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REFLECTIONS OF LOW FREQUENCY EQUATORIAL WAVES ON PARTIAL BOUNDARIES

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ABSTRACT

We develop a linear theory for the effects of partial boundaries on low frequency waves of the kind thought to be important in the seasonal and interannual variations of the equatorial circulation. The western partial boundary case (e.g. Brazil) differs from the eastern one (e.g., the Gulf of Guinea) by the presence of short Rossby waves trapped along the north-south part of the boundary which form a boundary current accomplishing the required meridional redistribution of the zonal mass flux. There is a discontinuity in the dynamic topography at the corner of the east-west current at this point.

Calculations for the world's equatorial ocean basin shapes are discussed. Calculations carried out for equatorial islands show that propagation of such low frequency waves will not be affected significantly by any island of the real equatorial ocean.

INTRODUCTION

The dynamical effect of the reversal sign of the Coriolis force at the equator makes it a very effective wave guide which supports planetary waves unique to the tropics. Plane waves solution to linearized equations have a faster propagation near the equator than they have at higher latitude. As a consequence, the equatorial ocean exhibits a strong and rapid response to seasonal and interannual variations in the wind stress.

We develop a complete linear theory for the effects of partial boundaries on the low frequency waves which play an important role in these variations. By partial boundaries, we mean a coast which presents discontinuities (or caps) and which is not a straight north-south coast. For example, we may think of the coast of the

Gulf of Guinea and Brazil in the Atlantic ocean, the New Guinea coast in the Pacific and the Somali coast in the Indian ocean. Furthermore, islands in the tropical oceans are a particular case of partial boundaries.

Our interest is in the effect of such boundaries on the wave motions essential to basin wide adjustment rather than on details of the perturbations near the boundaries.

We will solve the linear shallow water equations appropriate to a single baroclinic mode using the usual equatorial scaling, namely, the length scale : $L = (gH/\beta^2)^{1/4}$ (the equatorial radius of deformation) and the timescale $T = (gH\beta^2)^{-1/4}$ with H the equivalent depth of the baroclinic mode, g the acceleration due to gravity and β the meridional derivative of the Coriolis parameter.

Since we are interested in low frequency motion, the frequency is small compared to the equatorial scaling frequency so that :

1 - The mixed gravity wave and short Rossby waves cannot propagate very far into the interior before friction destroys them.

2 - Geostrophic balance holds in the meridional direction.

3 - The westward propagating long Rossby waves are approximately non dispersive.

We choose as our canonical problem the large t asymptotic flow that results when a wave source is switched on at t = O and remains steady thereafter ; as discussed in Cane and Sarachik (1976), the solution for a periodic forcing or any forcing can be deduced from the solution to this problem.

Cane and Sarachik (1976) (see also Anderson and Rowlands, 1976) have shown that for large t, the asymptotic motions are of 3 kinds :

(i) Equatorial Kelvin waves, propagating energy eastward with u and h proportional to ψ_o, the zeroth order Hermite function : $\psi_o = \pi^{-1/4} e^{-y^2/2}$. The large t response is steady for an H(t) time dependence and is independent of x..

(ii) Long Rossby waves, propagating energy westward. The equatorial Kelvin wave has v = O and the long Rossby waves have $v \simeq 0$; both satisfy the geostrophic relation

$$yu + \frac{\partial h}{\partial y} = 0 \qquad (1)$$

Again, the large t asymptotic form is independent of t and x.

(iii) Short Rossby waves (including the mixed Rossby-gravity wave) of the form (see Cane and Sarachik, 1977 p 404).

$$(u^s, v^s, h^s) = [-\frac{\partial}{\partial y}, \frac{\partial}{\partial x}, y] \{J_o(2\sqrt{xt})\ \chi(y)\} \qquad (2)$$

Note that as $t \to \infty$, $J_o(2\sqrt{xt}) \longrightarrow \delta(x)$; a sum of such modes is an ever thinning boundary layer trapped at x = 0.

The usual boundary conditions prescribed at the eastern and western full boundaries must be carefully applied to the case of partial boundary to ensure conservation of mass. For a full boundary, no normal flow exists at the longitude of the boundary, but if the boundary does *not* extend accross all latitudes in the basin, the condition of no normal flow can no longer be applied in the open ocean region.

