2.3

STATISTICAL ANALYSIS OF POINT AND AEREAL HOURLY RAINFALLS

(APPLICATION TO DESIGN FLOOD BY THE GRADEX METHOD)

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I. INTRODUCTION

In development projects on small rural river basins and in the growing field of urban hydrology, rainfall over periods of less than one day becomes of great interest. Hourly rainfall has a different spatial distribution than daily rainfall and also a different temporal distribution, especially in relation to non-negligible autocorrelation and the large number of zero values. The use of hourly rainfall information is handicapped by the scarcity of long terme recording raingages graphs as compared to daily rain gage records. Moreover the extreme flood estimation approach dealt with in this paper, called the Gradex method, requires an accurate knowledge of the temporal distribution of the average amount of precipitation for periods close to the watershed time of concentration.

For a region in France subject to particulary violent and sudden flooding, this paper presents an attempt to make best use of available information to estimate rainfall amounts of given return periods for different durations between one and twenty-four hours.

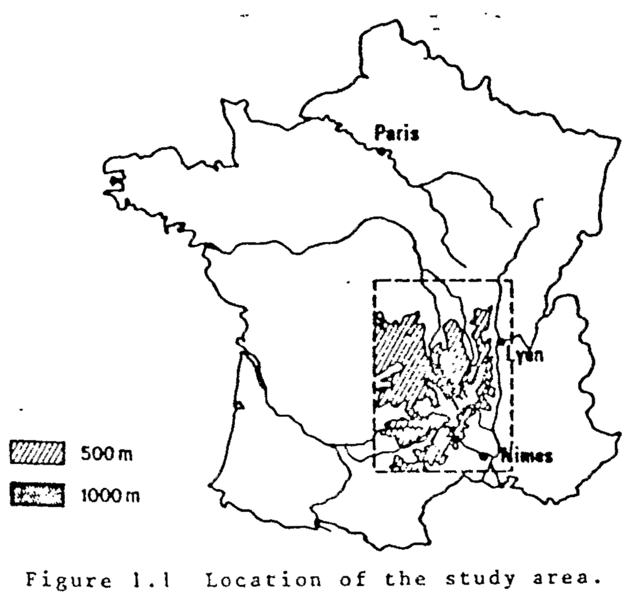


Figure 1.1 Location of the study area.

2. DISTRIBUTION OF EXTREME RAINFALL VALUES AT A SINGLE STATION

2. 1 - Gumbel distribution

It is generally accepted that the asymptotic behavior of probabilistic rainfall models must involve an exponential decay (SMITH-SCHREIBER 1973 ; SUMNER 1978 ; WOOLHISER 1979, 1982 ; REVFEIM 1982). The assumption can be justified by the following reasoning : assuming that the total amount of rainfall x failling over a unit time period (in the order of an hour or a day) comes from an inexhaustible source (the atmosphere) at a random rate, then the probability of this source delivering an additional quantity h is independent of the total amount emitted. Under these conditions :

$G(x + h) = G(x) \times g(h)$ for all x and $(x + h) > 0$

where G(x) is the probability of the rainfall exceeding x and g(h) is the probability of an additional amount exceeding h.

The only functions G and g that satisfy this functional equation are :

$$G(X) = K\, e^{-X/a} \qquad g(h) = e^{-h/a} \qquad 2.2$$

If the probabilistic rainfall model is a single exponential, it is easy to show that the maximum of N occurrences follows a Gumbel distribution. Assuming this is the case for rainfalls exceeding a level So :

$$N(X_i > x) = e^{-(x - x_o)/a} \qquad 2.3$$

$$n(S_o) = e^{-(S_o - x_o)/a} = N \qquad 2.4$$

where : n is the number of times that X_i has a value equal to or greater than x ;

N is the number of observations with $X_i \geq S_o$ for a given period.

A relative frequency can be defined :

$$n'(x) = \frac{n(x)}{N} = e^{-(x - S_o)/a} = P\,(X_i \geq x) \qquad 2.5$$

$$e^{-(x - x_o)/a} = N \times e^{-(x - S_o)/a} \qquad 2.6$$

which gives :

$$X_o = S_o + a \operatorname{Log} N \qquad 2.7$$

If we look at a random variable X, equal to the maximum obtained for N occurences of X_i, then :

$$P(X \leq x) = \left[P\ (X_i \leq x)\right]^N = \left[1 - \frac{n(x)}{N}\right]^N \approx e^{-n(x)} \qquad 2.8$$

$$P(X < x) = EXP\ (-EXP\ -\frac{(x - x_o)}{a}) \qquad 2.9$$

which is the Gumbel distribution.

The parameter a is the "gradex" (the exponential gradient).

For any model with an asymptotic exponential decay, it is always possible to find a level above which the distribution behaves as a single exponential and a number N sufficiently large to justify approximation 2.8. The Gumbel distribution is therefore generally valid and supports the conclusions of Hershfield (1960) based on a systematic study of rainfall all over the world.

One frequently used probabilistic rainfall model involves the sum of two exponentials (DUBAND 1967) :

$$F(x) = 1 - \gamma e^{-X/a} - \beta e^{-X/c} \qquad 2.10$$

with a > c, where a is the gradex.

In the model, rainfalls are considered to come from a mixture of two populations with one occurring less frequently but giving much higher maximums than the other. In many regions, the meteorological origin of precipitation corresponds to this scheme.

