Short Communication

[5]

SIMPLIFIED CALCULATION OF THE ZERO-PLANE DISPLACEMENT FROM WIND-SPEED PROFILES

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ABSTRACT


A simplified method is proposed to calculate zero-plane displacement from wind-speed profiles. This method is applied to a wheat crop; accuracy is discussed and the effects of instability are incorporated to improve the result.

INTRODUCTION

Under neutral atmospheric conditions, the wind-speed profile above vegetation can be described by:

\[ U = \frac{U_*}{k} \ln \left( \frac{z-d}{z_0} \right) \]  

(1a)

If \( U \) is measured at two different heights, this becomes:

\[ U_2 - U_1 = \frac{U_*}{k} \ln \left( \frac{(z_2-d)}{(z_1-d)} \right) \]  

(1b)

In these expressions, \( U \) is wind velocity; \( z \) height of measurement above the soil surface; \( U_* \) the friction velocity; \( z_0 \) roughness height; \( k \) von Karman's constant; and \( d \) the zero-level displacement caused by the vegetation.

Atmospheric instability causes deviations in these profiles. If the instability is slight, the wind-speed profile may be described by the formula given by Dyer and Hicks (1970):

\[ U = \frac{U_*}{k} \left[ \ln \left( \frac{(z-d)}{z_0} \right) + 4L^{-1}(z-d) \right] \]  

(2)

where \( L \) is Monin–Obukhov's length, which is negative in this case.

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Because $d$ also influences evapotranspiration knowledge of this zero-level displacement is of interest in hydrology. Its value, however, is rather difficult to calculate from measured wind-speed profiles. Though a rapid solution could be obtained by using the computer, a simple method is presented to evaluate $d$ from such measurements. It consists of three parts:

1. An initial estimate $d_0$ is obtained from the vegetation height $h$, using empirical formulae like:

$$d_0 = 0.64h \quad \text{(Cowan, 1968)}$$

or

$$\log d_0 = 0.9793 \log h - 0.1536 \quad \text{(Stanhill, 1969)}$$

where $d_0$ and $h$ are both expressed in cm.

2. Calculation of $d$ from wind-profile measurements, assuming neutral atmospheric conditions and using a convenient approximation to eq. 1b.

3. Correction for atmospheric instability, if necessary.

### NEUTRAL CONDITIONS

The method is based on the fact that the function $\ln \left[\frac{(z_2 - d)(z_1 - d)}{(z_2 - d)(z_1 - d)}\right]$ under particular conditions remains close to $A \left[(z_2 - d)(z_1 - d)\right]^{-0.5}$, where $A$ is a fixed number depending on the known levels $z_1$ and $z_2$ and on the unknown value of $d$.

Table I gives the value of $A$, for $z_1 = 1.5$, $z_2 = 3$, $d$ varying from 0 to 1 (all values expressed in m).

For $d$-values ranging from 0 to 1, $A$ displays only a variation of 6%.

A simple method for calculating $d$ is thus suggested: an estimated value $d_0$ is assigned a priori to $d$, e.g., obtained from the vegetation height $h$.

By using one of the above-mentioned formulae, $A$ can now be calculated.

<table>
<thead>
<tr>
<th>$d$ (m)</th>
<th>$\ln \left[\frac{(3 - d)(1.5 - d)}{(3 - d)(1.5 - d)}\right]$</th>
<th>$\left[(3 - d)(1.5 - d)\right]^{-0.5}$</th>
<th>$A$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.693</td>
<td>0.471</td>
<td>1.470</td>
</tr>
<tr>
<td>0.2</td>
<td>0.767</td>
<td>0.524</td>
<td>1.464</td>
</tr>
<tr>
<td>0.4</td>
<td>0.860</td>
<td>0.591</td>
<td>1.456</td>
</tr>
<tr>
<td>0.6</td>
<td>0.981</td>
<td>0.680</td>
<td>1.442</td>
</tr>
<tr>
<td>0.8</td>
<td>1.145</td>
<td>0.806</td>
<td>1.421</td>
</tr>
<tr>
<td>1.0</td>
<td>1.386</td>
<td>1</td>
<td>1.386</td>
</tr>
</tbody>
</table>
and one can write for three levels $z_1$, $z_2$ and $z_3$:

$$U_2 - U_1 = \Delta U = (U_*/k)A [(z_2 - d)(z_1 - d)]^{-0.5}$$

$$U_3 - U_2 = \Delta U' = (U_*/k)A' [(z_3 - d)(z_2 - d)]^{-0.5}$$

and

$$\Delta U/\Delta U' = (A/A')[(z_1 - d)/(z_3 - d)]^{-0.5}$$

and with $A'/A = a$:

$$d = [(a^2(\Delta U/\Delta U')^2 z_1 - z_3) /[a^2(\Delta U/\Delta U')^2 - 1] \quad (3)$$

For variations of $d$ from 0 to 1, Fig. 1 indicates values of $a^2$ related to $a^2_{0.5}$, the value corresponding to $d = 0.5$, for $z_1 = 1.5$ and $z_3 = 4.5$ and three intermediate levels 2, 2.5 and 3.

Though the variation in $a^2$ is smallest when the intermediate level $(z_2 - d)$ remains close to $[(z_3 - d)(z_1 - d)]^{0.5}$, it always remains small: an estimation error of ±0.1 in $d_0$ leads to relative deviations in $a^2$ ranging from 0.5% when $d$ is small to 3% when $d$ is near 1. Calculating $d$ with eq. 3 would result in an absolute error of 0.01 for $d \leq 0.2$ and 0.03 for $d \geq 0.8$.

**Example: wheat crop in Beauce**

This method was applied to wind-speed measurements performed above a dense wheat crop with a mean height $h = 1.10$ m, in Voves, France.

Seven profiles of wind speed measured over 10 min. were selected because they corresponded to near-neutral conditions.

Heights of wind-speed measurement were 1.2, 1.6, 2.0, 2.5, 3.0, 5.2, 7.2 and 10.2 m. The constants $A$ and $A'$ were calculated from an a priori estimate $d_0 = 0.7$ m and $d$ was calculated from the mean differences in wind speed.

**HOMOGENEITY TEST**

For given levels $z_1$ and $z_3$, $z_2$ being variable, eq. 3 shows that the product $b^2 = a^2 (\Delta U/\Delta U')^2$ must remain constant. With this test, one can detect a level $z_2$ of faulty measurements where the corresponding $b^2$-value systematically deviates from other values; in the present case, level 1.6 had to be eliminated.

**CALCULATING $d$**

The value of $d$ was calculated for various values of $z_1$ and $z_3$ while taking intermediate levels of $z_2$ into consideration each time and avoiding levels near $z_1$ and $z_3$. The value $z_1$ was selected successively equal to 1.2, 2.0 and 2.5 and $z_3$ equal to 5.2, 7 and 10.2 m.

Eq. 1 can be also written:

$$\frac{z_3}{(b^2 - 1)} = \left[ \frac{b^2 z_1}{(b^2 - 1)} \right] - d$$

where $b^2 = a^2 (\Delta U/\Delta U')^2 = (z_3 - d)/(z_1 - d)$

Putting $y = z_3/(b^2 - 1)$ and $x = z_1 b^2/(b^2 - 1)$, $\bar{d}$ is obtained by:

$$\bar{d} = (\Sigma x - \Sigma y)/n$$

$n$ being the number of couples $x, y$.

With 16 such couples, we obtain:

$$\bar{d} = 0.79 \quad \text{and} \quad \bar{d}/h = 0.72$$

where $d$ varies from 0.64 to 0.97 and its standard deviation is 0.10.

Fig. 2 shows the relationship between $x$ and $y$.