The plan of the remainder of this paper is as follow : in section 2, we solve the problem of a Kelvin wave and Rossby waves separately, for an eastern partial boundary. The case for a western partial boundary will differ from the eastern one by the presence of short Rossby waves trapped along the north-south part of the boundary and is discussed in section 3. In section 4, we will extend the results to a more complex geometry (for example a "zigzag" step coast). In section 5, we summarize our results and consider their applicability to the world ocean.

EASTERN PARTIAL BOUNDARY CASE

Incoming Kelvin wave

We first consider the case of a unit amplitude Kelvin wave impinging on a partial coast at $X = X_B$ extending from the latitude y = b at south to infinity at north (see figure 1). The part of the wave north of y = b will be reflected as a set of long Rossby waves as in the case of a full boundary. South of b and east of X_B, a transmitted Kelvin wave of amplitude T^K and short Rossby waves are allowed to propagate.

Therefore west of X_B, we may write :

$$u^W = u_R + \psi_o\ (y), \qquad (3)$$
$$h^W = h_R + \psi_o\ (y)\ ;$$

while east of X_B

$$u^E = u_s + T^K\psi_o\ (y), \qquad (4)$$
$$h^E = h_s + T^K\psi_o\ (y)\ ;$$

where (u_R, h_R) are components of long Rossby waves and (u_s, h_s) are components of short Rossby waves.

At $X = X_B$ and south of the corner, the matching conditions are that u and h be continuous at $X = X_E$, $y<b$ so that $u^E = u^W$ $h^E = h^W$ at $X = X_E$, $y<b$.

Making use of (2),

$$u^W = T^K\psi_o (y) - \frac{\partial \chi}{\partial y},$$

$$h^W = T^K\psi_o (y) + y\chi. \qquad (5)$$

Since (u^W, h^W) satisfies (1), substituting (5) into (3) yields :

$$y\left[T\psi_o (y) - \frac{\partial \chi}{\partial y}\right] + \frac{\partial}{\partial y}\left[T\psi_o (y) + y\chi\right] = 0$$

or, since the Kelvin wave is also geostrophic,

$$- y \frac{\partial \chi}{\partial y} + \frac{\partial}{\partial y}\left[y\chi\right] = 0$$

Therefore $\chi = 0$ for $y<b$; that means there is no reflected short Rossby wave and only Rossby waves and Kelvin wave are reflected and transmitted.

Integrating (1) across the boundary longitude yields :

$$\int_{b-}^{b+} yudy + h(b+) - h(b-) = 0$$

Therefore there can be no jump in h at y = b.

We now summarize conditions that must be satisfy at an eastern partial boundary :

(a) h and u must be continuous in x at the boundary longitude. South of the boundary, the Rossby modes must conspire to cancel the untransmitted part of the Kelvin wave to ensure continuity in x :

$$h_R = u_R = (T^K - 1)\psi_o \qquad \text{for } y<b \qquad (6)$$

(b) Above the boundary, the conditions are the same as for a full boundary, namely, no normal flow and the total h a constant :

$$u_R = -\psi_o$$

$$h_R = D^K - \psi_o \qquad \text{for } y>b \qquad (7)$$

(c) h is continuous in y at the corner $X = X_B$, $y = b$. Using the expressions (6) and (7) for h this implies

$$\overset{\text{south}}{h_R(b)} = (T^K - 1)\psi_o(b) = D^K - \psi_o(b) = \overset{\text{north}}{h_R(b)}$$

$$\text{or} \quad T^K\psi_o(b) = D^K \tag{8}$$

(d) The Rossby waves generated to the west of the boundary are orthogonal to the Kelvin mode (see Cane and Sarachik, 1979, Appendix A). This condition may be expressed in the form

$$[(\psi_o, \psi_o), (u_R, h_R)] = 0 \tag{9}$$

where the inner product is defined by

$$[(u_a, h_a), (u_b, h_b)] = \int_{-\infty}^{+\infty} (u_a u_b + h_a h_b)\,dy.$$

Substituting (6) and (7) into (9) yields

$$2(T^K - 1)\int_{-\infty}^{b} \psi_o^2 + D^K \int_b^{\infty} \psi_o - 2\int_b^{\infty} \psi_o^2 = 0$$

Finally using (8) and the normalization condition $\left[\int_{-\infty}^{+\infty} \psi_o^2 = 1\right]$,

$$T^K = \frac{2}{2\int_{-\infty}^{b} \psi_o^2\,dy + \psi_o(b)\int_b^{\infty} \psi_o(y)\,dy} \tag{10}$$

Equations (6), (7), (8) and (10) give the entire solution for the problem. We postpone discussion after the incoming Rossby wave case has been solved.