2. 2 - Small times periods

In our study involving the Cevennes region, problems were frequently encountered in fitting the Gumbel distribution to monthly maximum 1-hour and 2-hour rainfall data (figure 2.1)

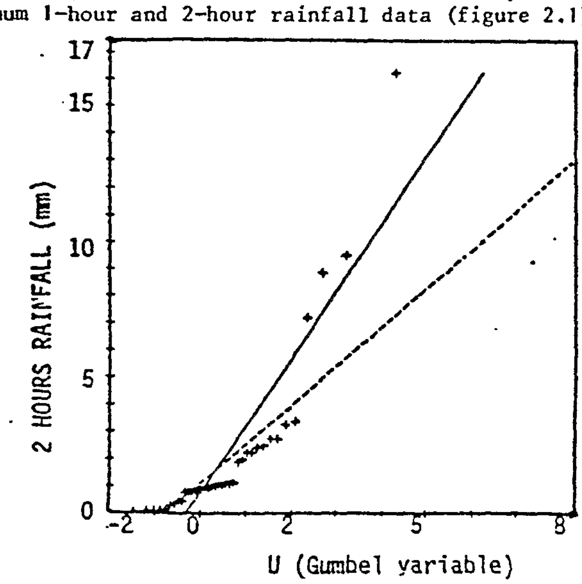


Figure 2.1 Extreme value distribution of two hours rainfall (.1mm);PUECHABON.
Solid line: Gumbel fit by moment method
Broken line:gumbel fit by M.L method.

A homogeneous season was used (3 autumn months) and the length of the series made it impossible to always select only the season maximum (e. g. 39 autumn months instead of 13 seasons).

For this short duration rainfall data, β in model 2.10 is large with respect to γ and N must therefore be very large to obtain maximums which follow a Gumbel distribution.

Table 2.1 Computed coefficients of mixed exponential (model 2.10).

	γ	β	F(o)	a	c
1-hour rainfall	.015	.060	.925	6.5	2.4
24-hour rainfall	.14	.19	.670	31.5	11.0
Station : Puechabon					

Although based on larger samples with a lower proportion of high values, model 2.10 shows a sampling distribution of the parameter "a" with a fairly high dispersion. Furthermore, there is a high autocorrelation for 1-hour and 2-hour rainfalls (figure 2.2) and fitting on decorrelated subsets is often highly dependent on the subset chosen.

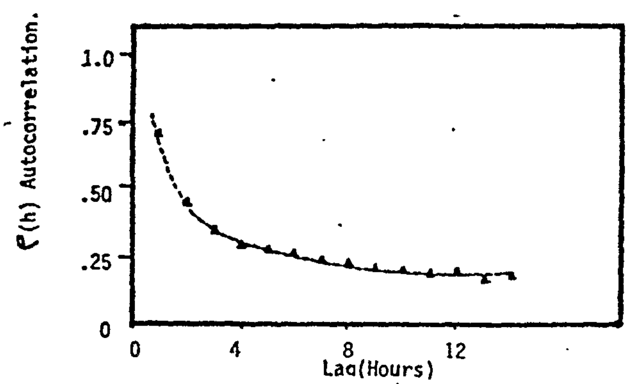


Figure 2.2

Autocorrelation of hourly rainfalls (Le Vigan)

For the 50 stations dealt with, we encountered these fitting problems in about ten cases, all located on the western or southern boundary of the region concerned. In these transition zones between two predominant climatic influences, the mixture of populations is in proportions such that, for short durations, the events of the more rare population occur only once every 3 to 10 years. This has already been observed for tropical regions subjected to extremely violent but rare cyclones (Bois 1969).

On the other hand, within the actual study zone, the Gumbel distribution appears to be well suited for all durations from 1 to 24 hours; it gives gradex estimations near, even if somewhat lower than those obtained by fitting model 2.10 to all the consecutive rainfalls.

3. EXTENSION TO SPATIAL RAINFALL

3. 1 - Calculation of spatial rainfall amounts

The average rainfall falling on a watershed during 1 hour can be calculated using a two-dimensional spline interpolation method (LEBEL, CREUTIN 1983).

The entire recording rain gage network of the Cevennes region (97 stations over about 15 000 km^2) was used for these calculations, which were carried out for four adjacent watersheds (Figure 3.1).

In reality, only 20 to 30 stations have significant weights. Furthermore, the temporal distribution study requires a base involving only those stations with a record lasting more than 10 years, i.e. 13 of the 30 stations (Table 3.1).

Table 3.1 : Influence of the 13 long term stations on the calculation of areal rainfall by spline method.

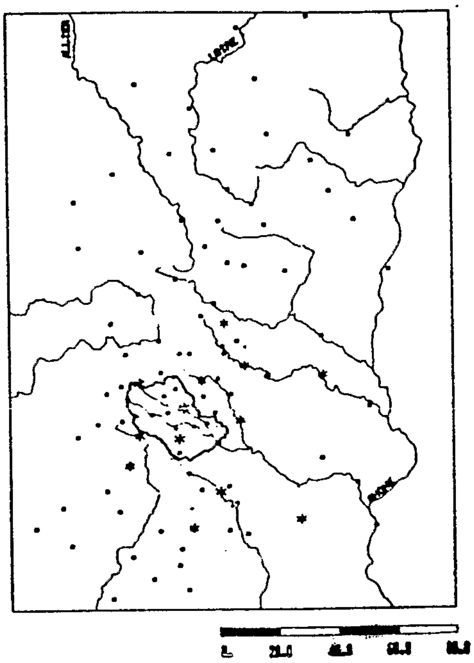
STATION (NUMB. & NAME)	GRADEX 1 HOUR (1/10 MM)	GRADEX 24 HOUR (1/10 MM)	WEIGHT GANDR (53 Km²)	WEIGHT GAJEA (165 Km²)	WEIGHT GAMIA (265 Km²)	WEIGHT GANDU (545 Km²)
201 ALES	131	376	- .014	- .004	.025	.078
206 COLLET	140	550	- .066	- .056	.111	.034
208 SOUDORGUE	139	572	- .001	.321	.056	.308
215 ST E V F	136	752	.164	.290	.544	.333
216 BARRE D C	91	518	.449	.168	.253	.148
217 MONTCLUS	85	356	.003	.001	.001	.002
219 BESSEGES	129	439	.002	.012	-.016	-.013
220 MALONS	74	501	- .031	- .014	-.003	-.004
223 QUISSAC	118	299	- .013	- .015	.003	.034
226 VAL.RAUG	94	648	.590	.373	.056	.125
227 LE VIGAN	83	394	- .109	- .081	-.031	-.031
252 NIMES	80	270	.003	.003	.001	-.004
263 VALFLAUN	79	256	.004	.002	ε	-.005

The spatial means were recalculated using the 13-station subnetwork and they appear to still give a good estimation of the hourly rainfall on each basin (LEBEL, CREUTIN 1983).

