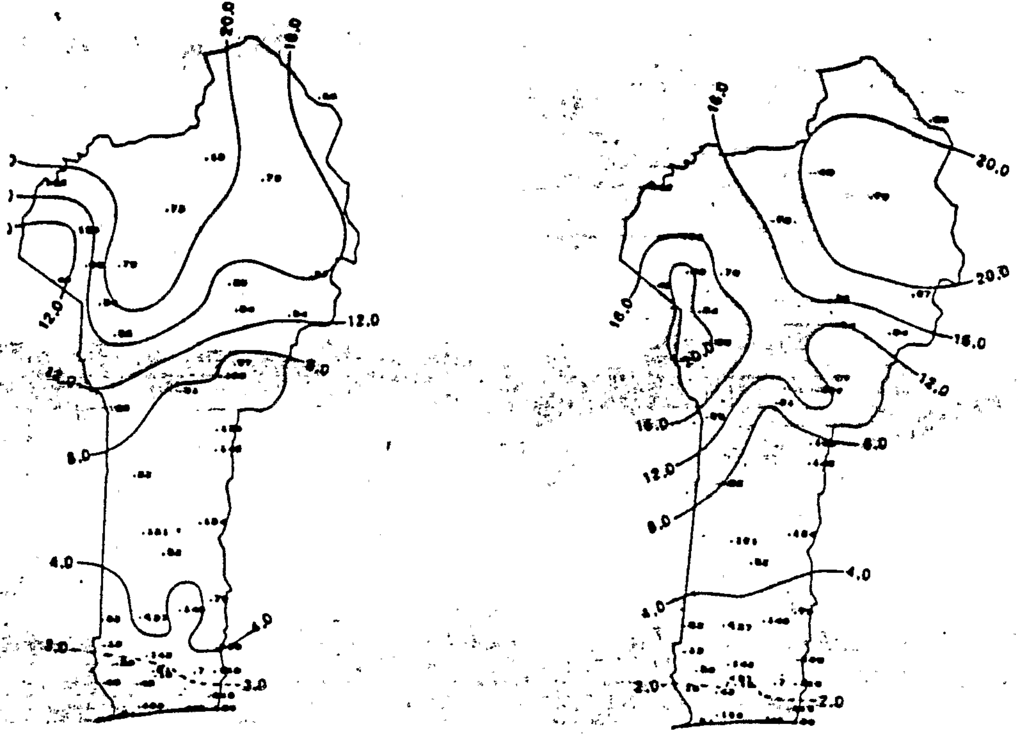
Figure 2. The leak distribution fitted to monthly data.



SCALE PARAMETER
(1/10 mm)