Incident Rossby wave motion

Another possible situation is to have a set of long Rossby waves propagating from the east and encountering the corner at y = b. Let $\hat{u}$ and $\hat{h}$ be the components of the incoming Rossby waves ($\hat{v} \simeq 0$). Since it is made up of long Rossby waves, the geostrophic relation (1) still holds so, as before, we may conclude that there are no short Rossby waves generated at the boundary. However, there is the possibility of a Kelvin wave (of amplitude T^R) being reflected eastward from the boundary. Hence for $X > X_B$, $u^E = T^R\psi_o + \hat{u}$ and $h^E = T^R\psi_o + \hat{h}$.

On the other hand there can be no Kelvin wave west of X_B; the solution there must be solely Rossby waves (u_R, h_R).

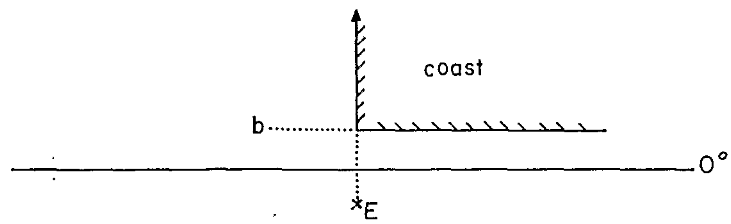

Fig.1: Diagram of partial boundary near equator used in calculating transmission coefficients of reflected planetary waves.

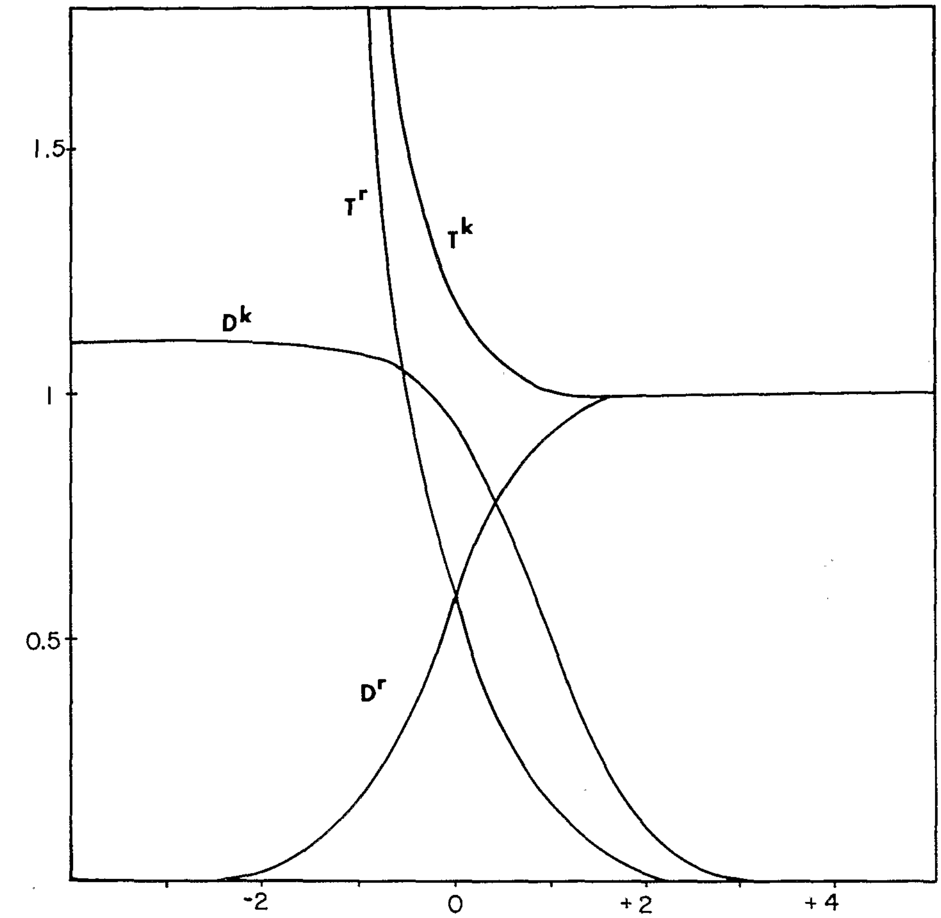

Fig.2: Transmission coefficients of Kelvin mode and height constants along upper wall for partial boundary as functions of distance b from the equator to the zonal coast. T^k transmission coefficient for incident Kelvin waves, T^r transmission coefficient for incident Rossby modes with unit amplitude at corner, D^k height constant set-up for incident Kelvin waves, D^r height constant for incident Rossby waves.