We therefore have four temporal series of spatial rainfalls over the period 1971-1980.

Table 3.2 : Maximum rainfalls (.1mm) on Gardon d'Anduze watershed (GANDU : 545 km^2)

DATE	MAXIMUM 1 H	jour	2 H	jour	4 H	jour	6 H	jour	12 H	jour	24 H	jour
1971 Septembre	67	19	115	19	181	19	222	19	283	19	293	19
Octobre	26	13	46	12	59	13	99	13	133	13	174	12
Novembre	48	7	82	6	135	6	158	6	158	6	242	9
1972 Septembre	154	7	259	7	368	7	493	7	667	7	734	6
Octobre	208	10	412	10	549	10	613	10	788	10	919	17
1973 Septembre	33	20	64	20	86	20	112	20	161	20	162	5
Octobre	130	2	296	2	417	2	507	2	860	2	1420	2
Novembre	212	5	383	5	577	5	709	5	936	5	1826	5
1974 Septembre	243	17	401	17	705	17	1020	16	1366	16	1807	16
Octobre	24	7	37	7	55	7	76	7	97	7	98	7
Novembre	73	14	137	14	253	14	334	14	585	14	1000	14
1975 Septembre	331	30	582	30	705	30	791	30	921	30	935	16
Octobre	30	16	60	16	78	16	90	16	91	16	92	16
Novembre	43	13	63	13	89	13	101	13	186	9	230	9
1976 Septembre	419	12	763	12	1222	12	1411	12	1615	12	2052	11
Octobre	105	25	202	25	350	25	479	25	737	25	1316	24
Novembre	153	9	229	9	353	9	519	9	664	9	697	9
1977 Septembre	29	24	48	24	69	24	125	24	213	24	259	24
Octobre	258	23	454	23	828	23	1052	23	1509	23	2592	22
Novembre	66	20	107	20	201	20	265	20	359	20	362	20
1978 Septembre	3	4	6	4	8	4	10	4	10	4	12	4
Octobre	173	17	259	17	335	17	336	17	338	17	340	17
Novembre	43	13	61	13	100	12	115	12	145	12	149	12
1979 Septembre	21	20	41	20	73	20	94	20	144	20	167	20
Octobre	200	4	315	4	497	4	642	4	865	25	1479	25
Novembre	14	14	27	14	46	14	56	14	58	14	78	14
1980 Septembre	107	21	196	21	326	21	444	20	764	21	1225	20
Octobre	198	16	316	16	425	16	531	16	687	16	751	16
Novembre	45	8	124	8	226	8	311	8	490	8	660	8



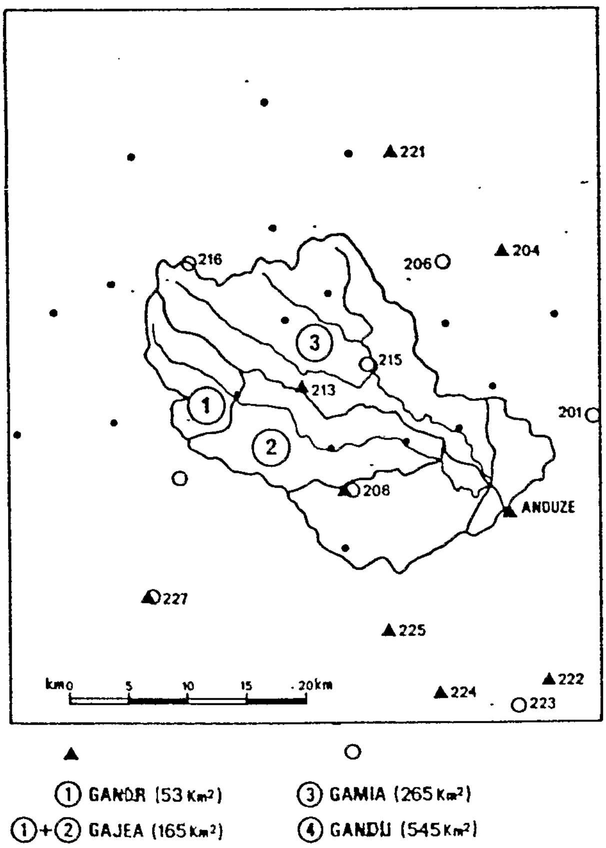


Figure 3.1
Top: the complete 97-network
(*: records of more than 10 years)
Bottom: the network composed of the 34 stations nearest Gardon d'Anduze.

3. 2 - Fitting a Gumbel distribution to spatial rainfall

A spatial rainfall value estimated by a weighted average of Gumbelian variables should not itself be Gumbelian unless all intercorrelations between stations are equal to 1.

However the Gumbel distribution is only a probabilistic model representing in our view the best approximation of the rainfall distribution at a single station. As shown in figure 3.2, the model remains applicable to series of mean rainfalls calculated on sufficiently small watersheds (< 1 000 km^2 in the Cevennes region).

