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Selection of calibration objective functions in the context of rainfall–runoff modelling in a sudanese savannah area

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Abstract In the context of rainfall–runoff modelling carried out on the sudanese savannah area in the northwest of the Ivory Coast, attempts are being made to reconstitute the flow at the outlets of catchments in 10 day time steps. By using algorithms with automatic fitting procedures for the parameters, it appeared necessary to make a choice concerning the calibration objective functions to be used. The paper presents the algorithms, data and objective functions that have been used. The results obtained from the calibrations made have been analysed. That analysis was done principally with the help of a comparative evaluation modulus which takes into account elements other than the value of the objective function alone and which enables the quality of the results to be picked out from a hydrological point of view. At the conclusion of the analysis, the objective function defined by Nash seems to stand out quite clearly in relation to the other formulae examined.

Sélection de fonctions critères dans le cadre d'une modélisation pluie-débit en zone de savane soudanaise

Résumé Dans le cadre de travaux de modélisation pluie-débit menés en zone de savane soudanaise dans le nord-ouest de la Côte d'Ivoire, on cherche à reconstituer les apports à l'exutoire du bassins versants au pas de temps décadaire. Du fait de l'utilisation d'algorithmes employant des procédures de calage automatique des paramètres, il est apparu nécessaire de procéder à un choix en ce qui concerne les critères numériques de calage à utiliser. Les auteurs présentent les algorithmes, les données et les critères qu'ils ont utilisés. Ils analysent ensuite les résultats obtenus à l'issue des calages entrepris. Cette analyse se fait essentiellement à l'aide d'un module d'évaluation comparative prenant en compte des éléments autres que la seule valeur du critère et permettant de caractériser la qualité des résultats d'un point de vue hydrologique. A l'issue de cette analyse, le critère défini par Nash semble s'imposer assez nettement par rapport aux autres formulations retenues.

Open for discussion until 1 February 1992

INTRODUCTION

As an aid to a development perspective when setting up projects with different concerns (irrigation, potable water supply, etc.), the modelling of the rainfall–runoff relationship is underway in the sudanese savannah area in the northwest of the Ivory Coast. The aim is the estimation of supplies in 10 day time steps, frequently used in agronomy. For this, lumped deterministic and conceptual models are used which link the precipitation to the amount of runoff. Such algorithms must be able to take into account the data of the Ivory Coast national network which are, in practice, the only elements available to developers for them to be able to undertake simulations over long periods.

The savannah area is characterized by a long dry season (November to April) with rain mainly concentrated from June to September. The runoff coefficients measured on the catchments studied are very low; they rarely exceed 12% and in some years can drop to 1 or 2%. Such conditions are, therefore, very exceptional for models which are generally designed for use in temperate zones.

Before initiating a systematic programme for simulating and reconstituting hydrometric series, a series of tests was performed in order to evaluate the quality of the results obtained with the various models used. Since the algorithms employed use automatic calibration procedures for fitting the various model parameters, it quickly appeared necessary to make a choice concerning the calibration objective functions.

Several of the objective functions were tested in order to determine among those selected that formulation best adapted to the nature of the problem (estimation of flow in 10 day time steps) and to the specificity of the data. Five objective functions were tested, using three different rainfall–runoff models and four catchments (but five distinct calibration periods). The interpretation of the tests was carried out with the help of a modulus for evaluating the quality of the calibrations, based on hydrological criteria, viz. correlation between observed depth of runoff and calculated depth of runoff, autocorrelation coefficient, volume balances, and estimation of the flood volume. The nature of the modulus will be examined in greater detail below.

MODELS, DATA AND OBJECTIVE FUNCTIONS USED

Lumped models

The lumped models used will be briefly described. More information will be found in the references quoted.

The CREC Model CREC is a conceptual model based on a relatively classical storage schema which enables the identification of a production function and of a transfer function (Combes, 1985; Guilbot, 1986; Servat & Dezetter, 1988). The CREC version used herein has 10 parameters (X_1, X_2, ..., X_{10}) and gives the possibility of surface runoff passing through a linear reservoir. The production function (X_3, X_4, X_7, X_8, X_9) takes into consideration the state of soil humidity using the level of water in a reservoir supplying evapotranspiration and gives the proportion of precipitated water in the runoff. The transfer function (X_1, X_2, X_5, X_6) consists of a rapid runoff term and a slow runoff term (represented by a decreasing exponential). X_6 is the value of a threshold which controls the outlet of the rapid runoff reservoir. The model works in daily time steps and calculates therefore an average daily discharge which is the sum of a possible surface runoff Q_j^S, a rapid flow Q_j^H and a slow flow Q_j^G.

The different parameters are optimized with the methods of Rosenbrock (1960) and Nelder & Mead (1964) used in sequence. Figure 1 presents the conceptual schema of the CREC model.

The MODGLO model This is a lumped conceptual model for which it is also possible to identify a production function and a transfer function (Servat, 1986; Dezetter, 1987). At the production function level, some physical mechanisms for rainfall–runoff transformation are called upon (consideration of the water retention capacity of soils: *CRT, DCRT, SH*, and of the infiltration processes: *BB, AA*) although numerous simplifying assumptions have been made. The transfer function consists of three parallel reservoirs. Each one is characterized by a supply coefficient (with the volumes resulting from the production function split between the reservoirs: *C1, C2, C3*) and a recession coefficient enabling the flow to be modulated in time (*COEFFQ1, COEFFQ2, COEFFQ3*). This model operates in daily time steps and therefore calculates an average daily discharge.

The MODGLO parameters are optimized according to the same procedure as for CREC. Figure 2 shows the conceptual schema of the MODGLO model.

The GR3 model The GR3 model is also a storage model (Edijatno & Michel, 1989). The structure of this algorithm is based on three concepts:

(a) the ground reservoir (*A*), whose only outlet is the removal of water by evapotranspiration and which controls the division of the net rainfall between itself and the routing submodel;

(b) an access time (*C*), which is the delay between net rainfall appearing and the time it gets into the second reservoir; and

(c) the water gravity reservoir (*B*) which receives delayed inputs (cf. above) and whose only outlet is the flow of the river with a recession of the quadratic type.

According to Edijatno & Michel (1989), this set of three operators each of which depends on a single parameter seems to be, at present, the simplest schema for giving an account of the rainfall–runoff transformation in the simplest way.

The GR3 parameters are optimized according to the same procedure as for the CREC and MODGLO models. Figure 3 presents the conceptual schema of the GR3 model.

