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Energy balance of heterogeneous terrain: averaging the controlling parameters

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ABSTRACT

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We address the problem of averaging surface fluxes of sensible and latent heat at a regional scale. We assume that surface temperature can be expressed, at this scale, as a simple areal averaging of the temperature of each surface making up the region. A theoretical development, based on the diffusion equations and the energy balance equation, allows us to derive a simple averaging process. The controlling parameters of surface fluxes, such as aerodynamic resistance, resistance to latent heat transfer and albedo, must be weighted by the relative area of each component multiplied by what we call an omega coefficient. This coefficient, specific to each component surface, is defined in terms of aerodynamic resistance, surface resistance and also involves air temperature. A numerical simulation is carried out. Three cases are examined in which the whole area is divided into two contrasting components. The averaging process based on the omega coefficients is tested against the real values of energy fluxes directly obtained by averaging the local fluxes. It is shown that the omega coefficients substantially improve the estimates of the regional fluxes with respect to a simple areal averaging of the controlling parameters.

INTRODUCTION

Since general circulation models and mesoscale climatic studies require estimates of energy fluxes at a large scale, from areas of 10 km^2 upwards, the problem of averaging surface energy fluxes at a regional scale constitutes an important issue of international collaborative research (Shuttleworth, 1988; Raupach, 1991). This point is particularly relevant in the context of the sahelian projects such as SEBEX (Wallace et al., 1991) and the future HAPEX–Sahel (Hoepffner et al., 1990). One of the main objectives of these projects is to provide adequate calibrations and parameterizations of surface fluxes at a grid square scale for use in global circulation models. These models, properly calibrated, can considerably improve our understanding of the link

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between vegetation degradation and climatic changes. But they are very sensitive to the description of land surface processes and their predictions can differ greatly according to the accuracy of the description (Cunnington and Rowntree, 1986).

Several calculation schemes, together with appropriate ground-based instrumentation, can provide fairly accurate determinations of the energy fluxes of sensible and latent heat above homogeneous surfaces, such as open water, bare soil or plant canopy, from an area of about a tenth of a square kilometer. To determine fluxes at a regional scale, the idea is to use the same calculation schemes as those used at a local scale. Depending on the calculation scheme used, such controlling parameters as surface temperature, aerodynamic resistance, surface resistance or albedo are needed to determine surface fluxes. These parameters have their own subgrid-scale variability, and we are confronted with the necessity of averaging them over a grid square. Because of the non-linear dependence of fluxes on these controlling parameters, the appropriate averaging process to be used is unknown. The problem addressed here is to try to determine a correct averaging procedure in order to scale up surface fluxes from local measurements to larger areas.

Recently, the concept of 'blending height' has been used as a pragmatic approach to calculate areally averaged surface fluxes of momentum and heat and to deduce effective parameters (Masson, 1988; Claussen, 1991). The 'blending height' has been defined by Wieringa (1986) as the height at which the characteristics of the flow become approximately independent of horizontal position. The approach used in this paper is based upon a very similar concept and upon the classical flux and energy balance equations. We also assume that the areal temperature can be legitimately approximated by an areally averaged temperature. Such a definition is close to the temperature inferred for a large area from remote sensing techniques, and is consistent with the one used at a grid scale in general circulation models.

THEORETICAL DEVELOPMENT

Assumptions and basic equations

The context of the model presented below is the classical modelling of the unstable planetary boundary layer or convective boundary layer (CBL) over flat land. In this type of modelling, a thin surface layer is assumed to be overlaid by a well-mixed outer layer, the characteristics of which are independent of height and horizontally homogeneous. This assumption is well accepted for homogeneous terrain and in certain conditions for heterogeneous terrain. Shuttleworth (1988) and Raupach (1991) have considered two classes of heterogeneous landscape: on one hand the surfaces with 'disorganized' variability or 'heterogeneous on the microscale', and on the other hand, surfaces with 'organized' variability or 'heterogeneous on the mesoscale'. In a landscape with microscale heterogeneity, the CBL does not respond to successive surface patches and develops as it would over homogeneous terrain, whereas in a landscape with mesoscale heterogeneity, the CBL adjusts to successive surface types (Raupach, 1991). The model presented in this paper applies to the first type of landscape, where the characteristics of the air in the well-mixed layer are assumed to be horizontally homogeneous (independent of local position) as over a homogeneous landscape. Therefore, we use the air characteristics in the outer layer, such as temperature and humidity, and the incoming radiations as reference values, transferable from one land unit to another (McNaughton and Jarvis, 1983).