Since there is no incident zonal velocity above the boundary, we have :

$$u_R = 0 \qquad h_R = D^R = \text{constant} \qquad \text{for } y>b \text{ at } X = X_B, \tag{11}$$

while continuity in x below the boundary means :

$$\begin{aligned} u_R &= \hat{u} + T^R\psi_o, \\ h_R &= \hat{h} + T^R\psi_o. \end{aligned} \qquad \text{at } X = X_B \qquad y<y_b \tag{12}$$

As in the previous case h must be continuous in y at the corner:

$$D^R = \hat{h}(b) + T^R\psi_o(b). \tag{13}$$

West of X_B, the Kelvin wave amplitude is zero. Therefore projecting a Kelvin wave onto the solution west of the boundary using (12) and (13) gives :

$$\hat{h}(b)\int_b^\infty \psi_o(y)\,dy + T^R\psi_o(b)\int_b^\infty \psi_o(y)\,dy + 2T^R\int_{-\infty}^b \psi_o^2(y) + $$
$$+ \int_{-\infty}^b (\hat{u}+\hat{h})\psi_o(y) = 0$$

Finally, solving for T^R, we get :

$$T^R = -\frac{\hat{h}(b)\int_b^\infty \psi_o\,dy}{2\int_{-\infty}^b \psi_o^2(y) + \psi_o(b)\int_b^\infty \psi_o(y)\,dy} \tag{14}$$

or $T^R = -\hat{h}(b)T^K\int_b^\infty \psi_o(y)\,dy$

We note that T^R and D^R (and by extension the entire solution for this case) depend only on the value of the height of the Rossby mode at y = b, i.e. on $\hat{h}(b)$.

Results

Figure 2 displays the solution coefficients T and D for an incoming Kelvin wave and for an incident Rossby wave motion of unit height amplitude at the corner. For the Kelvin wave case, we obtain the surprising result that the Kelvin wave transmission coefficient is greater than one : the curve asymptotes to one from above as b recedes to infinity to the north (b>0), but T^K goes to infinity as b increases south of the equator (i.e. as the corner is situated south of the equator and $b \to -\infty$). To understand this behavior, we must consider the entire mechanism.

First, the transmitted equatorial Kelvin wave can be considered as a coastal Kelvin wave. To see that the two are the same, we normalize the equatorial Kelvin wave to be one at y = b and define $\eta = y - b$, the distance from the coast. Therefore :

$$u = h = e^{b^2/2}\, e^{-y^2/2} = e^{-b^2/2}\, e^{-(b+\eta)^2/2} = e^{-b\eta}\, e^{-b^2/2}$$

η is small and $e^{-\eta^2/2} \simeq 1$

1/b is liked a (scaled) radius of deformation at y = b, so the wave amplitude behaves like the usual f-plane coastal Kelvin wave. The amplitude of this wave must be such as to make its height at the corner match the constant height set up among the boundary as given by (8).

In the other hand, as b goes south ($b \to -\infty$), the reflected Rossby waves have the same structure as they would if the barrier were infinite. In particular, the height sets up to have the same value as for an infinite coastline case. The process that brings this set up has been discussed by Anderson and Rowlands (1976) and Cane and Sarachik (1977). Mass is carried towards the poles in a meridional current that is like a coastal Kelvin wave and is geostrophically balanced. The transport in this current at y = b is given by :

$$V\ (y=b) = \int_{-\infty}^{X_E} v\ (y=b)\, dx = \frac{D}{b}$$

If we now calculate the transport under the boundary carried by the transmitted Kelvin, we find by asymptotic expansions as $b \to \infty$, that :

$$T^K \int_{-\infty}^{b} \psi_o \approx \frac{D}{b} = \sqrt{2}\ \pi^{1/4} \qquad \text{as } b \to -\infty$$

The mass transport around the corner is just what the coastal Kelvin wave under the boundary is able to supply.