AUGUST

SEPTEMBER



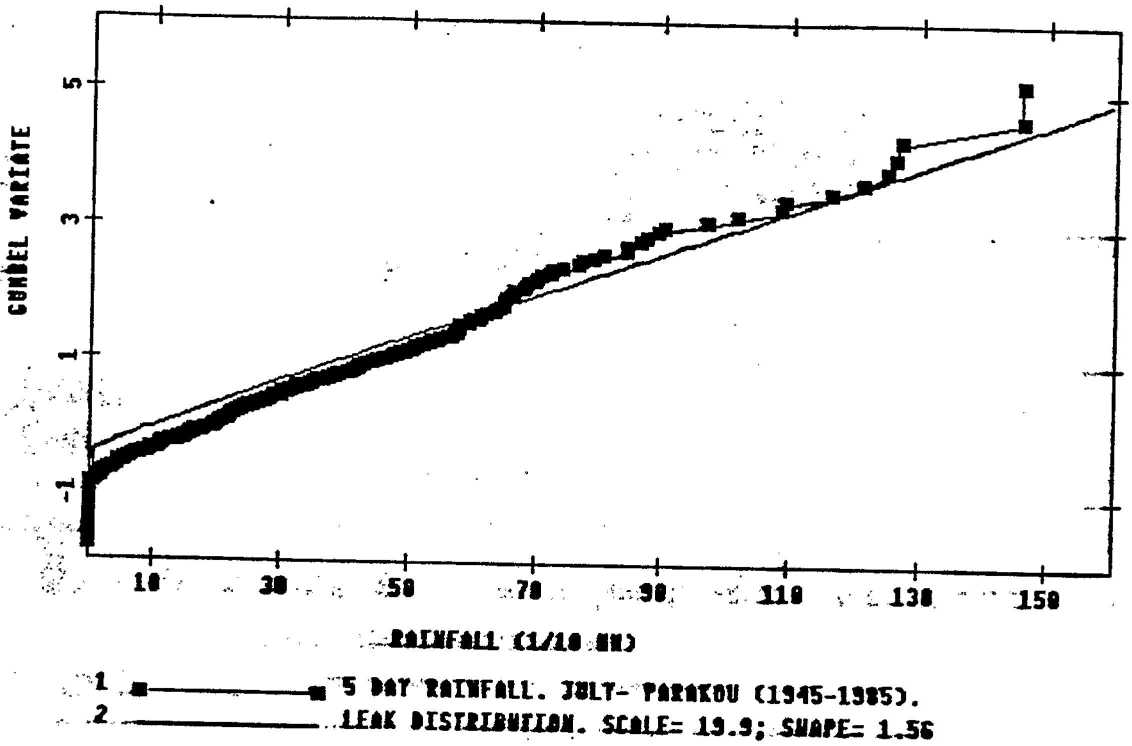
SHAPE PARAMETER

JULY

AUGUST

Figure 3. Maps of the scale and shape parameters over Benin.

5 DAY LEAK DISTRIBUTION DEDUCED FROM THE MONTHLY LEAK DISTRIBUTION.



10 DAY LEAK DISTRIBUTION DEDUCED FROM THE MONTHLY LEAK DISTRIBUTION.

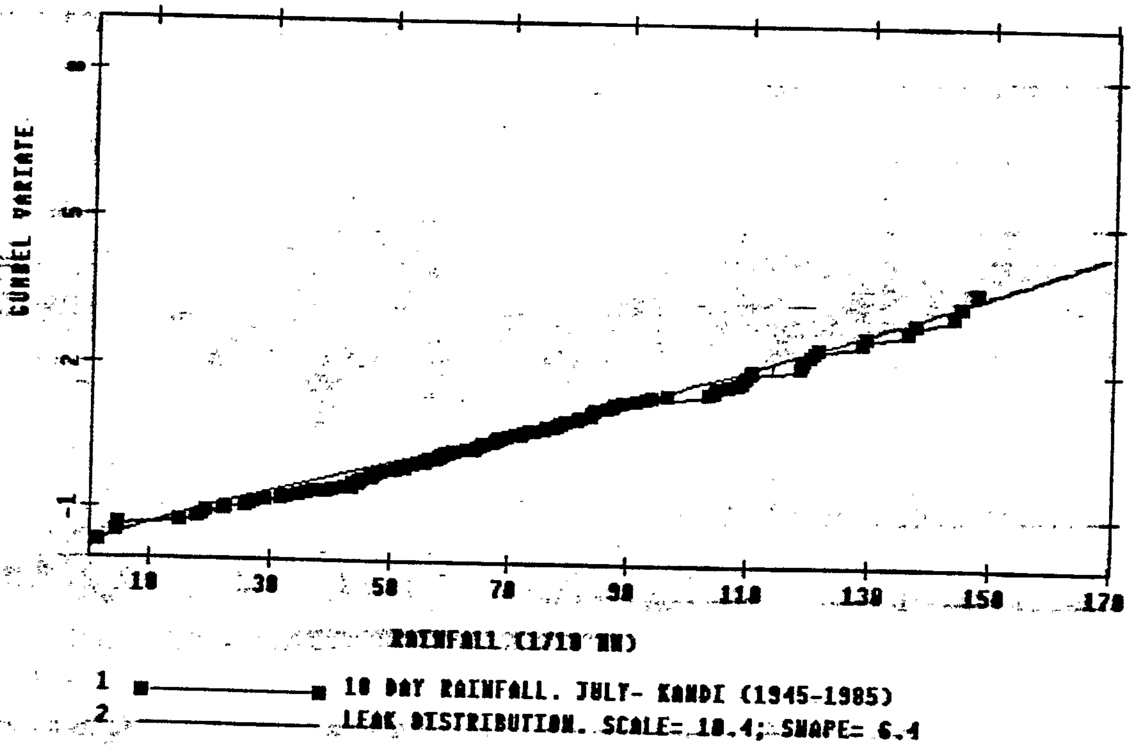


Figure 4. Five day leak distribution deduced from the monthly leak distribution.