The data used

Four catchments situated in the northwest of the Ivory Coast were used at

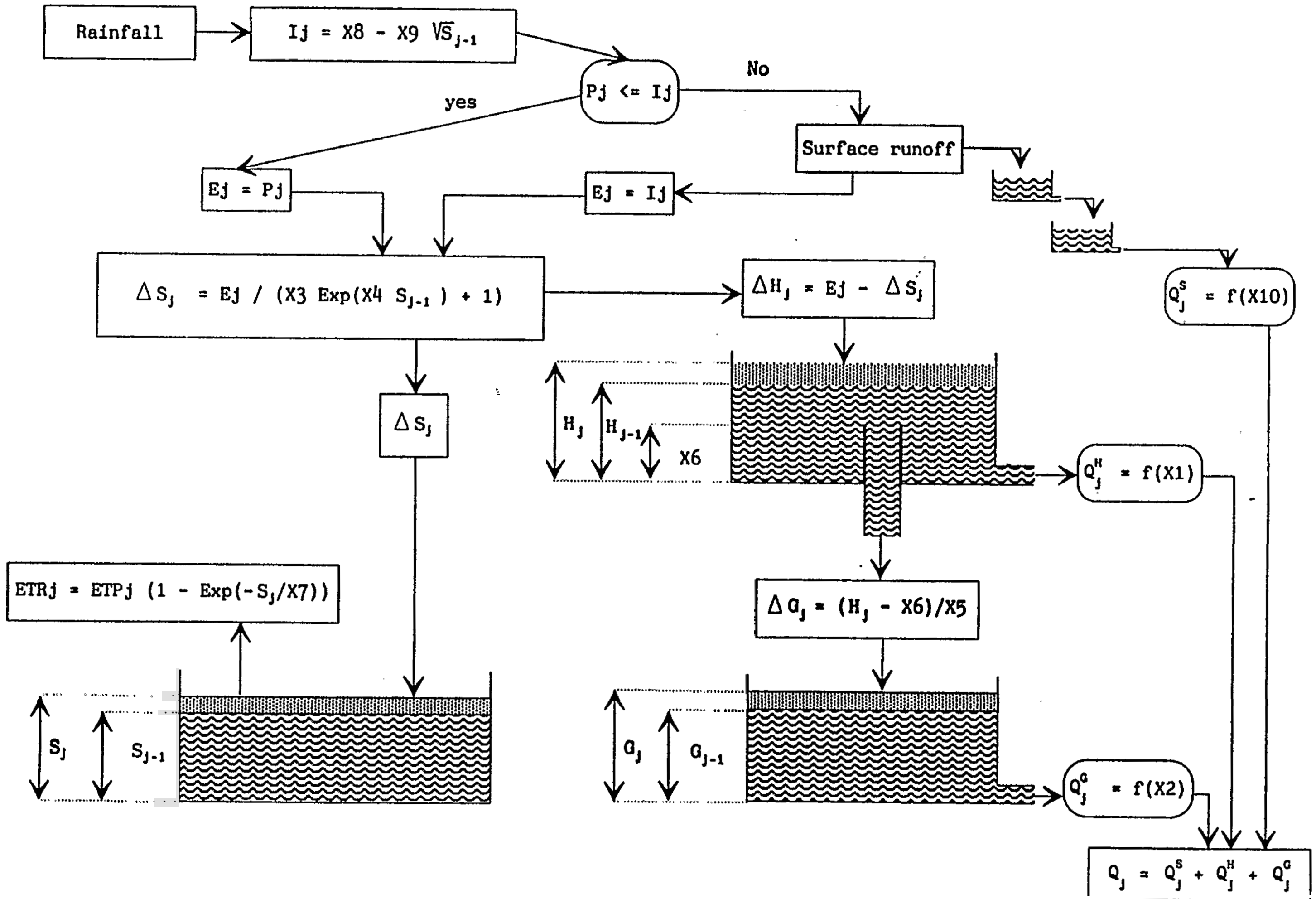

Fig. 1 Conceptual schema of the CREC model.

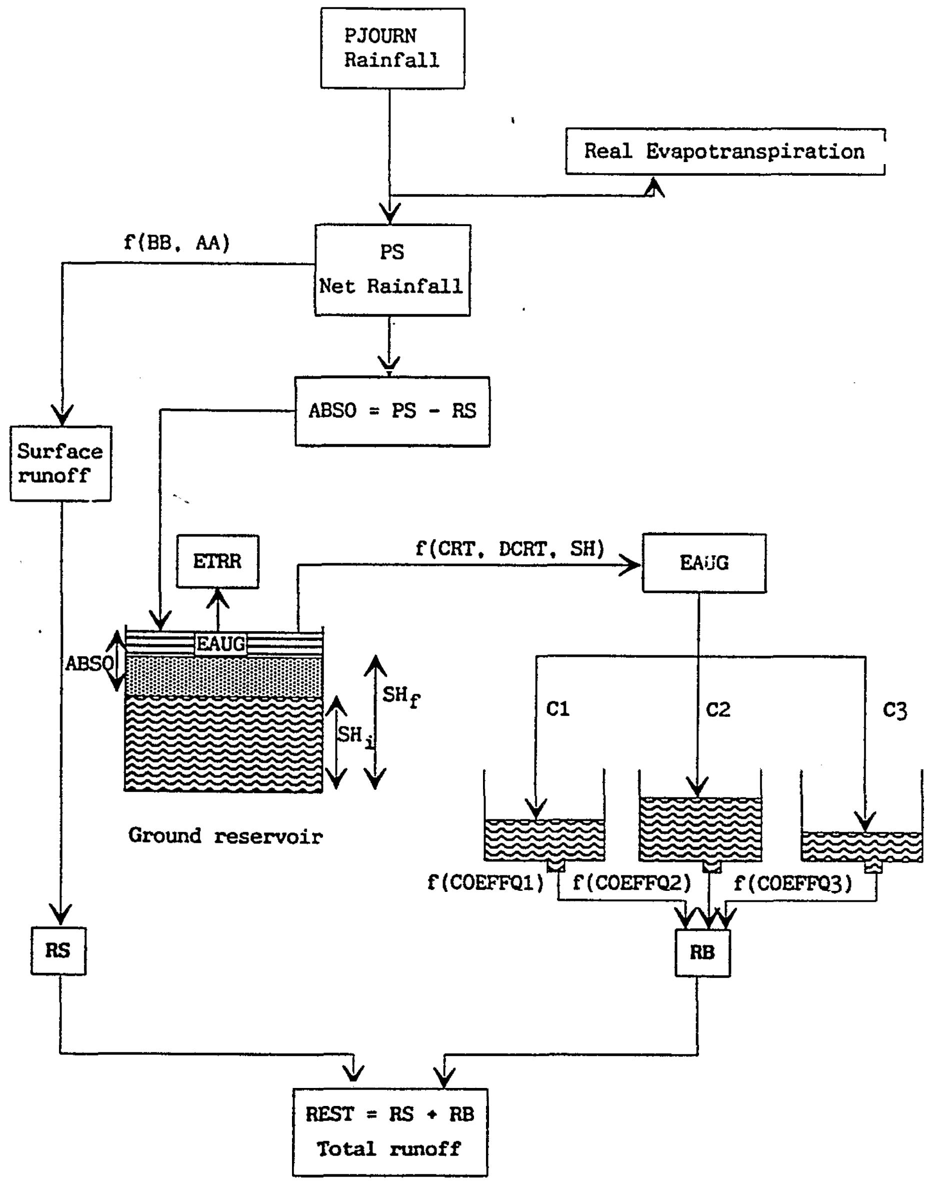

Fig. 2 Conceptual schema of the MODGLO model.

this stage of the programme (Fig. 4).

The Bagoé at Guingérini This catchment has an area of 1042 km^2. It is situated in the Niger basin, east of Odienné. The calibration period covers the years 1981, 1982 and 1983. The annual characteristics of precipitation and runoff have been grouped together in Table 1. 1983 appears to be a very dry year with a very low runoff. This was confirmed throughout the Ivory Coast where the drought was felt very severely.