We make the assumption that the different patches are large enough so that effects of local advection at the upwind edges of the various patches are negligible, so that the classical one-dimensional diffusion equations of sensible and latent heat can be used over each land unit. These equations are written as

$$H = \rho c_p (T_s - T_a)/r_a \tag{1}$$

$$\lambda E = (\rho c_p/\gamma)[e^*(T_s) - e_a]/(r_a + r_s) \tag{2}$$

where ρ is the mean air density, c_p is the specific heat of air at constant pressure, γ is the psychrometric constant, T_a and e_a are the temperature and the vapor pressure of the air at a reference level z_r, which can be chosen near the bottom of the mixed layer, T_s is the surface temperature, $e^*(T)$ is the saturated vapor pressure at temperature T, and r_a is the aerodynamic resistance to transfer from the surface patch to the well-mixed layer. The definition and determination of r_a integrated over the whole patch has been discussed by Raupach (1991). r_s is the surface resistance, which is classically used to describe the transfer process through a non-saturated surface as well for a vegetated canopy (Monteith, 1973) as for a bare soil (Monteith, 1981; Shuttleworth and Wallace, 1985). Latent heat and sensible heat fluxes are linked by the energy balance equation

$$H + \lambda E = R_n - G \tag{3}$$

where G is the soil heat flux and R_n is the net radiation which can be detailed as

$$R_n = (1 - \alpha)R_s + \varepsilon(R_l - \sigma T_s^4) \tag{4}$$

where α and ε are the albedo and the emissivity of the surface, R_s and R_l are the incoming short-wave and long-wave radiation, and σ is the Stefan–Boltzmann constant. Latent heat flux can be determined in another way, by making use of the Penman–Monteith equation, obtained by eliminating surface temperature between eqns. (1), (2) and (3) (Monteith, 1973). This equation is written as

$$\lambda E = [s(R_n - G) + \rho c_p D_a/r_a]/[s + \gamma(1 + r_s/r_a)] \quad (5)$$

where D_a is the vapor pressure deficit of the air in the well-mixed layer and s is the slope of the saturated vapor pressure curve determined at the temperature of the air. If the latent heat flux is calculated by eqn. (5) the sensible heat flux can be determined as a residual term of the energy balance equation.

The basic idea for estimating fluxes at a regional scale consists of making use of the same diffusion equations as are used at the local scale (equations cited above), but with all the magnitudes specified at a regional scale. A good estimate of the average surface temperature of a large area can be obtained by means of satellites and airborne sensors. As a first approximation, the areal temperature given by these instruments (denoted by angle brackets) can be expressed as the mean value of the surface temperature of the components making up the area, weighted by their relative area

$$\langle T_s \rangle = \Sigma_i\, a_i T_{s,i} \qquad \text{with} \qquad \Sigma_i\, a_i = 1 \quad (6)$$

where a_i is the fractional area of the component i and Σ_i represents a summation over all the individual components of the whole area. The problem is to determine what type of averaging must be used for the controlling parameters (r_a, r_s, α, ε and G). Since the relationships are not linear, it is not evident whether the simple areal averaging used for surface temperature can be used for the other controlling parameters.

The problem addressed in the following paragraph is to try to work out an averaging procedure to obtain the effective controlling parameters of the whole area as a function of the controlling parameters of each land unit. Two basic assumptions are made. The areal surface temperature is given by eqn. (6), and the climatic environment at a reference height (T_a, e_a, R_s, R_l) is supposed not to vary. Since emissivity does not vary markedly among the natural surfaces and much less than the other parameters, we will also assume this parameter to be constant over the whole area.

Deriving the effective parameters

In natural environments the difference between surface temperature and air temperature is generally small. So, it is legitimate to linearize $e^*(T)$ in eqn. (2) and T_s^4 in eqn. (4) in such a way that

$$e^*(T_s) - e_a = s(T_s - T_a) + D_a \quad (7)$$

where s and D_a have been previously defined, and

$$T_s^4 = T_a^4 + 4T_a^3(T_s - T_a) \quad (8)$$

By analogy with the equation used for heat transfer, it is convenient to define a notional resistance to radiative transfer r_0 (Monteith, 1973)

$$r_0 = \rho c_p / 4\varepsilon\sigma T_a^3 \tag{9}$$

which allows one to write

$$\varepsilon\sigma T_s^4 = \varepsilon\sigma T_a^4 + \rho c_p (T_s - T_a)/r_0 \tag{10}$$