The method of moments can be used to estimate the parameters of the Gumbel distribution, with the parameter Teta calculated for a sample of monthly maximums (3 per year for the autumn season). The 10-year rainfall is obtained using the following transformations :

$$TETA = TETA + GRADEX * Log_N\ (VA) \quad 3.1$$

$$P10 = TETA - L_N(-L_N\ (.9)) * GRADEX \quad 3.2$$

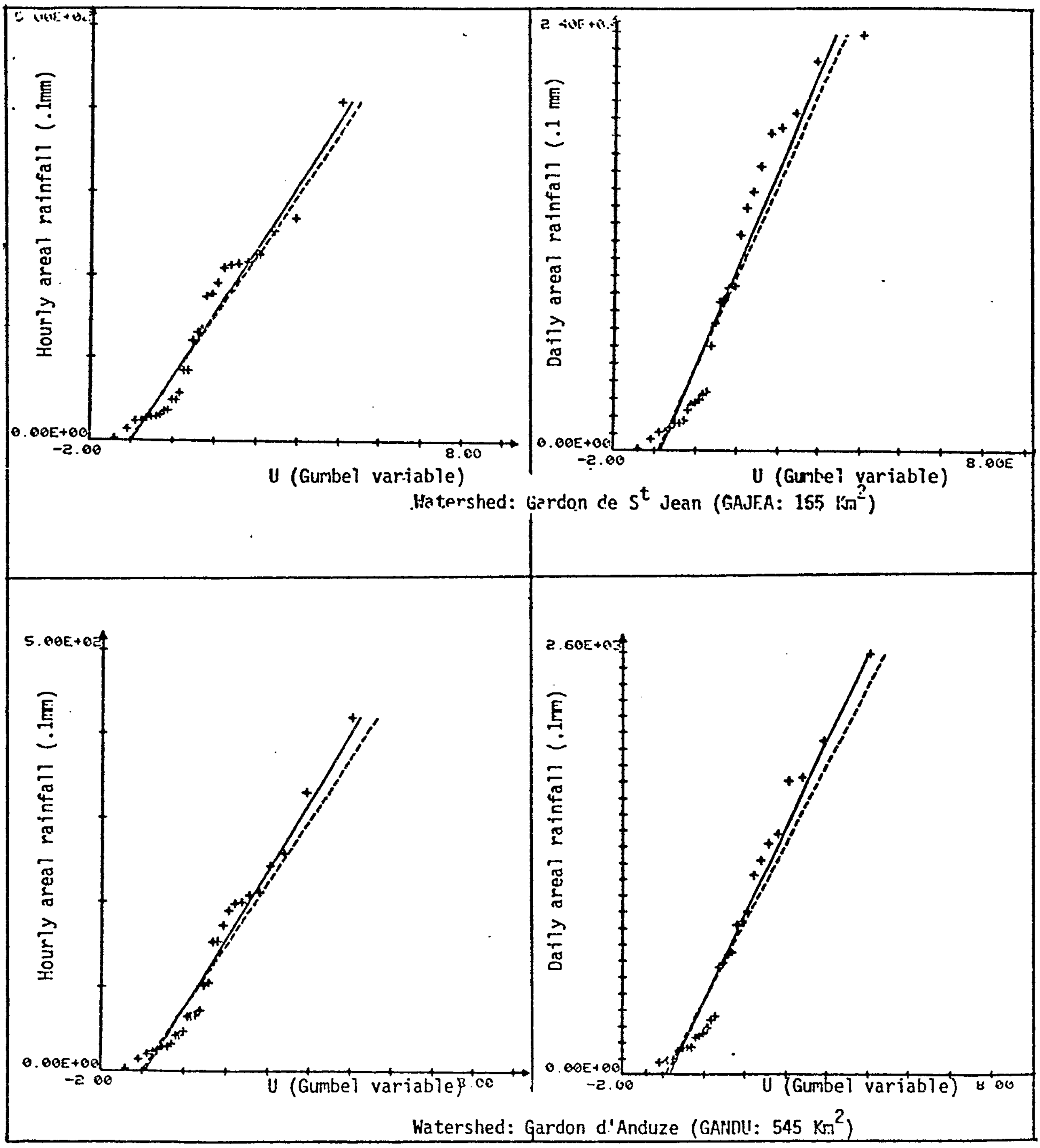


Figure 3.2 : Extreme values distribution of areal rainfall on two watersheds.

Solid line : Moment method fit. Broken line : Maxima likelihood fit.
The quality of fit is as good as the commonly obtained fit on stations.

Table 3.3 : Areal gradex (.1mm) calculated by fitting a Gumbel distribution on areal rainfalls estimated by spline method on a 13 long term stations network.

	GRADEX MOMENT	GRADEX M.VRA.	TETA MOMENT	TETA M.VRA.	F.10 MOMENT	F.10 M.VRA.
1H.	79.	74.	74.	73.	338.	320.
2H.	128.	120.	121.	120.	549.	521.
4H.	187.	170.	177.	179.	805.	747.
6H.	244.	219.	228.	230.	1044.	965.
12H.	350.	328.	331.	330.	1503.	1427.
24H.	617.	515.	426.	445.	2492.	2169.

Watershed: GANDR($53\ Km^2$)

	GRADEX MOMENT	GRADEX M.VRA.	TETA MOMENT	TETA M.VRA.	F.10 MOMENT	F.10 M.VRA.
1H.	88.	77.	77.	79.	373.	335.
2H.	156.	134.	131.	134.	652.	584.
4H.	250.	204.	190.	201.	1029.	884.
6H.	294.	253.	248.	254.	1231.	1101.
12H.	301.	351.	340.	348.	1625.	1523.
24H.	583.	516.	446.	451.	2397.	2179.