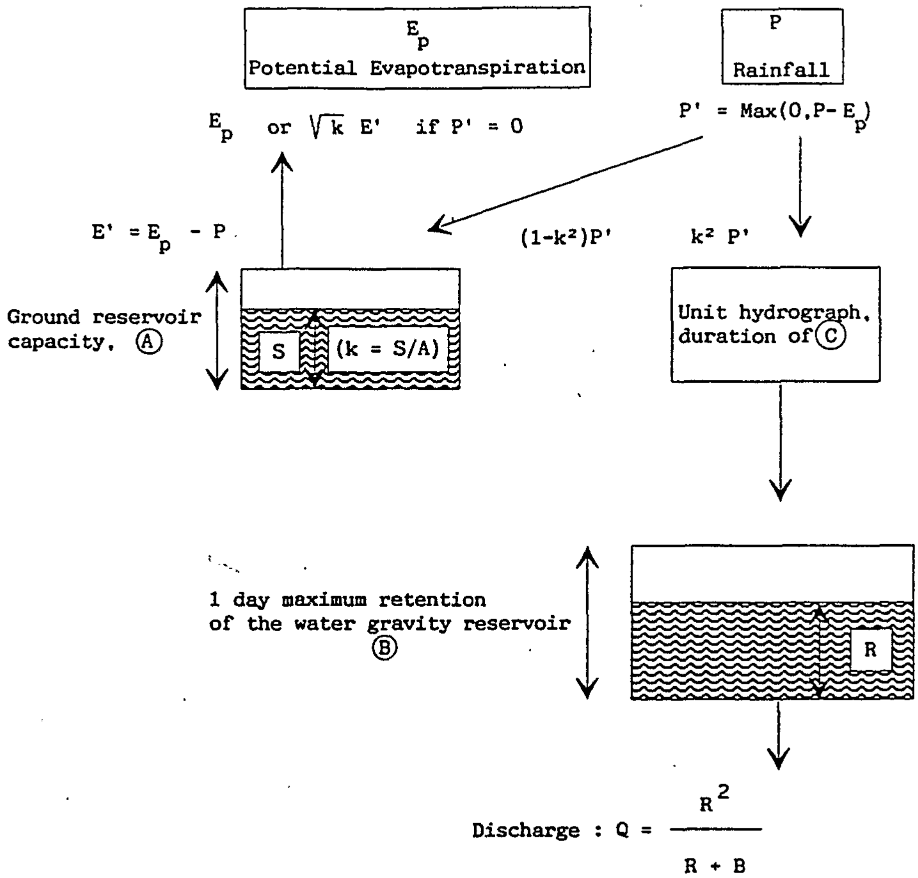

Fig. 3 Conceptual schema of the GR3 model.

The Bagoé at Kouto This catchment has an area of 4700 km^2. It includes the Bagoé at Guingérini catchment, and is also in the Niger catchment, east of Odienné. Calibrations were carried out over two distinct periods: 1973 to 1976 and 1981 to 1985. The annual characteristics of each of those periods have been grouped together in Tables 2 and 3. The years 1983 and 1984 show particularly low runoff depths and runoff coefficients.

The Bou at Boron The Bou at Boron catchment has an area of 3710 km^2. It is in the Bandama catchment, southwest of Korhogo. Calibrations were made over the period 1981 to 1985. Their characteristics have been reproduced in Table 4. The runoff coefficients for the years 1982 to 1984 are particularly low with, in particular, the value of 1.1% in 1983 which leads one to expect serious modelling difficulties.

The Lafigue at Badikaha road The Lafigue at Badikaha road catchment has an area of 443 km^2. It is in the Bandama catchment. The town of

Korhogo is situated in the catchment. Calibrations were done over the period 1981 to 1984. The main features have been reproduced in Table 5. The pronounced drought and the very low runoff coefficient for 1983 will once again be noted.

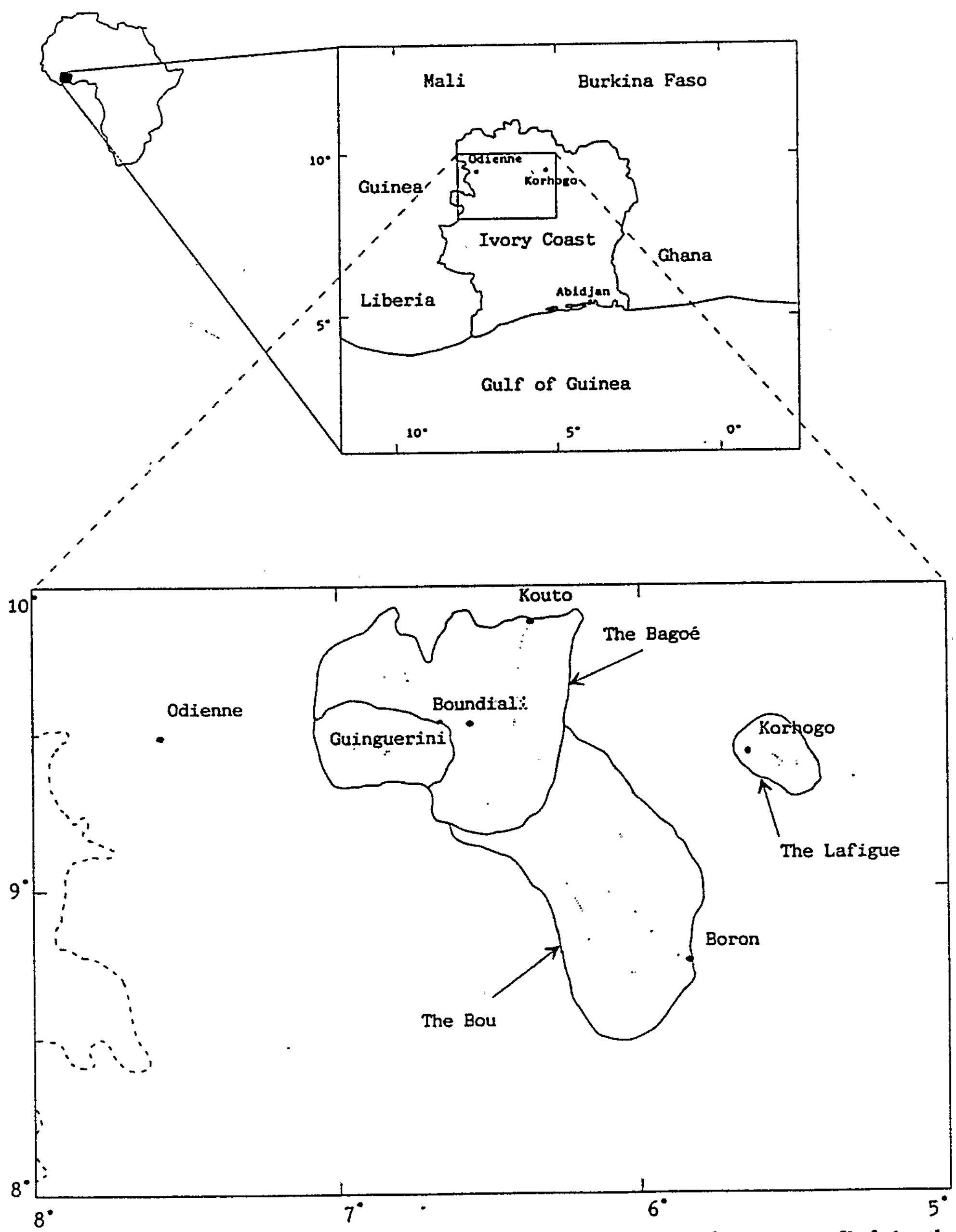

Fig. 4 Map of the situation of the four catchments studied in the northwest of the Ivory Coast.