Taking into account the linearizations described above, the energy balance equation (eqn. (3)), in which each term has been previously detailed, can be rewritten in the following way

$$T_s = T_a + (\omega/\rho c_p)[(1 - \alpha)R_s + \varepsilon(R_l - \sigma T_a^4) - G] - (\omega/\gamma)D_a/(r_a + r_s) \tag{11}$$

where ω is defined by

$$\omega = 1/[1/r_0 + 1/r_a + s/\gamma(r_a + r_s)] \tag{12}$$

Equation (11) can be written for each component of the whole area, i.e. each individual area, as well as for the whole area. When eqn. (11) is applied to an individual area denoted by subscript i, T_s, r_a, r_s, α, G and ω are, respectively, replaced by $T_{s,i}$, $r_{a,i}$, $r_{s,i}$, α_i, G_i and ω_i, and T_a, D_a, R_s, and R_l remain unchanged because the climatic conditions are assumed to be the same for the whole area.

Substituting in eqn. (6) $T_{s,i}$, defined by eqn. (11), we obtain the following expression for the areal surface temperature

$$\langle T_s \rangle = T_a + (1/\rho c_p)[R_s \Sigma_i a_i \omega_i (1 - \alpha_i) + \varepsilon(R_l - \sigma T_a^4)\Sigma_i a_i \omega_i - \Sigma_i a_i \omega_i G_i] - (D_a/\gamma)\Sigma_i a_i \omega_i/(r_{a,i} + r_{s,i}) \tag{13}$$

Applying eqn. (11) to the whole area and identifying T_s with the areal temperature defined by eqn. (6), the matching of eqns. (11) and (13) leads to the following relationships

$$(1 - \alpha)\omega = \Sigma_i a_i \omega_i (1 - \alpha_i) \tag{14}$$

$$\omega = \Sigma_i a_i \omega_i \tag{15}$$

$$G\omega = \Sigma_i a_i \omega_i G_i \tag{16}$$

$$\omega/(r_a + r_s) = \Sigma_i a_i \omega_i/(r_{a,i} + r_{s,i}) \tag{17}$$

We have four equations with four unknown variables, r_a, r_s, α and G. It is easy to infer the following expressions

$$1/r_a = (\Sigma_i a_i \omega_i / r_{a,i})/\Sigma_i a_i \omega_i \tag{18}$$

$$1/(r_a + r_s) = [\Sigma_i a_i \omega_i/(r_{a,i} + r_{s,i})]/\Sigma_i a_i \omega_i \tag{19}$$

$$\alpha = (\Sigma_i a_i \omega_i \alpha_i)/\Sigma_i a_i \omega_i \tag{20}$$

$$G = (\Sigma_i a_i \omega_i G_i)/\Sigma_i a_i \omega_i \tag{21}$$

where ω_i is given by eqn. (12). The weighting factors which have to be used to calculate the regional values of resistance, albedo and soil heat flux are $a_i \omega_i$. They take into account not only the relative area (a_i) of each component but also involve a given combination (ω_i) of their resistances. We have to stress that the areal surface conductance ($1/r_s$) cannot be directly calculated as the mean value of the elementary surface conductances ($1/r_{s,i}$), weighted by $a_i \omega_i$, as the aerodynamic conductance, but has to be derived from eqn. (19), r_a being calculated by eqn. (18).

The omega coefficients involve three resistances, r_0, r_a and r_s. For a given region, radiative resistance r_0 is the same for all coefficients and depends only on air temperature T_a. Its value is fairly low, about $3 \times 10^{-3}\,\mathrm{s\,m^{-1}}$. When temperature increases, r_0 decreases and, since s increases with temperature, ω decreases also. The relative variation of ω as a function of the relative variation of either r_a or r_s can be easily calculated from eqn. (12):

$$\mathrm{d}\omega/\omega = [\omega(1 + s/\gamma)/r_a](\mathrm{d}r_a/r_a) \tag{22}$$

$$\mathrm{d}\omega/\omega = [\omega(s/\gamma)/r_s](\mathrm{d}r_s/r_s) \tag{23}$$

These relationships show that ω is an increasing function of both r_a and r_s. But all the conditions being equal, ω increases more rapidly with r_a than with r_s since the coefficient of eqn. (22) is greater than the coefficient of eqn. (23), r_a being generally much smaller than r_s and $1 + s/\gamma$ being greater than s/γ. Practically this result means that, the greater the aerodynamic and surface resistances of a subarea are, the more important the weight of this subarea with respect to its relative area is. Said in a different way, that means that the role played by a surface in the regional fluxes turns out to be more important as its aerodynamic and surface resistances increase.