As b goes north ($b \to +\infty$), the amplitude of the transmitted Kelvin wave goes to one and the height along the north-south coast goes to zero. There is no transmitted Rossby wave.

For an incoming Rossby wave motion under the corner, away from the equator to the north, the height along the wall is simply the height of the incident Rossby modes at the corner and no Kelvin wave is reflected. This is to be expected since the height along the wall will have an increasing smaller projection on the equatorial Kelvin mode as the boundary recedes from the equator.

THE WESTERN PARTIAL BOUNDARY CASE

Incoming Kelvin wave

We now deal with a western partial boundary extending from $y = -\infty$ to $y = b$ at a longitude $X = X_W$ (figure 3). The only possible motion generated at the corner are long Rossby waves to the west (above the east-west coast), and Kelvin waves plus boundary-trapped short Rossby waves to the east of the corner.

West of the longitude X_W, the solution is made up of the incoming Kelvin wave (assumed to have unit amplitude), and a set of long Rossby waves reflected at the boundary at X_W. The part of the Kelvin wave not reflected in Rossby waves is transmitted with an amplitude T^K. Equations (3) and (4) still hold. Both u and h are continuous in x, so that $u^E = u^W$ and $h^E = h^W$ for $y>b$. Since u and h are in geostrophic balance at $X = X_W$ $y>b$, we can say as in section 2 that there is no disturbance created by short Rossby waves and

$$h^R = u^R = (T^K - 1)\psi_o(y)$$

At $X = X_W$ and $y<b$, the boundary condition is $u^E = 0$; hence

$$\left.\begin{aligned} u^E &= T^K\psi_o - \chi_y = 0 \; ; \\ h^E &= T^K\psi_o + y\chi \end{aligned}\right\} \quad X = X_W \qquad y<b$$

This condition leads to :

$$\chi(y) = T^K\int_{-\infty}^{y} \psi_o(y) + C, \qquad C = \chi(-\infty) = 0 \; ; \tag{15}$$

$$\text{and } h^E = T^K\psi_o(y) + y\, T^K\int_{-\infty}^{y} \psi_o(y) \qquad \text{at } X = X_W \quad y<b \tag{16}$$

So at $X = X_W$, $y = b_+$, $h^E = T^K\psi_o(b)$,

$y = b_-$, $h^E = T^K\psi_o(b) + bT^K\int_{-\infty}^{b}\psi_o(y)$.

Discontinuities in h are thus possible at $y = b$. Since west of the boundary u and h are in geostrophic balance for all y, we must allow for the possibility on an infinite zonal velocity at $y = b$

to balance the jump in h : i.e. $u = A\delta(y - b)$ at $y = b$. Hence west of X_B, we have

$$u^W = T^K\psi_o(y) + A\delta(y - b) \quad ; \quad h^W = T^K\psi_o(y) \tag{17}$$

Using the orthogonality and completness properties of the eigen functions of the shallow water equations, we may project the Kelvin wave on the solution east of the barrier :

$$2T\int_{-\infty}^{+\infty}\psi_o^2 = 2T\int_b^{\infty}\psi_o^2 + A\psi_o(b) + T\int_{-\infty}^{b}\{\psi_o(y) + y\int_{-\infty}^{y}\psi_o(y')dy'\}\psi_o(y)dy \ ;$$

using the fact that $y\psi_o = -\psi_o'$ and integrating the last term by parts leads to :

$$A = T\int_{-\infty}^{b}\psi_o dy \tag{18}$$

Note that it follows from (2), (15) and (18) that

$$\int_{X_W}^{-\infty} V(y=b)\,dx = -A$$

That is, the eastward transport A in the boundary current at the corner is all carried southward by the boundary current along the north-south coast. There is no net flux in this boundary layer (see Cane and Sarachik, 1977) ; its only role is to redistribute zonally the mass flux so that the Kelvin wave may carry it off.

Projecting the Kelvin wave west of the boundary where the Kelvin wave amplitude is known to be one leads to :

$$T^K = \frac{2\int_b^{\infty}\psi_o^2}{2\int_b^{\infty}\psi_o^2 + \psi_o(b)\int_{-\infty}^{b}\psi_o(y)dy} \tag{19}$$

Incident Rossby wave motions

In this section, we are looking at the effects of the western corner on an incoming Rossby wave.