Table 1 Precipitation and runoff annual characteristics of the Bagoé at Guingerini catchment (1981–1983)

Year	1981	1982	1983
Depth of rain (mm)	1412.0	1454.0	1103.7
Depth of runoff (mm)	299.5	233.9	52.1
Runoff coefficient (%)	21.2	16.1	4.7

Table 2 Annual precipitation and runoff characteristics of the Bagoé at Kouto catchment (1973–1976)

Year	1973	1974	1975	1976
Depth of rain (mm)	1424.0	1827.4	1463.1	1290.9
Depth of runoff (mm)	141.3	126.7	183.8	111.9
Runoff coefficient (%)	9.9	6.9	12.6	8.7

Table 3 Annual precipitation and runoff characteristics of the Bagoé at Kouto catchment (1981–1985)

Year	1981	1982	1983	1984	1985
Depth of rain (mm)	1340.2	1315.9	970.6	1146.6	1376.8
Depth of runoff (mm)	221.1	166.5	45.0	67.1	223.4
Runoff coefficient (%)	16.5	12.7	4.6	5.9	16.2

Table 4 Annual precipitation and runoff characteristics of the Bou at Boron catchment

Year	1981	1982	1983	1984	1985
Depth of rain (mm)	1052.9	1076.4	852.6	1055.6	1437.5
Depth of runoff (mm)	96.9	42.0	9.6	33.4	134.2
Runoff coefficient (%)	9.2	3.9	1.1	3.2	9.3

Table 5 Annual precipitation and runoff characteristics of the Lafigue at Badikaha road

Year	1973	1974	1975	1976
Depth of rain (mm)	1198.2	1170.7	835.6	1410.1
Depth of runoff (mm)	195.5	115.4	35.0	160.7
Runoff coefficient (%)	16.3	9.9	4.3	11.4

The objective functions tested

The automatic optimization of the parameters of a model requires the use of an objective function, i.e. a reference numerical "quantity" enabling the calibration to be improved. The choice of an objective function is not, however, without influence on the kind and quality of calibration at the end of the process.

In the context of this study several objective function formulations were tested in order to judge their performances and to select one for further systematic operations. The objectives set were the following:

(a) reconstitution as precisely as possible of the volume of floods in the rainy season (let it be recalled that the objective of the studies carried out was to determine water supplies);
(b) restoration of hydrograph dynamics; and
(c) no time lag between observed and calculated hydrographs.

Less importance was attached to the precise reconstitution of low flows, often very low as a result of long periods without or with very low precipitation, and generally quite simple to reproduce with storage models.

There is a very large number of objective functions in the literature. It was not the intention to carry out an exhaustive study thereof which is, in practice, not possible. Five different objective functions were therefore studied, three of which were the subject of numerous uses in hydrological modelling. The remaining two were set up considering elements which seemed important.

The Crec objective function This objective function, so-called for it is the one which was originally used in the CREC model, (Combes, 1985) is expressed as follows:

$$1/N \cdot \Sigma\left[|1 - (Q_c/Q_o)| \cdot |1 - (Q_o/Q_{mo})|\right]$$

with: N = number of observations;
Q_c = calculated discharge;
Q_o = observed discharge; and
Q_{mo} = observed mean discharge.

This expression tends towards 0 when Q_c tends towards Q_o.

The CrecBi objective function This objective function is none other than the Crec objective function to which has been added a balance term. The formulation therefore becomes:

$$1/N \cdot \Sigma\left[|1 - (Q_c/Q_o)| \cdot |1 - (Q_o/Q_{mo})|\right] + 1/N \cdot |\Sigma(Q_o - Q_c)/Q_{mo})|$$

This expression tends towards 0 when Q_c tends towards Q_o.

Initially, this balance term was used as a mutiplier weighting coefficient. Its influence was therefore dominating and harmed the quality of the reconstituted hydrograph dynamics. It was finally taken into consideration in the form given, i.e. an additive element whose size is comparable to the first term of the objective function.

The Fortin objective function This was put forward by Fortin *et al.*, (1971). Its expression is as follows:

$$1/N \cdot \Sigma \left| \left[(Q_c - Q_o)/Q_o \right) \cdot (1 + (|Q_o - Q_{mo}|/Q_{mo}) \right] \right|$$

This expression tends towards 0 when Q_c tends towards Q_o.

The Nash objective function First proposed by Nash (1969) and again by Nash & Sutcliffe (1970), the formulation of this objective function is:

$$1 - \left[\Sigma (Q_c - Q_o)^2 / \Sigma (Q_o - Q_{mo})^2 \right]$$

This expression tends towards 1 when Q_c tends towards Q_o.

It is easy, as far as it is concerned, to draw an analogy with a regression analysis. The term $\Sigma(Q_o - Q_{mo})^2$ corresponds to a form of the variance of the observed series. The term $\Sigma(Q_c - Q_o)^2$ can be likened to a form of residual variance. The formulation of the Nash objective function thus expresses a kind of "efficiency" (or "yield") in a model similar in the R^2 of a regression analysis.

From a technical point of view a modified form of the Nash objective function was used:

$$\Sigma (Q_c - Q_o)^2 / \Sigma (Q_o - Q_{mo})^2$$

This expression tends towards 0 when Q_c tends towards Q_o. It does not give the proportion of the variance explained by the model but the percentage of the residual variance compared to the total variance.

The SExpER objective function (sum of the exponentials of relative deviations) This objective function brings in a term which makes it very sensitive to changes in Q_c:

$$1/N \cdot \Sigma \left[\exp(|Q_c - Q_o|/Q_o) \cdot (Q_o/Q_{mo}) \right]$$

The purpose of the weighting term Q_o/Q_{mo} is to give greater weight to high values (since the average observed runoff is generally very low, the Q_o/Q_{mo} relationship reaches significant values during high flow periods but low values when water levels are low).

This expression tends towards 1 when Q_c tends towards Q_o but, concerned with homogeneity, the deviation from 1 has been minimized.

The above five objective functions were used in calibrating each of the three models presented. The optimizations were performed with the following constraints:

(a) a limited number of iterations for Rosenbrock's method (50 × P where P is the number of parameters to be optimized simultaneously);

(b) an end-of-run criterion based on the stop of progress for Nelder & Mead's method;
(c) the same initial parameters in each case; and
(d) the same intervals of variation for the parameters in each case.

Examination of the optimal solutions found can be envisaged from several points of view. In the studies reported here, the systematic use of a module for evaluating the quality of the hydrographs thus reconstituted was favoured. Data were therefore available for enabling the performances of this or that objective function to be appraised either globally or depending on the model used.

RESULTS OF THE CALIBRATIONS

The calibrations, carried out in daily time steps (Leviandier & Ma, 1987), used a 10 day time step basis, which is the frequency selected concerning simulation and reconstitution. All the values taken by the objective functions at the end of the calibration phases have been regrouped in Table 6.

Several remarks are necessary:
(a) a simple interpretation of the values of the objective functions is difficult. Only the Nash function can be linked directly to a statistical measure, viz. the percentage of residual variance compared to the total variance observed. This varies, depending on the case, between 13% and 63%;
(b) the combination of the GR3 model and the SExpER objective function did not appear to function correctly since in four out of five cases there was no convergence. This can be attributed to a set of causes linked together, difficult to differentiate, and among which are the objective function expression, the initial parameters, and the methodology used; and

Table 6 Objective function (OF) values resulting from the calibrations

OF	*Model*	*Bagoé Kouto 1973-1976*	*Bagoé Kouto 1981-1985*	*Bagoé Guingerini 1981-1983*	*Bou 1981-1985*	*Lafigue 1981-1984*
Crec	*CREC*	*0.968*	*0.667*	*0.769*	*1.163*	*0.796*
	MODGLO	*0.894*	*0.717*	*0.765*	*0.863*	*0.807*
	GR3	*6.914*	*1.305*	*5.249*	*1.098*	*0.761*
CrecBi	*CREC*	*1.173*	*0.736*	*0.806*	*1.359*	*0.881*
	MODGLO	*0.943*	*0.621*	*0.782*	*1.121*	*0.903*
	GR3	*8.305*	*1.793*	*5.682*	*1.602*	*0.873*
Fortin	*CREC*	*1.877*	*1.362*	*1.385*	*2.279*	*1.512*
	MODGLO	*1.404*	*1.047*	*1.114*	*1.456*	*1.460*
	GR3	*8.424*	*1.914*	*5.803*	*1.789*	*1.452*
Nash	*CREC*	*0.334*	*0.137*	*0.184*	*0.197*	*0.353*
	MODGLO	*0.369*	*0.133*	*0.185*	*0.205*	*0.490*
	GR3	*0.632*	*0.131*	*0.178*	*0.199*	*0.389*
SExpER	*CREC*	*1.388*	*1.581*	*1.255*	*1.733*	*1.228*
	MODGLO	*0.955*	*0.507*	*0.512*	*1.040*	*1.355*
	GR3	*NC*	*NC*	*NC*	*NC*	*1.015*

NC: no convergence.