**SLIGHTLY UNSTABLE CONDITIONS**

Let $\lambda = (z_3 - d)/(z_1 - d)$, which is equivalent to $b^2$ for neutral stability; the error of $d$ as caused by deviations in the wind speeds can now be estimated by:
\[ \delta d \approx [2\lambda(z_1 - d)/(\lambda - 1)] \left[ \delta (\Delta U'/\Delta U)/(\Delta U/\Delta U') \right] \]

Fig. 2. Application of the method to the case of a wheat crop \((h = 1.1)\).

In cases of slight instability, such deviations are described by the second term in eq. 2.

Let:

\[ \phi = \delta (\Delta U'/\Delta U)/(\Delta U/\Delta U') = \delta (\Delta U)/(\Delta U) - \delta (\Delta U'/\Delta U') \]

where \(\delta\) denotes deviations caused by instability and where the values themselves refer to the “neutral” case.

From eq. 2:

\[ \delta (\Delta U)/\Delta U \approx (4/L)(z_2 - z_1)/\ln [(z_2 - d)/(z_1 - d)] \]

and

\[ \delta (\Delta U'/\Delta U) \approx (4/L)(z_3 - z_2)/\ln [(z_3 - d)/(z_2 - d)] \]

with \(c = (z_2 - d)/(z_1 - d)\), hence:

\[ c - 1 = (z_2 - z_1)/(z_1 - d) \quad \text{and} \quad c - \lambda = (z_2 - z_3)/(z_1 - d) \]
and

\[ F(c, \lambda) = (c - 1)/\ln c - (c - \lambda)/\ln (c/\lambda) \]

This finally yields:

\[ \phi = (4/L)(z_1 - d)F(c, \lambda) \]

For the correction in \( d \) to be applied to account for instability, we find:

\[ \delta d = 8\lambda(z_1 - d)^2 F(c, \lambda)/(\lambda - 1)L \]

\( F(c, \lambda) \) can be calculated for several values of \( c \) and \( \lambda \); it is always < 0 and \( |F(c, \lambda)| \) increases with \( c \) when \( \lambda \) is given, and also with \( \lambda \), when \( c \) is given.

Then, \( \delta d \) is always > 0 and, finally, it is smallest when \( (z_1 - d) \) is small, for any value of \( c \).

For \( L = -200 \), the Table II gives the values of \( \delta d \).

**TABLE II**

Values of \( \delta d \) for \( L = -200 \)

<table>
<thead>
<tr>
<th>( \delta d )</th>
<th>( z_1 - d = 0.5 )</th>
<th>( z_1 - d = 1.3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( z_2 - d )</td>
<td>( z_3 - d )</td>
<td>( z_3 - d )</td>
</tr>
<tr>
<td>0.9</td>
<td>0.03</td>
<td>0.05</td>
</tr>
<tr>
<td>1.3</td>
<td>0.04</td>
<td>0.05</td>
</tr>
<tr>
<td>1.8</td>
<td>0.04</td>
<td>0.06</td>
</tr>
<tr>
<td>2.3</td>
<td>0.05</td>
<td>0.06</td>
</tr>
<tr>
<td>4.5</td>
<td>0.08</td>
<td>0.10</td>
</tr>
</tbody>
</table>

In general, the smallest corrections, and hence the best estimate of \( d \) are obtained with the lowest levels of \( z_1 \).

In the foregoing example, one can see on Fig. 2 the effect of instability; when \( z_1 \) (and also \( x \)) increases, \( \delta d \) increases.

During the measurements, \( L \) was about \(-200\) m. The corrections \( \delta d \) from Table II can be used, and a new estimate of \( \bar{d} \) gives the improved result:

\[ \bar{d} = 0.69 \quad \text{and} \quad \bar{d}/h = 0.63 \]

Now the standard deviation of \( d \) is only 0.06.

The value of \( \bar{d} \) is actually very close to value \( d_0 = 0.7 \), chosen a priori from Cowan’s (1968) and Stanhill’s (1969) estimates.
REFERENCES