Again, the only possible motions generated at the corner are a set of long Rossby waves to the west, reflected Kelvin wave and boundary-trapped short Rossby waves to the east. As in the previous section, there is no perturbation created by short Rossby waves above the latitude of the corner ($\chi = 0$ for $y>b$). We write the solution as

$$\left.\begin{aligned} u^r &= \hat{u} + T^R\psi_o \\ h^r &= \hat{h} + R^R\psi_o \end{aligned}\right\} \quad \text{at } X = X_W \qquad y>b \tag{20}$$

$$\left.\begin{aligned} u^E &= 0 = \hat{u} + T^R\psi_o - \frac{\partial \chi}{\partial y} \\ h^E &= \hat{h} + T^R\psi_o + y\chi \end{aligned}\right\} \quad \text{at } X = X_W \qquad y<b \tag{21}$$

Therefore, $u^E = 0$ on the wall implies

$$\chi(y) = \int_{-\infty}^{y} (\hat{u} + T^R\psi_o) \qquad \text{for } y<b \tag{22}$$

As in the previous section, , there is a discontinuity for h at y = b and an infinitesimally thin boundary current must be allowed for to balance it :

$$\begin{aligned} u^R &= \hat{u} + T^R\psi_o(y) + B\delta(y - b) \\ h^R &= \hat{h} + T^R\psi_o(y) \\ h^E &= \hat{h} + T^R\psi_o(y) + y\int_{-\infty}^{b} (\hat{u} + T^R\psi_o)\,dy \end{aligned} \tag{23}$$

Projecting the Kelvin wave on the solution east of the boundary leads to :

$$B = \int_{-\infty}^{b} (\hat{u} + T^R\psi_o)\,dy \tag{24}$$

The zero projection of the Kelvin wave on the Rossby waves west of the boundary leads to :

$$\int_{-\infty}^{+\infty} (u^R,\ h^R)(\psi_o\ ,\ \psi_o)\,dy = 0 = \int_{b}^{\infty} (\hat{u} + \hat{h})\psi_o dy + 2T^R\int_{b}^{\infty} (\psi_o^2)\,dy + B\psi_o(b)$$

and with equation (24) :

$$T^R = -\frac{\int_{b}^{\infty} (\hat{u} + \hat{h})\psi_o(y)\,dy + \psi_o(b)\int_{-\infty}^{b} \hat{u}\,dy}{2\int_{b}^{\infty} \psi_o^2(y) + \psi_o(b)\int_{-\infty}^{b} \psi_o(y)\,dy} \tag{25}$$

Results

Figure 3 shows the results obtained for the western boundary case. For an incoming Kelvin wave, A^K the amplitude of the boundary