(c) except for the combination with SExpER it will be noted that the GR3 model had some difficulties with calibrations for the Bagoé at Kouto (1973–1976) and for the Bagoé at Guingérini. It is, however, difficult to differentiate among the respective influences at work, e.g. the model itself, the objective function and the "model–objective function" association in these bad results, all the more so when one considers the good results of the GR3 model on the Bagoé at Guingérini with the Nash objective function.

Examining only the values taken by the objective functions at the end of the calibrations is not enough to judge the quality of the calculated hydrographs. Apart from the Nash objective function, it is difficult to appreciate the quality of the results. The objective functions are, in fact, a help with the calibration in so far as they constitute an objective to be reached. Their formulations are not, however, without influence on the shape and values of the series calculated. One objective function will have an important effect on low levels of flow, another one on the flood level peak. Their convergence will be more or less quick and precise and their relevance can be variable depending on the algorithm (and therefore the equations) with which they are associated. As a result, if the value of any objective function can enable some solutions to be eliminated or rejected, other elements of appraisal must be determined, enabling the quality of the hydrograph as a whole to be judged from a hydrological point of view. The first of these elements which comes to mind is the study of the hydrographs of the observed and calculated chronological series.

The Bagoé at Guingérini (1981–1983)

In all cases, 1983, a year with a particularly low runoff coefficient (4.7%), seems overestimated. Its influence on the calibration process is certainly important and has, as a result, a slight and systematic under-evaluation of the hydrographs calculated in 1981 and 1982. However, the level of adequacy reached in those two years lets one suppose that the algorithms used are able to reproduce correctly the hydrographs observed in "normal" years. This is what Fig. 5 shows, for example.

The Bagoé at Kouto (1973–1976)

All the calibrations obtained on the Bagoé at Kouto (1973–1976) are of very average quality whatever the objective functions and models considered. They all have the same characteristics, namely:

(a) the 1974 hydrograph is over-estimated; and
(b) the 1973, 1975 and 1976 hydrographs are under-estimated.

The year 1974 has a relatively low runoff coefficient (6.9%) which can introduce a significant bias into the optimization process. This could explain the systematic dysfunction that 1974, on the one hand, and 1973, 1975 and 1976 on the other hand, show (Fig. 6).

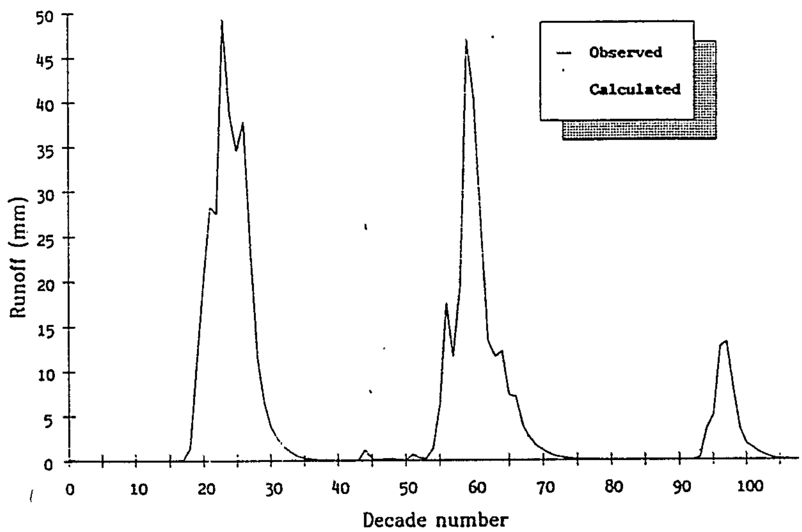

Fig 5 The Bagoé at Guingérini, 1981–1983, CREC model and Nash objective function: observed and calculated hydrographs for a 10 day time step.

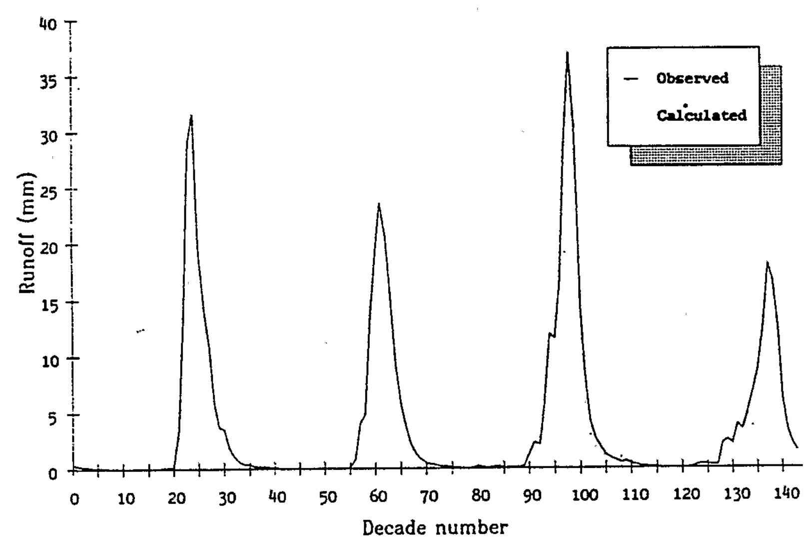

Fig. 6 The Bagoé at Kouto, 1973–1976, MODGLO model and Nash objective function: observed and calculated hydrographs for a 10 day time step.

The Bagoé at Kouto 1981–1985

The hydrographs calculated are generally of quite good quality despite the consideration of two years with very low runoff coefficients, in 1983 and 1984. The years 1981 and 1982 are generally slightly underestimated whereas 1985 is shown to be slightly in excess (Fig. 7, MODGLO and CrecBi). Few or no time lags are observed. The hydrograph dynamics are good, particularly when the water level is recessing, despite, sometimes, a surplus when the water is low (CREC and Nash, cf. Fig. 8).

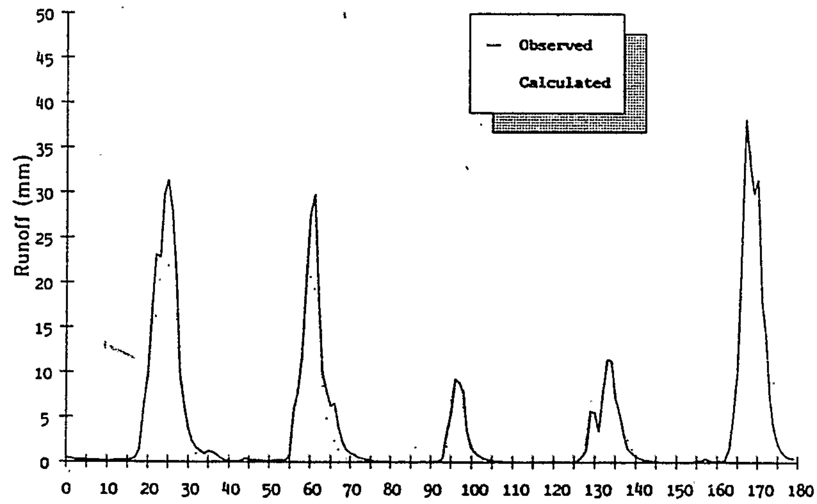

Fig. 7 The Bagoé at Kouto, 1981–1985, MODGLO model and CrecBi objective function: observed and calculated hydrographs for a 10 day time step.