Watershed: GAMIA($265\ Km^2$)

	GRADEX MOMENT	GRADEX M.VRA.	TETA MOMENT	TETA M.VRA.	F.10 MOMENT	F.10 M.VRA.
1H.	77.	73.	161.	156.	335.	322.
2H.	141.	132.	209.	279.	606.	576.
4H.	207.	190.	428.	411.	894.	838.
6H.	252.	240.	536.	520.	1104.	1061.
12H.	341.	342.	742.	734.	1508.	1503.
24H.	562.	524.	1103.	1057.	2368.	2236.

Watershed: GAJEA($165\ Km^2$)

	GRADEX MOMENT	GRADEX M.VRA.	TETA MOMENT	TETA M.VRA.	F.10 MOMENT	F.10 M.VRA.
1H.	81.	74.	75.	75.	346.	322.
2H.	143.	126.	128.	130.	605.	553.
4H.	210.	192.	176.	201.	926.	843.
6H.	265.	237.	251.	254.	1139.	1055.
12H.	346.	328.	349.	347.	1508.	1446.
24H.	533.	403.	453.	455.	2239.	2071.

Watershed: GANDU($545\ Km^2$)

M.VRA.: Maximum likelihood method.

Note that there is no reduction in values as watershed size increases, whatever the duration. The calculations were also made using the maximum likelihood method (broken line). The difference between the two fits shows the importance of the break observed in the lower part of the point swarm. This break is due to the presence of maximums coming from the first population and tends to disappear if annual maximums are used.

3. 3 - Estimation of spatial Gradex without calculating mean rainfalls

It often happens that long temporal series of spatial rainfall values cannot be calculated but several long point series with Gumbelian distributions are available.

To be on the safe side, we may decide to calculate the spatial gradex according to the following formula :

$$g_A^o = \sum_{i=1}^{n} \lambda i\ gi \qquad 3.3$$

This is equivalent to considering that all inter-station correlations are equal to 1.

This assumption is pessimistic and the modeling of the structure of the spatial field (e.g. variogram or correlogram) becomes useful.

The variance of a linear combination of random variables can be written :

$$\sigma^2(\Sigma\ \lambda_i\ X_i) = \sum_i \sum_j \lambda i\ \lambda j\ \mathrm{COV}\ (X_i\ Xj) \qquad 3.4$$

Consider a spatial rainfall estimated by :

$$P_A = \Sigma\ \lambda i\ Pi \qquad \begin{array}{l} Pi \text{ : rainfall at a station i} \\ \lambda_i \text{ : weighting of this station} \end{array} \qquad 3.5$$

We are looking for :

$$g_A = K * \sigma_A \quad \text{with :}$$

σ_A : temporal standard deviation of spatial rainfalls

g_A : gradex of spatial rainfalls

K : 0.78 for the method of moments — 3.6

$$\sigma_A^2 = \sum_i \sum_j \lambda i\ \lambda j\ \mathrm{COV}\ (Pi\ Pj)$$

The problem is therefore reduced to the estimation of COV (Pi Pj).

This is possible using a generalized covariance model or a variogram model with a finite variance. In this case :

$$\gamma(h) = \sigma_s\ (1 - P(h) \qquad 3.7$$

$\gamma(h)$: variogram model

p(h) : correlogram model

σ_s : spatial variance of the random field.

The identification of a variogram model characterized by a shape parameter β and a scale parameter α, does not require a very long temporal series, only a certain number of intense rainfall events from a period of less than 10 years (LEBEL, CREUTIN 1983).

For the sperical model ($\gamma(h) = \sigma_o + (\sigma_s - \sigma_o) * \left[1,5 \frac{h}{dP} - 0,5 \left(\frac{h}{dP}\right)^3\right]$ the unit variogram is obtained by expressing $\sigma_s = 1$ and $\sigma_o = \sigma_{s/c}$ with c > 1.

This gives the relationship :

$$\alpha = \sigma_s \qquad 3.8$$

and we can write :

$$\gamma(h) = 1 - P(h) \quad \text{or} \quad P(h) = 1 - \gamma(h) \qquad 3.9$$

For a pair of stations (i,j) separated by a distance h_{ij}, the covariance is :

$$COV\,(P_i,P_j) = (1 - \gamma ij) * \;_i\;_j \qquad 3.10$$

and furthermore :

$$gi = K\sigma i \qquad gj = K\sigma j \qquad 3.11$$

gi and gj being the gradex values at stations i and j.

Substituting (3.6), (3.7) and (3.11) in (3.10), we obtain :

$$\frac{1}{K^2} g_A^2 = \frac{1}{K^2} \Sigma\Sigma\, \lambda i\, \lambda j (1 - \gamma ij)\, gi\, gj \qquad 3.12$$

$$g_A = (\sum_{ij}\sum \lambda i\, \lambda j\, (1 - \gamma ij)\, gigj)^{1/2} \qquad 3.13$$

Furthermore : $\bar{X}_A = \Sigma\lambda i\bar{P}i$ 3.14

giving : $\theta_A = \bar{X}_A - 0,577\, g_A$ 3.15

The two parameters of the distribution are thus identified and this calculation is valid for any distribution which is a function of only the first two moments of the random variable.

3. 4 - Application to the Cevennes region

From a series of 103 hourly rainfall fields studied first separately and then using climatological approach, the spherical model was found to be the theoretical variogram model best adapted to our raw variograms (figure 3.3).

This model has the following form :

$$\gamma(0) = 0$$

$$\gamma(h) = \sigma_o + (\sigma_s - \sigma_o) * (1,5\frac{h}{dP} - 0,5(\frac{h}{dP})^3) \qquad o < h < dP$$

$$\gamma(h) = \sigma_s$$

σ_s is the spatial variance of the field, which is easy to calculate.
σ_o (nugget) and dp (range) are however more difficult to establish since their estimation often depends on the method used to fit the theoretical model (BASTIN and G.EVERS 1983).