MODEL PREDICTIONS

A numerical simulation was carried out to assess the impact of the omega coefficients on the regional value of fluxes. The two methods described before were tested. The first one consists of making use of eqns. (1) and (2), the areal temperature being determined by eqn. (6). The second one is to calculate the latent heat flux by means of the Penman–Monteith equation (eqn. (5)), and to determine the sensible heat flux as a residual term of the energy balance equation. In this second method, the areal temperature given by eqn. (6), and used as a basic assumption, is not explicitly present in the flux equations.

We have simulated three simple cases, in which the whole area is divided into two distinct components of equal area, under two different climatic conditions. These three cases are described in Table 1 and the two sets of climatic conditions in Table 2. For each case and each set of climatic con-

TABLE 1

Description of the three examples considered in the numerical simulations

Case	Parameters	Area 1	Area 2
(1)		Irrigated crop	Dry crop
	h_c (m)	0.5	0.5
	r_a ($s\,m^{-1}$)	223/u	223/u
	r_s ($s\,m^{-1}$)	100	1000
	α	0.20	0.20
(2)		Forest	Short vegetation
	h_c (m)	10	0.5
	r_a ($s\,m^{-1}$)	50/u	223/u
	r_s ($s\,m^{-1}$)	100	100
	α	0.15	0.15
(3)		Forest	Lake
	h_c (m)	10	0
	r_a ($s\,m^{-1}$)	50/u	632/u
	r_s ($s\,m^{-1}$)	100	0
	α	0.15	0.05

h_c, vegetation height; r_a, aerodynamic resistance; r_s, surface resistance; α, albedo. u is the wind velocity in meters per second at 30 m.

ditions, we have calculated the values of the regional fluxes in the two different ways described above, taking into account or not taking into account the omega coefficients. The values of the fluxes were compared with the values directly obtained by a simple areal averaging of the local fluxes (H_i and λE_i), calculated by the same method as the one used to determine the regional fluxes. These values are considered to be the true values and are used as references.

The surface resistance (r_s), the vegetation height (h_c) and the albedo (α) of each component of the whole area are specified as input. Aerodynamic resistance of each subarea is calculated in conditions of neutral atmospheric stability by the expression:

$$r_a = \ln^2[(z_r - d)/z_0]/k^2 u \tag{24}$$

TABLE 2

The two sets of climatic conditions used in the simulation

Case	R_s ($W\,m^{-2}$)	R_l ($W\,m^{-2}$)	T_a (°C)	e_a (hPa)	u ($m\,s^{-1}$)
(a)	800	300	25	15	6
(b)	400	300	15	10	3

The reference height, at which the air characteristics are taken, is 30 m. R_s, solar radiation; R_l, long-wave downward radiation; T_a, air temperature; e_a, vapor pressure; u, wind velocity.

TABLE 3

Comparison of regional surface fluxes ($W\,m^{-2}$) calculated with and without the omega coefficients

Case	Flux	Reference values	Averaging with ω coefficients	Simple areal averaging
Climatic conditions (a)				
Case (1)	λE	199	206 (+4%)	245 (+23%)
	H	238	238 (0%)	238 (0%)
Case (2)	λE	326	312 (−4%)	330 (+1%)
	H	171	184 (+8%)	264 (+54%)
Case (3)	λE	409	405 (−1%)	402 (−2%)
	H	130	150 (+15%)	290 (+123%)
Climatic conditions (b)				
Case (1)	λE	80	83 (+4%)	94 (+18%)
	H	108	108 (0%)	108 (0%)
Case (2)	λE	133	125 (−6%)	138 (+4%)
	H	92	99 (+8%)	143 (+55%)
Case (3)	λE	161	143 (−11%)	175 (+9%)
	H	81	105 (+30%)	232 (+186%)

Sensible heat flux (H) and latent heat flux (λE) are determined by the basic diffusion eqns. (1) and (2). The percentages are calculated with respect to the reference values.

where k is von Karman's constant (0.41), u is the wind velocity at the reference height z_r, d is the zero plane displacement and z_0 is the roughness length. d and z_0 are calculated as a fixed fraction of the vegetation height (h_c), following Monteith (1973):

$$z_0 = 0.13h_c \qquad \text{and} \qquad d = 0.63h_c \tag{25}$$

If the subarea is bare soil or open water, z_0 alone is introduced as input and set to 1 mm. Surface emissivity ε is taken as constant and set to 0.98. Soil heat flux is calculated as 5% of climatic net radiation (net radiation where surface temperature T_s is replaced by air temperature T_a). The surface temperature of each component is calculated by means of eqn. (11).