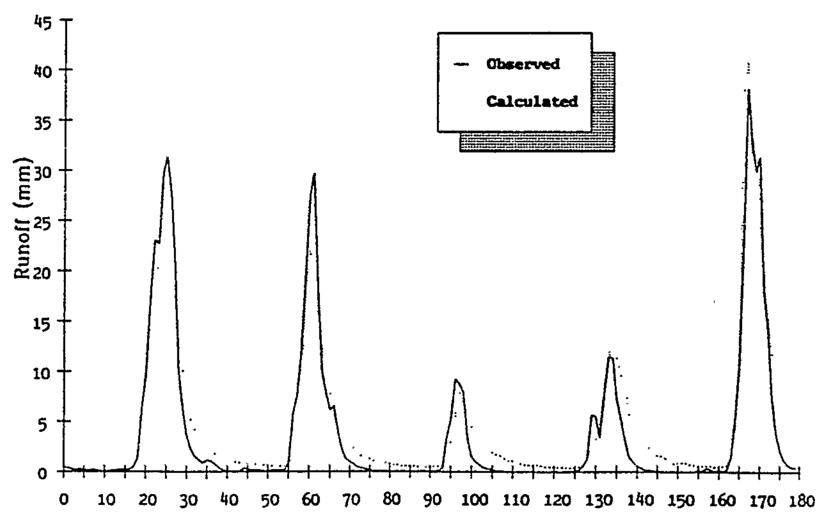

Fig. 8 The Bagoé at Kouto, 1981–1985, CREC model and Nash objective function: observed and calculated hydrographs for a 10 day time step.

The Bou at Boron 1981–1985

The runoff coefficients for the years 1982–1984 are particularly low (3.9%, 1.1%, 3.2%). 1985 was much rainier than the four preceding years (more than 33% of additional rainfall). This chronological series shows therefore extreme and opposite behaviour which make the task of optimizing the parameters more difficult.

Given the distinctive features of the period available for calibration, the results obtained with the combination CREC and Nash are fairly satisfactory. Only 1981 is greatly underestimated (Fig. 9). This same trend is found everywhere, more or less pronounced (Fig. 10).

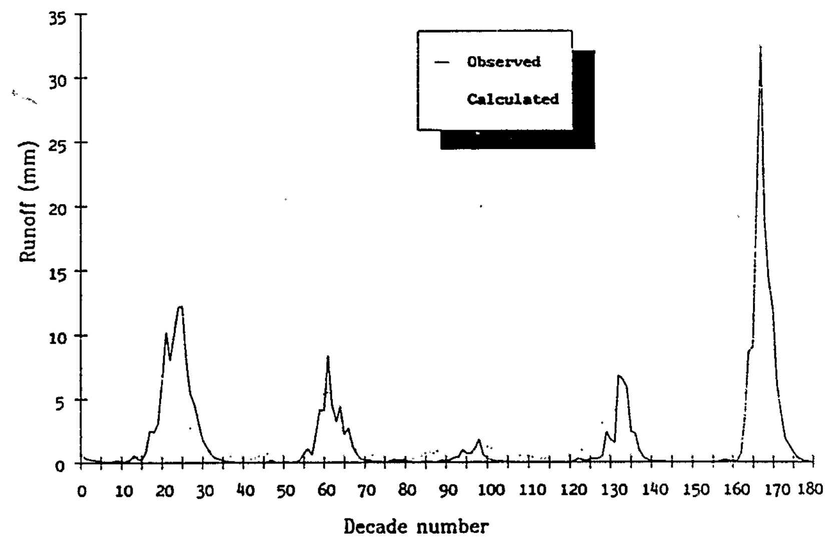

Fig. 9 The Bou at Boron, 1981–1985, CREC model and Nash objective function: observed and calculated hydrographs for a 10 day time step.

The Lafigue at the Badikaha road 1981–1984

It is the CREC and Nash combination (Fig. 11) that enables the best calculated hydrograph to be obtained. 1983 poses once again problems which are shown to be difficult to solve with the algorithms used.

BEHAVIOUR OF OBJECTIVE FUNCTIONS

Graphic characterization

Whatever the model used, examination of the plots of observed and calculated

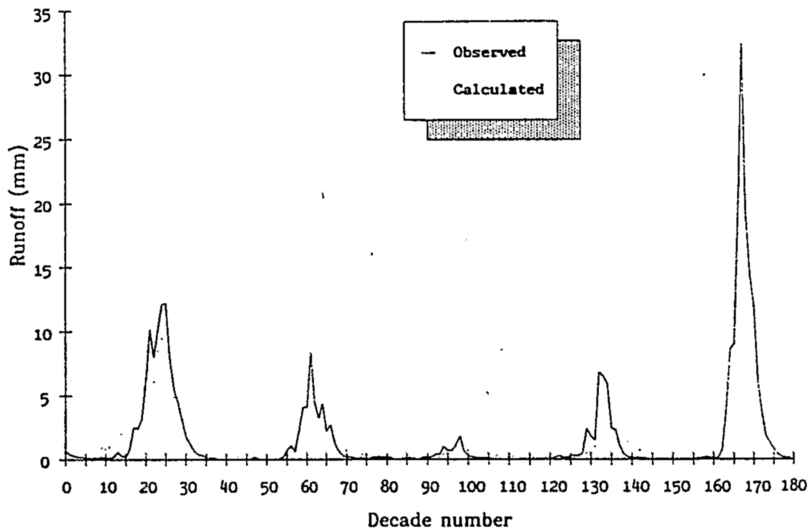

Fig. 10 The Bou at Boron, 1981–1985, GR3 model and Crec objective function: observed and calculated hydrographs for a 10 day time step.

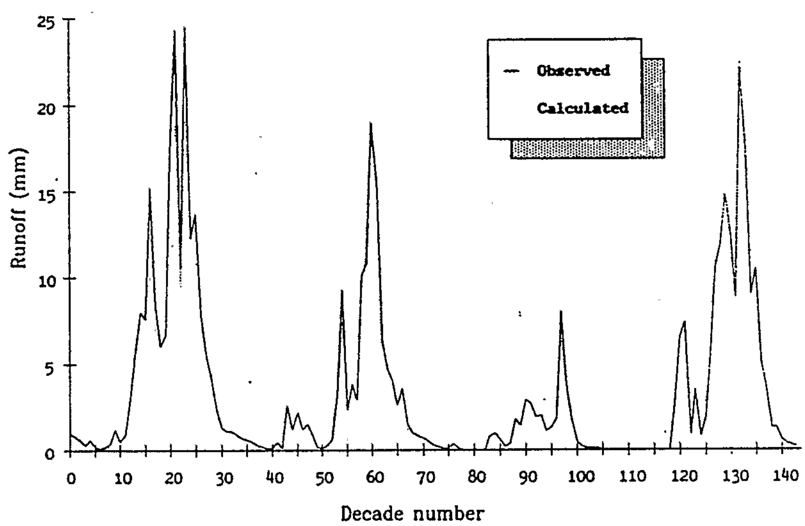

Fig. 11 The Lafigue at Badikaha road, 1981–1984, CREC model and Nash objective function: observed and calculated hydrographs for a 10 day time step.

series brings out some common points relating the behaviour of the objective function:

(a) the Crec objective function is shown to cause a little "reducing" in so far as the calculated hydrograph has a volume often lower than that observed. It respects, however, the dynamics of the hydrographs quite well;

(b) consideration of the balance term in a purely additive form in CrecBi certainly gives it excessive influence. This results, in some cases, in compensation by the balance term (and therefore the volume of flood) which is reflected in one year only;

(c) the behaviour of the Fortin objective function is similar to the Crec one in the sense that it is generally quite "reducing";

(d) the Nash objective function is shown to be outstanding concerning hydrograph dynamics and flood peaks. Its behaviour is, on the other hand, worse when low flows are concerned; and

(e) the SExpER objective function has a behaviour which is quite similar to those of the Crec and Fortin objective functions.