For 103 fields, approximately 75 % have a well defined sill with the range falling between 20 and 30 kms and 2/3 between 25 and 30 kms. The climatological variogram in figure 3.3 has a range of 27 to 30 kms and a zero nugget.

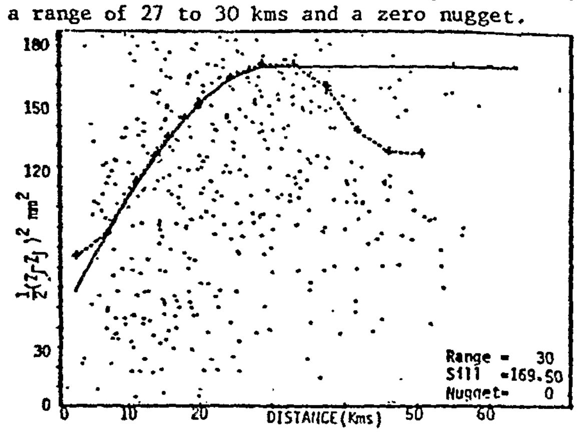


Figure 3.3

CLIMATOLOGICAL VARIOGRAM FOR 49 FIELDS

Selecting 3 values for the range (20, 25 and 30 kms) and 3 values for the nugget (0, 20 and 40 % of the variance σ_s), we obtain 9 possible spherical variograms.

Using the gradex values from the 13 long-record stations, we calculated the spatial gradex and P10 values for the 4 basins described in section 3.1 using formula 3.13 and the 9 proposed variograms. The nugget effect is slight : for the largest basins, the differences between the P10 and P100 estimations are less than 4 % for a constant range. We therefore consider that the nugget is zero ; this assumption can only be on the safe side since it always leads to overestimations (greater intercorrelations between stations).

The best spatial gradex estimation is obtained for a range of 25 kms for the 1-hour and 2-hour durations, and 30 kms for the 4-hour and 6-hour durations.

This difference is normal since the spatial fields are more structured when the hours of rainfall are cumulated, thereby giving a greater range.

A rigorous solution would employ, for each duration, the variogram fitted using the rainfalls of the corresponding duration. The 12 and 24 hour durations cannot be considered here since the range is likely around 50 kms.

In a region of very rugged topography where intense precipitation events are associated with complex meteorological phenomena, the large overestimations resulting from the use of formula 3.1 stress the importance of making best use of available information on the spatial rainfall frequency distributions.

Table 3.4 : COMPARISON OF AREAL GRADEX (.1MM) CALCULATED BY 3 METHODS

1. Gradex calculated on areal rainfall
2. Gradex estimated by a sperical variogram
3. Gradex estimated by the weighted mean of the point gradex
(correlation between every couple of stations equal to 1.0)

	Areal gradex computed on areal rainfall							Estimation of the areal gradex for 2 values of the range (spherical model) The last colums of each table consist the areal gradex estimated as the weighted mean of the point gradex									
								Range : 25 Kms			Weighted mean		Range : 30 Kms			Weighted mean	
	GRADEX MOMENT	GRADEX M.VRA.	TETA MOMENT	TETA M.VRA.	P.10 MOMENT	P.10 M.VSA.	GRADEX	GRADEX	P10	P100	GRADEX	P10	GRADEX	P10	P100	GRADEX	P10
								Watershed : Gardon de MIALET (GAMIA : 265 Km2)									
1H	88	77	77	79	373	335	1H.	91	421	636	124	513	96	435	661	124	513
2H	156	134	131	134	652	584	2H	149	674	1026	197	807	156	694	1062	197	807
4H	250	204	190	201	1029	884	4H	235	1028	1582	305	1222	245	1056	1634	305	1222
6H	294	253	248	254	1231	1101	6H	277	1216	1867	367	1466	290	1253	1935	367	1466
12H	381	351	348	348	1625	1523	12H	354	1560	2392	478	1904	372	1610	2485	478	1904
24H	583	516	446	451	2397	2179	24H	490	2124	3277	656	2584	514	2191	3401	656	2384
								Watershed : Gardon d'ANDUZE (GANDU : 545 Km2)									
1H	81	74	75	75	346	322	1H	81	395	586	126	520	07	411	617	126	520
2H	143	126	128	130	605	553	2H	132	630	942	201	819	141	656	990	201	819
4H	218	192	196	201	926	843	4H	191	897	1348	286	1161	204	934	1415	286	1161
6H	265	239	251	254	1139	1055	6H	226	1065	1597	343	1389	242	1110	1680	343	1389
12H	346	326	349	347	1508	1446	12H	286	1359	2033	441	1789	308	1419	2143	441	1789
24H	533	483	453	455	2239	2071	24H	404	1868	2817	615	2455	433	1950	2970	415	2435

EXTREME VALUE FLOOD ANALYSIS

4. APPLICATION TO EXTREME VALUE FLOOD ANALYSIS: the Gradex method

This method uses the rainfall gradex as a basis for the extrapolation of flood volumes.

Rainfall gradex values are equivalent to runoff as long as common integration times greater than or equal to the base time of the basin overland flow hydrograph are adopted for both ; rainfall peaks of shorter durations are spread out and damped by transfers within the drainage system.

The Gradex method in no way involves the extrapolation of a runoff coefficient, or a rainfall-discharge correlation, fitted to a sample of observed floods (which would be meaningless in view of the dominant role of the random "retention" in this low part of the distribution). Instead it is assumed that the marginal runoff coefficient tends towards one in the case of extreme value rainfalls due to the gradual saturation of the soil and the blocking of infiltration paths.

