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Short Communication

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SIMPLIFIED CALCULATION OF THE ZERO-PLANE DISPLACEMENT FROM WIND-SPEED PROFILES

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ABSTRACT

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A simplified method is proposed to calculate zero-plane displacement from wind-speed profiles. This method is applied to a wheat crop; accuracy is discussed and the effects of instability are incorporated to improve the result.

INTRODUCTION

Under neutral atmospheric conditions, the wind-speed profile above vegetation can be described by:

$$U = (U_*/k) \ln [(z-d)/z_0] \tag{1a}$$

If U is measured at two different heights, this becomes:

$$U_2 - U_1 = (U_*/k) \ln [(z_2-d)/(z_1-d)] \tag{1b}$$

In these expressions, U is wind velocity; z height of measurement above the soil surface; U_* the friction velocity; z_0 roughness height; k von Karman's constant; and d the zero-level displacement caused by the vegetation.

Atmospheric instability causes deviations in these profiles. If the instability is slight, the wind-speed profile may be described by the formula given by Dyer and Hicks (1970):

$$U = (U_*/k)[\ln \{(z-d)/z_0\} + 4L^{-1}(z-d)] \tag{2}$$

where L is Monin—Obukhov's length, which is negative in this case.

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Because d also influences evapotranspiration knowledge of this zero-level displacement is of interest in hydrology. Its value, however, is rather difficult to calculate from measured wind-speed profiles. Though a rapid solution could be obtained by using the computer, a simple method is presented to evaluate d from such measurements. It consists of three parts:

(1) An initial estimate d_0 is obtained from the vegetation height h, using empirical formulae like:

$$d_0 = 0.64h \qquad \text{(Cowan, 1968)}$$

or

$$\log d_0 = 0.9793 \log h - 0.1536 \qquad \text{(Stanhill, 1969)}$$

where d_0 and h are both expressed in cm.

(2) Calculation of d from wind-profile measurements, assuming neutral atmospheric conditions and using a convenient approximation to eq. 1b.

(3) Correction for atmospheric instability, if necessary.

NEUTRAL CONDITIONS

The method is based on the fact that the function $\ln [(z_2 - d)/(z_1 - d)]$ under particular conditions remains close to $A[(z_2 - d)(z_1 - d)]^{-0.5}$, where A is a fixed number depending on the known levels z_1 and z_2 and on the unknown value of d.

Table I gives the value of A, for $z_1 = 1.5$, $z_2 = 3$, d varying from 0 to 1 (all values expressed in m).

For d-values ranging from 0 to 1, A displays only a variation of 6%.

A simple method for calculating d is thus suggested: an estimated value d_0 is assigned a priori to d, e.g., obtained from the vegetation height h.

By using one of the above-mentioned formulae, A can now be calculated.

TABLE I

Values of A for $z_1 = 1.5$, $z_2 = 3$, d is variable

d (m)	$\ln [(3-d)/(1.5-d)]$	$[(3-d)(1.5-d)]^{-0.5}$	A
0	0.693	0.471	1.470
0.2	0.767	0.524	1.464
0.4	0.860	0.591	1.456
0.6	0.981	0.680	1.442
0.8	1.145	0.806	1.421
1.0	1.386	1	1.386

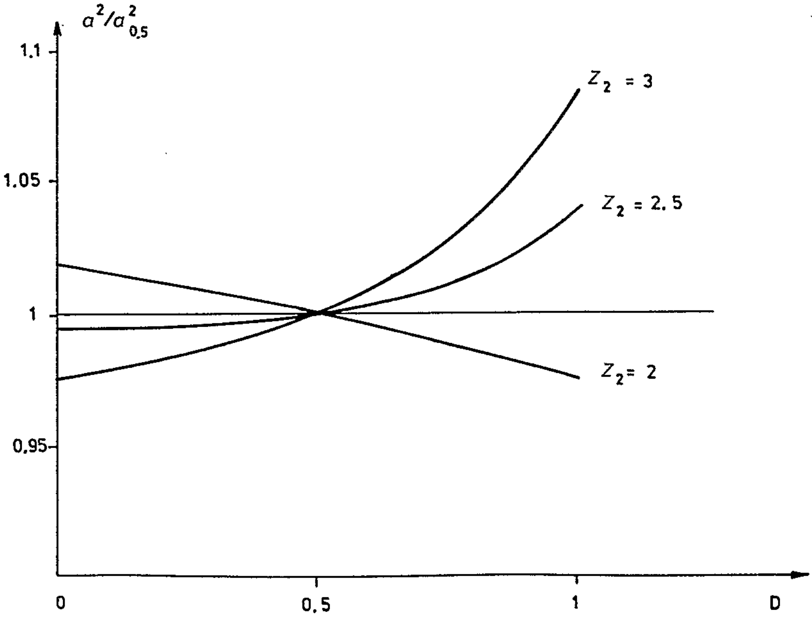

Fig. 1. $a^2/a^2_{0.5}$ with $z_1 = 1.5, z_3 = 4.5$ and three values of z_2.