Hydrological characterization

Starting from the assumption that in each case an optimal calibration was obtained for each "model–objective function" combination, attempts to evaluate the performance of the objective functions themselves for a series of observations and for a given model were made.

For this, a comparative evaluation modulus of the quality of the calibrations obtained was defined with elements selected according to hydrological characteristics. This approach is quite close to that of Diskin & Simon (1977), but it is drawn up with reference to hydrological elements and not only to the value of the objective function.

The comparative evaluation modulus This comparative evaluation modulus is built up around several elements for assessing the calibration.

(a) *The coefficient of correlation between observed and calculated depths of runoffs* This enables the consideration, in particular, of the time lags between observed and calculated hydrographs and, to a less extent, of the quantitative deviations between the two series.

(b) *The deviation between the coefficients of autocorrelation observed and calculated at lag 2* Deviations at lag 1 are not very important. Autocorrelation coefficients at lag 2 were therefore used as evidence of the dynamics of the hydrographs, particularly when the water level is recessing.

(c) *Two coefficients for evaluating the volume balance*
Balance 1 = $|\sum (L_o - L_c)|$
Balance 2 = $\sum|(L_o - L_c)|$
with: L_o = observed depth of runoff (mm); and
L_c = calculated depth of runoff (mm).
Balance 1 and Balance 2 tend towards 0 when L_c tends towards L_o.

These two balances are calculated over the whole calibration period. Balance 1 alone is not enough, since numerical compensations can intervene which will give a low value to this coefficient without the calculated hydrograph necessarily being of good quality. Balance 2 on its own is not enough since slight oscillations on either side of the observed curve can be accepted after all. These would give Balance 2 a high value even if the general shapes of the observed hydrograph and the volume of the floods were closely matched.

(d) *The index for the reconstitution of the volume of the floods (IRVF)* With regard to the applied objectives of these modelling studies, the reconstitution of floods has been favoured rather than of low waters. For the area concerned, and in order to be limited in time, the depth of runoff between 1 July and 31 October has been defined as the flood. Table 7 shows that, concerning the catchments studied, the depth of runoff during this period is always about 80% or more of the total depth of runoff. Appraisal of the overall reconstitution of flood volumes after the various calibrations is done with the help of the index for the reconstitution of the volumes of the floods (*IRVF*). This index is defined over the whole period under consideration. For each year the percentage deviation between observed and calculated floods is calculated. For each year, the absolute value of this deviation is weighted by a coefficient expressing the importance of the annual flood compared to the whole period flood. The *IRVF* is the sum of the weighted deviations. An observed series of *n* years gives:

$$IRVF = \sum_{i=1}^{n} \left[\frac{\text{flood, year } i}{\sum_{i=1}^{n} \text{annual floods}} \times 100 \left| \frac{\text{Calc. flood, year } i}{\text{Obs. flood, year } i} - 1 \right| \right]$$

i.e.

$$IRVF = \sum_{i=1}^{n} w_i \times |\text{"observed − calculated" deviation (\%), year } i|$$

The *IRVF* tends towards 0 when the calculated flood tends towards the observed flood. For information, a systematic error of 10% on the annual floods of a given series corresponds to a value of 10 for the *IRVF*.

Table 7 Runoff from 1 July to 31 October over the whole periods

Catchment and period	*Total runoff (mm)*	*Total runoff 1 July-31 October (mm)*	*Runoff % 1 July-31 October*
Bagoé at Kouto 1981-1985	*723.0*	*668.5*	*92.5*
Bagoé at Kouto 1973-1976	*563.1*	*485.0*	*86.1*
Bagoé at Guingérini 1981-1983	*585.3*	*552.1*	*94.3*
Bout at Boron 1981-1985	*316.1*	*290.9*	*92.0*
Lafigue road Badikaha 1981-1984	*506.9*	*386.9*	*76.3*

Evaluation of the objective functions

(a) **Methodology** Having defined a comparative evaluation for the performances of the objective functions which is representative of hydrological characteristics, a methodology modelled on that of Diskin & Simon (1977) was then followed.

For a given period and model, the procedure was:

(i) the values of the different terms of the modulus are calculated for each of the objective functions;

(ii) for each term of the modulus, a performance classification of the objective functions is established, and a number is given (1 for the best rank, 5 for the worst rank);

(iii) for each objective function, the different numbers obtained are added, thus yielding a "grade" between 5 and 25 (5 in the best case and 25 in the worst).

As an example, Table 8 presents this "grading" for the Bagoé at Guingérini.

Table 8 Values of the terms of the objective functions evaluation modulus: example of the Bagoé at Kouto (1981–1985), GR3 model

	Objective functions				
	Crec	*Fortin*	*CrecBi*	*Nash*	*SExpER*
Correlation coefficient	*0.889*	*0.887*	*0.882*	*0.955*	*0.936*
Autocorrelation difference	*0.082*	*0.078*	*0.082*	*0.021*	*0.001*
Balance 1	*388.4*	*341.7*	*292.2*	*8.8*	*183.7*
Balance 2	*418.8*	*378.9*	*347.8*	*235.0*	*312.7*
IRVF	*54.5*	*47.9*	*40.8*	*14.6*	*34.7*
	Objective functions (classification)				
	Crec	*Fortin*	*CrecBi*	*Nash*	*SExpER*
Correlation coefficient	*3*	*4*	*5*	*1*	*2*
Autocorrelation difference	*5*	*3*	*4*	*2*	*1*
Balance 1	*5*	*4*	*3*	*1*	*2*
Balance 2	*5*	*4*	*3*	*1*	*2*
IRVF	*5*	*4*	*3*	*1*	*2*
Sum ("grade")	*23*	*19*	*18*	*6*	*9*

(b) **Interpretation.** Considering all the cases studied, Table 9 brings out the respective classifications of each objective functions and their frequency. The Nash objective function clearly seems to dominate from a study of Table 9 since it is ranked first 12 times out of 15, and at the worst it is ranked third. After it, the CrecBi and Fortin objective functions, respectively ranked 12 and 9 times among the first three seem to have the best behaviour.

A more detailed study can be carried out by studying the behaviour of the objective functions in relation to each of the evaluation modulus elements:

Table 9 Rank frequencies for the different objective functions; the equally placed objective functions have not been differentiated

	Objective functions (rank frequencies)				
Rank	*Crec*	*Fortin*	*CrecBi*	*Nash*	*SExpER*
1	*-*	*1*	*2*	*12*	*1*
2	*1*	*4*	*6*	*2*	*5*
3	*5*	*4*	*4*	*1*	*-*
4	*6*	*3*	*3*	*-*	*2*
5	*3*	*2*	*1*	*-*	*7*

(i) *the correlation coefficient* (Table 10) The Nash objective function seems very superior to the others concerning the correlation between observed and calculated runoffs. It comes first 12 times out of 15 and it shows the lowest interval between values (0.760–0.955). The other objective functions have performances which are quite similar to each other but at a much lower level;

Table 10 Rank frequencies of the objective functions for the correlation coefficient between observed and calculated depths of runoff