During high water periods, the runoff volume is equal to the rainfall less a random quantity, the retention, which tends towards an upper limit, related to the initial dryness of the soil, as the rainfall increases. When basin saturation is approached, any increase in precipitation ΔP tends to produce an equal increase in discharge, i.e. $\Delta Q \longrightarrow \Delta P$ and the discharge extreme deviations take on the same order of magnitude as for the rainfall.

Now let's look more closely at these different points. First of all, in the quadrant (P, Q), the upper limit of the observed points is the first bisector as can be seen in figure 4 a ; the amount of water that flows cannot exceed the amount that falls.

In actual fact, it is possible for Q to exceed P in relatively rare cases involving rainfall reinforced by abundant snowmelt. This snowmelt contribution to very large floods is nevertheless limited to values in the order of 30 or 40 mm by the necessary heat exchanges and in no way weakens the following arguments.

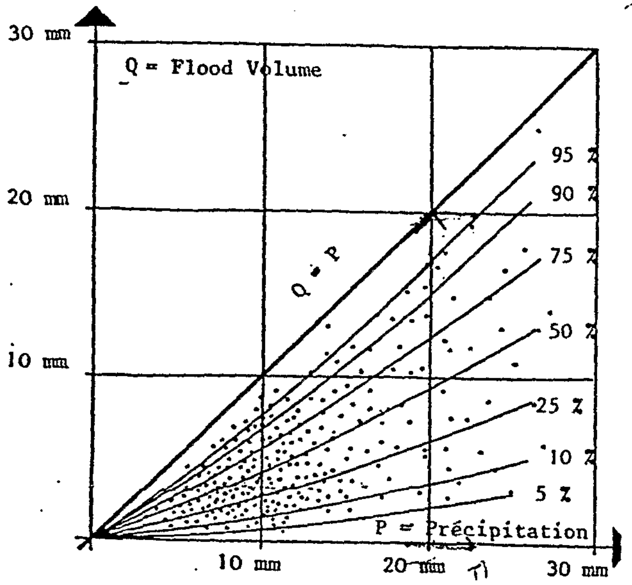


Figure 4. a

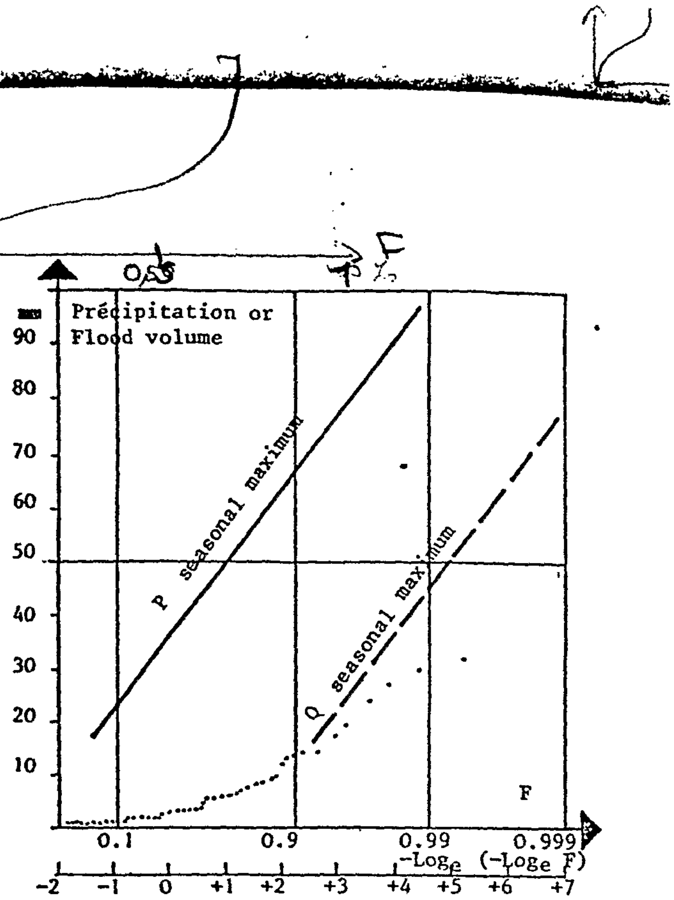


Figure 4. b

For a given value of P, a certain distribution of Q is observed. P – Q depends on numerous complex and interrelated physical factors characterizing the state of the basin before the rainfall Pi generating the flood. It is logical to consider the retention (P – Q) as a random variable with a distribution that can be related to P by the quantiles 5 %..., 95 %... (figure 4a) whose loci, as P varies, form the "quantile curves."

As basin saturation is approached, any additional precipitation tends to be fully incorporated in the discharge ; the quantile curves therefore straighten and finally become parallel to the quadrant bisector with the location of each asymptote depending on the initial basin conditions. The related distribution Hp(Q) tends to become homoscedastic.

Retention certainly has an upper limit amounting to between 50 and 200 mm depending on watershed morphology and type of soil. Accurate knowledge of this maximum retention capacity is unnecessary ; only its average value, during major precipitation events, matters.

Knowing the marginal distribution F(P) and the related distribution Hp(Q), the marginal discharge distribution is given by :

$$G(Q) = \int_{P=0}^{P=\infty} H(Q) \ . \ dF(P)$$

If a sufficiently large sample of rainfall-flood data is available to determine Hp(Q), then G(Q) can be established by summing up Hp(Q) per interval of P.

On conventional Gumbel probability paper, it can thus be shown that the extreme value flood curve is concave towards increasing Q values and asymptotic to a straight line parallel to the precipitation distribution (Figure 4. b).

The distance between the two distribution is not the maximum capacity of the dry ground voids, but simply the statistical expectation of the difference between the water stored and delivered by the basin during the base time. The most realistic approach for estimating this distance is to rely on the least poorly known part of the extreme flood distribution, usually under return periods from one to several decades.