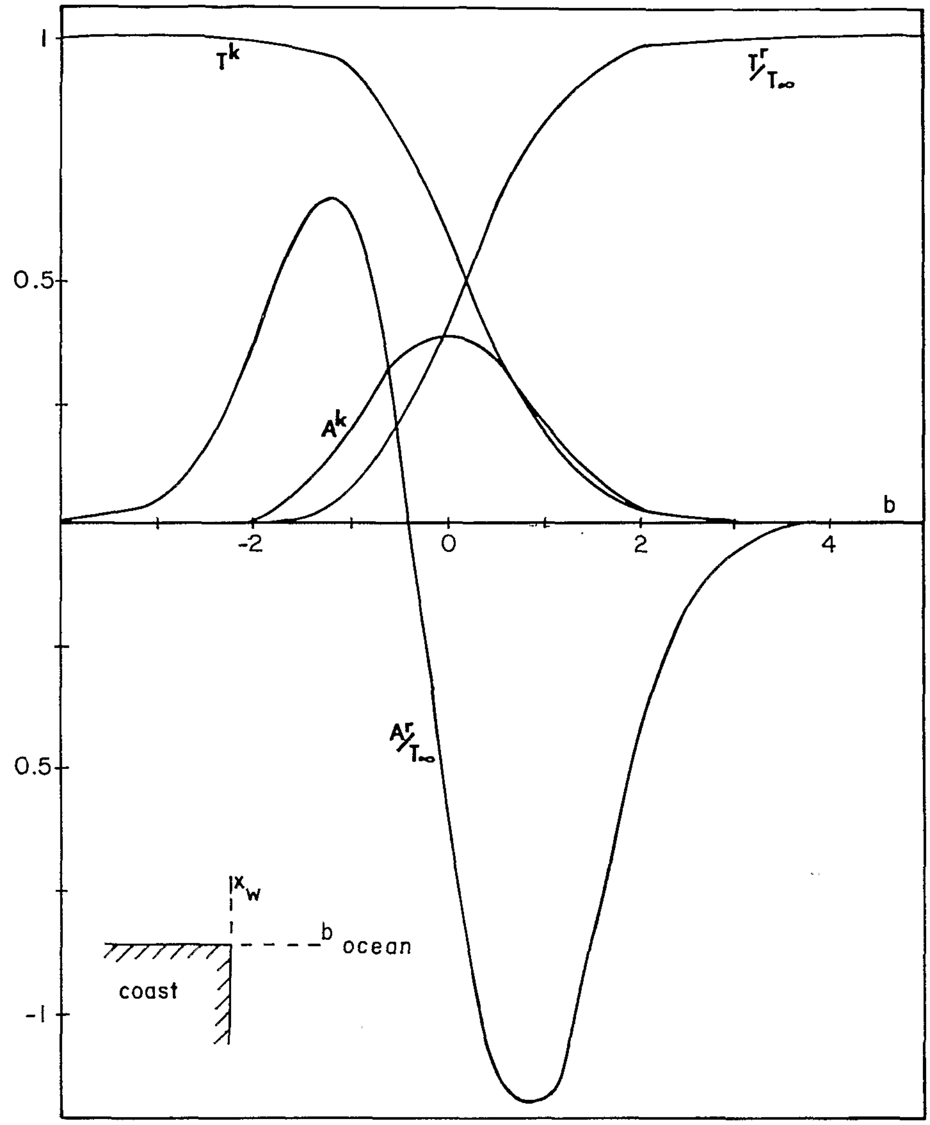

Fig.3: Transmission coefficient (T) and amplitude of the zonal current (A) at the latitude of the western corner as function of the latitude of the zonal coast. Subscript (**k**) applies to an incident Kelvin wave and subscript (**r**) for an n = 1 incident Rossby wave. T_∞ *is the value of* T^k as $b \to +\infty$

current at the corner is maximum when the corner is situated at the equator and becomes negligible when the position of the corner is of the order of two radii of deformation : for $b \lesssim -2$, the equatorial Kelvin wave does not feel the corner and propagates further far without significant change in amplitude ($T^K \approx 1$). As b increases northward from the equator, the amplitude of the boundary current and the transmission coefficient both decrease very rapidly at first becoming negligible by $b \simeq 2$. By this latitude, there is almost no equatorial Kelvin wave any more.

The results in figure 3 are for the n = 1 Rossby wave and have been normalized to the value of T^R at infinity, i.e. the value of T^R when the boundary extends from $-\infty$ to $+\infty$. As b goes to $+\infty$, $T^R_\infty \longrightarrow 2^{-1/2}\ \pi^{-1/4}\int_{-\infty}^{+\infty} \hat{u}dy$: as the barrier becomes infinite all of the incident mass flux is returned in the reflected Kelvin wave (see Cane and Sarachick, 1977). The north-south boundary current redistributes meridionnally the incoming mass flux to make this possible ; it has no net mass flux. In contrast to the eastern boundary case, where the amplitude of the reflected Kelvin wave depends only on the value of height at the corner, the western boundary case is quite complicated. As noted above, as the barrier becomes infinite, the answer depends on the total zonal mass flux. For a partial barrier, equation (25) shows that the response depends on the structure of the incident motions as well as on the zonal mass flux incident on the barrier. Note that the first term in the numerator is small for b >> 1 since ψ_o is then small. It is also small for b << -1 because the orthogonality of the Kelvin and Rossby modes then implies that the integral is small.

To know how well the n^{th} mode is transmitted past the corner, we also compute the transmission factor γ that is the ratio of the energy in the n^{th} mode west of the corner to the incident energy. $(T^K)^2$ is the comparable measure for the Kelvin wave case. Projecting the n^{th} Rossby mode on the solution west of the corner, we obtain :

$$\gamma = \frac{\int_b^\infty (\hat{u}^2 + \hat{h}^2)dy + B\hat{u}(b) + T^R\int_{+b}^{+\infty}\psi_o(\hat{u} + \hat{h})}{\int_{-\infty}^{+\infty}(\hat{u}^2 + \hat{h}^2)dy}$$