	Objective functions (rank frequencies)				
Rank	*Crec*	*Fortin*	*CrecBi*	*Nash*	*SExpER*
1	*1*	*2*	*-*	*12*	*-*
2	*1*	*3*	*5*	*2*	*4*
3	*7*	*3*	*2*	*-*	*3*
4	*5*	*6*	*3*	*-*	*2*
5	*1*	*1*	*5*	*1*	*6*
Minimum value	*0.382*	*0.382*	*0.570*	*0.760*	*0.470*
Maximum value	*0.944*	*0.946*	*0.946*	*0.955*	*0.943*

(ii) *deviations between the observed and calculated autocorrelation coefficients at lag 2* (Table 11) The Crec and Fortin objective functions do not seem very outstanding concerning autocorrelation. It is, on the other hand, quite difficult to separate the other three. It will simply be noted that the Nash objective function gives the lowest variation interval;

(iii) *Balance 1 and Balance 2* (Tables 12 and 13) The CrecBi and Nash objective functions appear quite clearly as the two objective functions which best respect the volumes (even if the Nash objective function does not, on occasions, reach the level of precision of CrecBi). It is, however, with the Nash objective function that, 11 times out of 15, the sum of the absolute deviations between observed and calculated values (Balance 2) is the lowest. The combination with Balance 1 also indicates that there are fewer numerical

Table 11 Rank frequencies of the objective functions for the deviations between the observed and calculated autocorrelation coefficients at lag 2

	Objective functions (rank frequencies)				
Rank	*Crec*	*Fortin*	*CrecBi*	*Nash*	*SExpER*
1	-	-	*5*	*6*	*4*
2	*3*	*4*	*3*	*2*	*3*
3	*5*	*3*	*2*	*1*	*4*
4	*4*	*7*	*2*	*3*	-
5	*3*	*1*	*3*	*3*	*4*
Minimum value	*0.016*	*0.007*	*0.002*	*0.003*	*0.001*
Maximum value	*0.716*	*0.716*	*0.183*	*0.123*	*0.203*

Table 12 Rank frequencies of the objective functions for the term Balance 1

	Objective functions (rank frequencies)				
Rank	*Crec*	*Fortin*	*CrecBi*	*Nash*	*SExpER*
1	-	-	*12*	*3*	-
2	*1*	-	-	*11*	*3*
3	*2*	*8*	*3*	-	*2*
4	*9*	*6*	-	*1*	-
5	*3*	*1*	-	-	*10*
Minimum value	*20.0*	*18.8*	*0.1*	*6.1*	*67.6*
Maximum value	*792.7*	*729.0*	*723.7*	*138.7*	*692.2*

Table 13 Rank frequencies of the objective functions for the term Balance 2

	Objective functions (rank frequencies)				
Rank	*Crec*	*Fortin*	*CrecBi*	*Nash*	*SExpER*
1	*2*	*3*	*1*	*10*	-
2	*1*	*4*	*3*	*2*	*4*
3	*7*	*3*	*3*	-	*3*
4	*3*	*5*	*1*	*3*	*2*
5	*2*	-	*7*	-	*6*
Minimum value	*146.2*	*154.9*	*179.3*	*149.1*	*163.0*
Maximum value	*826.3*	*779.0*	*774.5*	*412.5*	*435.8*

compensations than for the other objective functions; and

(iv) *IRVF* (Table 14) Concerning the index of the reconstitution of the volume of floods (*IRVF*), the Nash objective function shows an obvious superiority. It never reaches very high values and is quite clearly ahead of the other objective functions. The calibration using this objective function seems therefore to enable a better representation of the flood.

Table 14 Rank frequencies of the objective functions for the term IRVF

Rank	Objective functions (rank frequencies) Crec	Fortin	CrecBi	Nash	SExpER
1	-	-	2	13	-
2	2	4	2	1	5
3	4	5	5	1	1
4	6	5	2	-	3
5	3	1	4	-	6
Minimum value	19.6	24.7	12.6	14.6	25.7
Maximum value	154.7	144.1	142.9	48.6	119.2

BEHAVIOUR OF OBJECTIVE FUNCTIONS WITH GENERATED DATA

It is difficult to state to what extent the previous results are biassed by the fact that none of the models used describes the physical processes perfectly, and that lumped models do not take into consideration the areal irregularities of the input and of the parameters. In order to avoid this, a supplementary numerical experiment was carried out with input/output data and parameters of "perfect" models. The use of these generated data and the application of the calibration procedures with the different objective functions would produce parameter estimates which, when compared with the original ones, yield results not containing the above mentioned bias.

Data have been generated from the rainfalls and the catchment of the Bagoé at Kouto for 1981–1985. After the calibrations of the models were performed, results were interpreted following the same methodology as used above. Since the "observed data" were generated via the models themselves, the quality of the calibrations is excellent and they show very low differences between observed and calculated hydrographs. Table 15 brings out the respective classification of each objective function and its frequency. The Nash objective function still dominates since it is ranked first twice and second once (with the MODGLO model). As next best, it is confirmed that the Fortin objective function has the best behaviour (once first, twice third). Moreover the Nash objective function is always ranked first regarding *IRVF*, which is very important from a project management viewpoint.

CONCLUSION

The objective functions that have been used can be put into three categories:

(a) the first category groups together the Crec, Fortin and SExpER objective functions which are built up around the relative deviation between

Table 15 Rank frequencies for the different objective functions after calibration from generated data; the equally placed objective functions have not been differentiated

	Objective functions (rank frequencies)				
Rank	Crec	Fortin	CrecBi	Nash	SExpER
1	-	1	-	2	-
2	-	-	1	1	2
3	1	2	-	-	-
4	2	-	-	-	-
5	-	-	2	-	1

observed and calculated depth of runoff, and weighted by coefficients which are different;

(b) the second category concerns the CrecBi objective function, none other than the Crec one to which a balance term has been added; and

(c) the third category is represented by the Nash objective function whose formulation is linked to a classic statistical measure.

The periods for which calibrations were carried out are often critical for classical storage models of the type used here. They are, in fact, characterized by an exceptionally dry period observed principally in 1983, but also in 1984. The runoff coefficients are extremely low, with the depth of runoff representing only an almost negligible part of the water balance. This points to the likely importance of evapotranspiration which is unfortunately too often neglected in the rainfall–runoff models prepared by hydrologists.

Despite this distinctive feature, several lessons of a systematic nature can be learned from this study. There is nothing exceptional about the performances of the different models used, but generally speaking, they can be considered satisfactory. Some difficulties may be noted for the three algorithms used in "starting up again" after the dry season. The first floods are often badly reproduced. The long period of desaturation which is observed in the north of the Ivory Coast (little or no rain for several weeks, even several months) poses serious problems for the models built up according to a storage type schema. It is, moreover, difficult to integrate such an exceptional year as 1983 into a short calibration period.

Concerning the objective functions several points may be noted:

(a) the Crec, Fortin and SExpER objective functions have very similar behaviours which may be qualified as "reducing" in so far as the calculated hydrograph often has lower values than the observed one;

(b) in CrecBi, the consideration of a balance term in the form of an additive element has been shown to be relatively disadvantageous, since it often leads to numerical compensations compared to the observed hydrograph; and

(c) the Nash objective function behaves well on the whole although it shows some weakness with low flows.

A systematic study using an evaluation modulus for the performances of the objective functions has confirmed the first conclusions. Table 9 clearly shows that the Nash objective function stands out as the one which enables, overall, the best calibration. This is confirmed when each element of the modulus is taken separately. A study performed with generated data, to avoid bias due to the lumped models themselves, corroborates these conclusions.

With regard to the objectives aimed at (most precise estimation possible of the floods in the rainy season, restoration of the dynamics of the hydrographs, no time lags between observed and calculated hydrographs), the nature of the available data and the regional and climatic context (sudanese savannah area), it therefore appears clearly that it is the use of the Nash objective function which enables one to attain the best results.

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