Experience gained from applications to numerous basins (more than 300 applications have been made in France since 1965) shows that the 100-year and 1000-year floods calculated using the Gradex method are compatible with estimations, when available, of catastrophic flood flows which took place on certain rivers in past centuries.

The few decades of actual discharge measurements which are generally available is insufficient to determine the direction of extrapolation but can be used only to give a good estimate of the y-intercept, the starting point, for the distribution of extreme floods. Conventional distributions (e.g. log-normal or log-Pearson III), totally unadapted to distant extrapolations, can be sufficient if the problem is limited to the 10-year or 50-year flood.

If however an estimate of the 1000-year or 10,000-year flood is required, for instance in the case of a reservoir located upstream of an inhabited region, then only rain gage records, summarized by the Gradex, can give the necessary information. Furthermore it is an error to state, as MASSON & BEDIOT (1981), that nothing can be done with less than 30 years of observations. Twenty or even 10 years of good quality rainfall records is sufficient for such an estimation, which tells us more about the risks of extreme floods than 50 or even 100 years of stream gage records. And this is by no means the least advantage of the Gradex method.

To proceed from the distribution of extreme values of average flood flows (flood volume) to that of peak flood flows, the averages can be multiplied by a similarity factor equal to $\overline{P}$, the mean value of the peak to mean flow ratios observed for known floods. This is an adequate approximation if the shape of the flood is a random variable independent of its magnitude, as is generally confirmed by samples of knowns floods. It can also be shown that the estimation procedure is robust with respect to the integration time chosen ; a doubling of this time results in a change of less than 5 % in the calculated 1000-year peak flows.

5. CONCLUSION

Even a few years record of hourly Rainfall is enough to fit a spatial variogram , which allows to estimate the Gradex of the average rainfall on a watershed. Provided a proper time of integration be chosen, about the time length of the overland flow hydrograph, the gradex of rainfall is now considered by French hydrologists as the logical extrapolation guide to extreme floods flows, more significant than any statiscal parameter of the floods distribution (e.g. log-skew), more realistic than any assessment of the so-called P M P.

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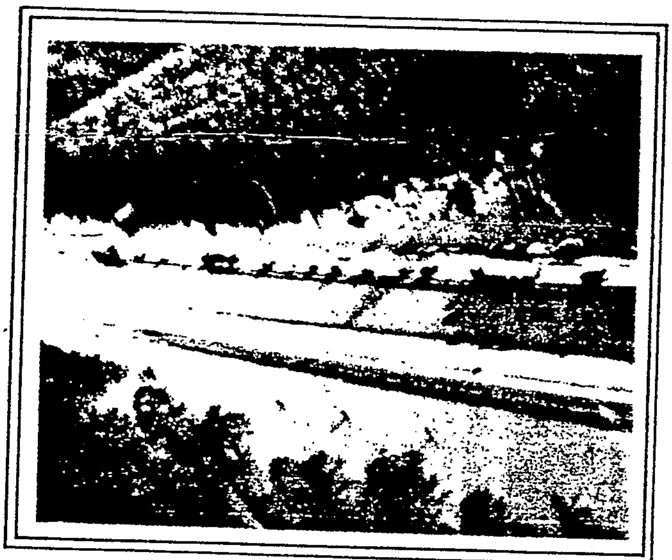
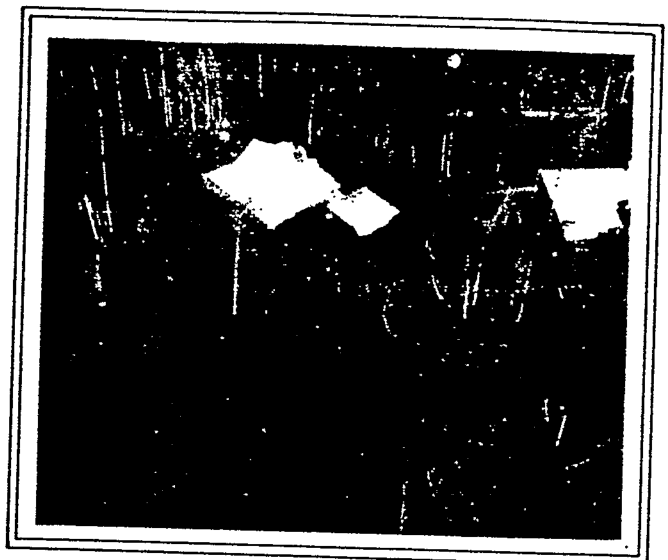

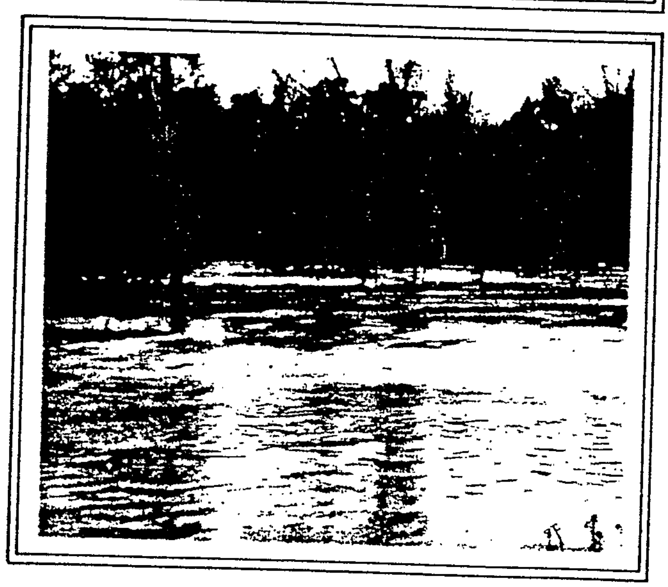

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