In Table 3, fluxes have been calculated by means of eqns. (1) and (2). In case (1), the aerodynamic resistance of both components is the same. Therefore, it is evident that the omega coefficients do not affect the regional value of this resistance and that the sensible heat flux remains the same. But we can notice that the omega coefficients, by increasing the value of the surface resistance, substantially improve the estimate of the regional evaporation. In case (2), the omega coefficients increase the regional aerodynamic resistance, which considerably improves the estimate of the sensible heat flux (from +54% to +8% with climate (a) and from +55 to +8% with climate (b)). The evapotranspiration rate is slightly reduced by the omega coefficients but not improved. In case (3), as in case (2), the estimate of the sensible heat flux is considerably improved by the omega coefficients, from +123 to +15% under

TABLE 4

Comparison of regional surface fluxes ($W m^{-2}$) calculated with and without the omega coefficients

Case	Flux	Reference values	Averaging with ω coefficients	Simple areal averaging
Climatic conditions (a)				
Case (1)	λE	189	189 (0%)	213 (+13%)
	H	236	236 (0%)	212 (−10%)
Case (2)	λE	320	307 (−4%)	301 (−6%)
	H	171	183 (+7%)	189 (+11%)
Case (3)	λE	394	396 (+1%)	346 (−12%)
	H	130	150 (+15%)	179 (+38%)
Climatic conditions (b)				
Case (1)	λE	75	75 (0%)	83 (+11%)
	H	107	107 (0%)	100 (−7%)
Case (2)	λE	129	122 (−5%)	121 (−6%)
	H	92	99 (+8%)	100 (+9%)
Case (3)	λE	145	132 (−9%)	119 (−18%)
	H	80	105 (+31%)	106 (+32%)

Latent heat flux (λE) is determined by means of the Penman–Monteith equation and sensible heat flux (H) as a residual term of the energy balance equation.

condition (a) and from +186 to +30% under condition (b). The estimate of the latent heat flux is improved in condition (a) but it is not improved in condition (b). In Table 4 the latent heat flux is calculated by means of the Penman–Monteith equation and the sensible heat flux as a residual term of the energy balance equation. The omega coefficients improve the estimate of the regional fluxes in each case. The improvement is particularly substantial in the first case and in the third case. In case (2) an improvement exists for both fluxes but it is slight.

These numerical simulations clearly show the impact of the omega coefficients on the calculation of fluxes at a regional scale. Significant errors can be introduced by using the basic diffusion equations (eqns. (1) and (2)) with a simple averaging of resistances (up to 186% in the examples simulated). Using the omega coefficients to average the local values of the controlling parameters considerably reduces the amplitude of these errors. And when the omega coefficients do not improve the estimates with respect to a simple areal averaging, the difference is always very slight and both estimates are generally fairly good. The Penman–Monteith equation is less sensitive to the averaging procedure than the basic diffusion equations. Even with a simple areal averaging the estimates obtained by the Penman–Monteith equation are never very bad and generally are much more correct than those obtained by the first method. However, with this method also, the omega coefficients improve the estimates with respect to a simple areal averaging.

CONCLUSIONS

The energy balance of heterogeneous landscape made up by different patches has been assessed in order to determine the effective controlling parameters. The classical flux and energy balance equations employed at the canopy scale have been transposed at a larger scale, and the areal surface temperature of the whole area has been approximated by a simple areal averaging of the surface temperature of each subarea. In these circumstances, it has been shown that the effective controlling parameters are expressed as weighted mean of the parameters specific to each subarea. The weighting coefficients are the product of the relative area of each component by an omega coefficient, which involves a given combination of the resistances characterizing each component. The role played by the omega coefficients is to give more relative weight to the patches with great resistances. The numerical results confirm that the effective parameters calculated in this way improve the estimates of the areal fluxes with respect to a simple areal averaging, whatever method is used to calculate the fluxes (basic flux equations or Penman–Monteith equations).

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