and one can write for three levels z_1, z_2 and z_3:

$$U_2 - U_1 = \Delta U = (U_*/k)A\,[(z_2 - d)(z_1 - d)]^{-0.5}$$

$$U_3 - U_2 = \Delta U' = (U_*/k)A'\,[(z_3 - d)(z_2 - d)]^{-0.5}$$

and

$$\Delta U/\Delta U' = (A/A')\,[(z_1 - d)/(z_3 - d)]^{-0.5}$$

and with $A'/A = a$:

$$d = [a^2(\Delta U/\Delta U')^2 z_1 - z_3]/[a^2(\Delta U/\Delta U')^2 - 1] \qquad (3)$$

For variations of d from 0 to 1, Fig. 1 indicates values of a^2 related to $a^2_{0.5}$, the value corresponding to $d = 0.5$, for $z_1 = 1.5$ and $z_3 = 4.5$ and three intermediate levels 2, 2.5 and 3.

Though the variation in a^2 is smallest when the intermediate level $(z_2 - d)$ remains close to $[(z_3 - d)(z_1 - d)]^{0.5}$, it always remains small: an estimation error of ± 0.1 in d_0 leads to relative deviations in a^2 ranging from 0.5% when d is small to 3% when d is near 1. Calculating d with eq. 3 would result in an absolute error of 0.01 for $d \leqslant 0.2$ and 0.03 for $d \geqslant 0.8$.

Example: wheat crop in Beauce

This method was applied to wind-speed measurements performed above a dense wheat crop with a mean height $h = 1.10$ m, in Voves, France

(48°30′N, 1°E), during observation of lower atmospheric layers in July 1977 by B. Itier (pers. commun., I.N.R.A.).

Seven profiles of wind speed measured over 10 min. were selected because they corresponded to near-neutral conditions.

Heights of wind-speed measurement were 1.2, 1.6, 2.0, 2.5, 3.0, 5.2, 7.2 and 10.2 m. The constants A and A' were calculated from an a priori estimate $d_0 = 0.7$ m and d was calculated from the mean differences in wind speed.

HOMOGENEITY TEST

For given levels z_1 and z_3, z_2 being variable, eq. 3 shows that the product $b^2 = a^2(\Delta U/\Delta U')^2$ must remain constant. With this test, one can detect a level z_2 of faulty measurements where the corresponding b^2-value systematically deviates from other values; in the present case, level 1.6 had to be eliminated.

CALCULATING d

The value of d was calculated for various values of z_1 and z_3 while taking intermediate levels of z_2 into consideration each time and avoiding levels near z_1 and z_3. The value z_1 was selected successively equal to 1.2, 2.0 and 2.5 and z_3 equal to 5.2, 7 and 10.2 m.

Eq. 1 can be also written:

$$z_3/(b^2 - 1) = [b^2 z_1/(b^2 - 1)] - d$$

where $b^2 = a^2(\Delta U/\Delta U')^2 = (z_3 - d)/(z_1 - d)$

Putting $y = z_3/(b^2 - 1)$ and $x = z_1 b^2/(b^2 - 1)$, $\bar{d}$ is obtained by:

$$\bar{d} = (\Sigma x - \Sigma y)/n$$

n being the number of couples x, y.

With 16 such couples, we obtain:

$$\bar{d} = 0.79 \qquad \text{and} \qquad \bar{d}/\bar{h} = 0.72$$

where d varies from 0.64 to 0.97 and its standard deviation is 0.10.

Fig. 2 shows the relationship between x and y.

SLIGHTLY UNSTABLE CONDITIONS

Let $\lambda = (z_3 - d)/(z_1 - d)$, which is equivalent to b^2 for neutral stability; the error of d as caused by deviations in the wind speeds can now be estimated by:

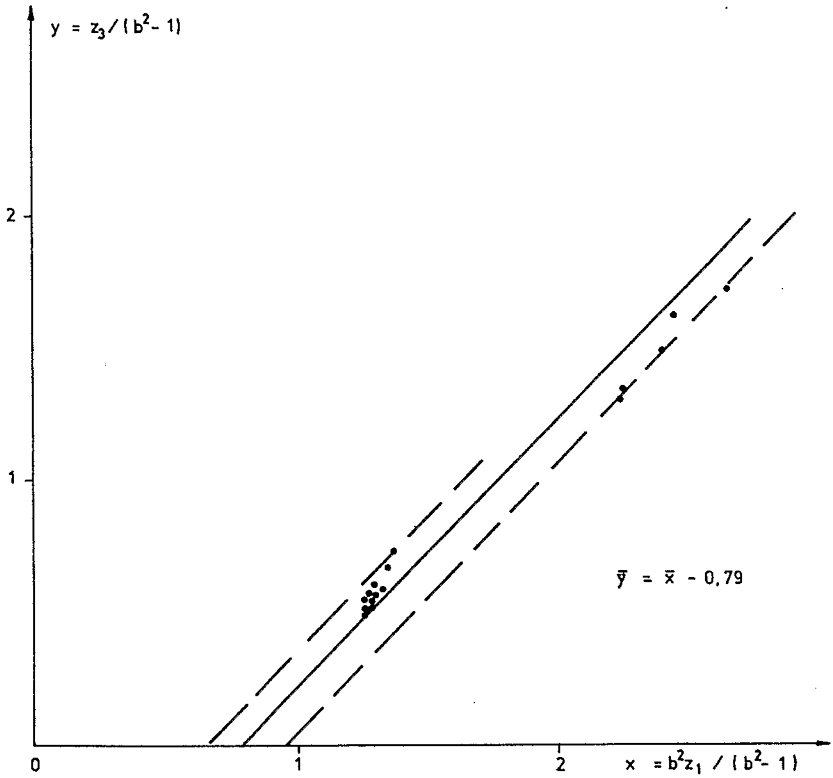

Fig. 2. Application of the method to the case of a wheat crop ($h = 1.1$).

$$\delta d \simeq [2\lambda(z_1 - d)/(\lambda - 1)]\,[\delta(\Delta U/\Delta U')/(\Delta U/\Delta U')]$$

In cases of slight instability, such deviations are described by the second term in eq. 2.

Let:

$$\phi = \delta(\Delta U/\Delta U')/(\Delta U/\Delta U') = \delta(\Delta U)/(\Delta U) - \delta(\Delta U')/\Delta U'$$

where δ denotes deviations caused by instability and where the values themselves refer to the "neutral" case.

From eq. 2:

$$\delta(\Delta U)/\Delta U \cong (4/L)(z_2 - z_1)/\ln[(z_2 - d)/(z_1 - d)]$$

and

$$\delta(\Delta U')/\Delta U \cong (4/L)(z_3 - z_2)/\ln[(z_3 - d)/(z_2 - d)]$$

with $c = (z_2 - d)/(z_1 - d)$, hence:

$$c - 1 = (z_2 - z_1)/(z_1 - d) \qquad \text{and} \qquad c - \lambda = (z_2 - z_3)/(z_1 - d)$$

and

$$F(c,\lambda) = (c-1)/\ln c - (c-\lambda)/\ln(c/\lambda)$$

This finally yields:

$$\phi = (4/L)(z_1 - d)F(c, \lambda)$$

For the correction in d to be applied to account for instability, we find:

$$\delta d = 8\lambda(z_1 - d)^2 F(c, \lambda)/(\lambda - 1)L$$

$F(c, \lambda)$ can be calculated for several values of c and λ; it is always < 0 and $|F(c, \lambda)|$ increases with c when λ is given, and also with λ, when c is given.

Then, δd is always > 0 and, finally, it is smallest when $(z_1 - d)$ is small, for any value of c.

For $L = -200$, the Table II gives the values of δd.

TABLE II

Values of δd for $L = -200$

δd	$z_1 - d = 0.5$			$z_1 - d = 1.3$		
$z_3 - d$ / $z_2 - d$	4.5	6.5	9.5	4.5	6.5	9.5
0.9	0.03	0.05	0.06			
1.3	0.04	0.05	0.07			
1.8	0.04	0.06	0.08	0.10	0.14	0.19
2.3	0.05	0.06	0.08	0.11	0.15	0.20
4.5		0.08	0.10		0.21	0.24

In general, the smallest corrections, and hence the best estimate of d are obtained with the lowest levels of z_1.

In the foregoing example, one can see on Fig. 2 the effect of instability; when z_1 (and also x) increases, δd increases.

During the measurements, L was about -200 m. The corrections δd from Table II can be used, and a new estimate of $\bar{d}$ gives the improved result:

$$\bar{d} = 0.69 \qquad \text{and} \qquad \bar{d}/\bar{h} = 0.63$$

Now the standard deviation of d is only 0.06.

The value of $\bar{d}$ is actually very close to value $d_0 = 0.7$, chosen a priori from Cowan's (1968) and Stanhill's (1969) estimates.

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