Figure 4 shows the transmission coefficient for the first 6 Rossby modes. For the n^{th} mode, the value approaches zero as $b \sim (2n + 1)^{1/2}$ which is the turning latitude. At that latitude,

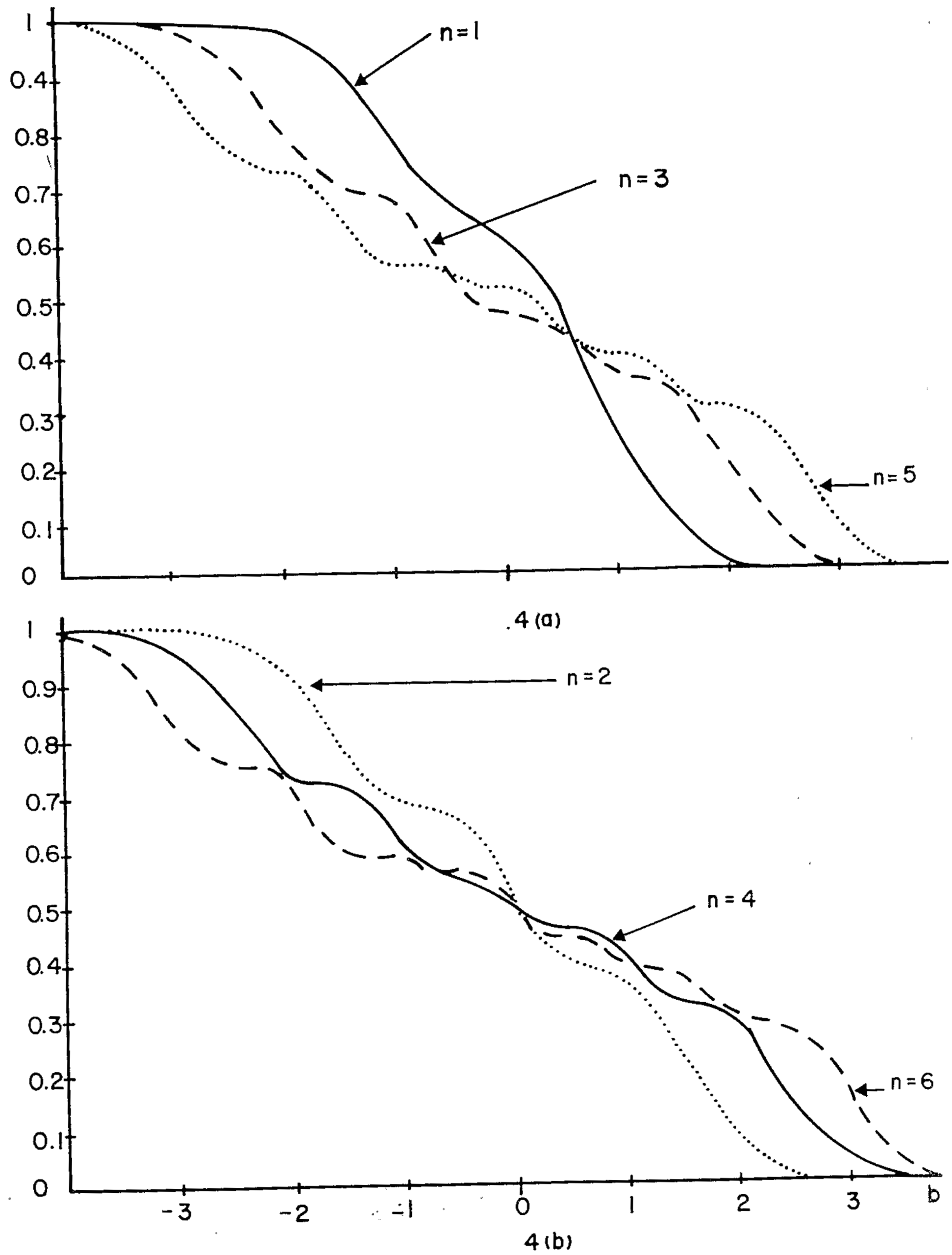

Fig. 4(a)_4(b): The transmission factor γ for the first 6 Rosby waves in the case of western partial boundary in function of the latitude b of the corner. (a) symetric modes, n odd
(b) antisymetric modes, n even

the barrier is extended enough to block the incident motion. The bumps in the curves are due to the oscillatory nature of the u component.

GENERALIZATION

The configuration of oceanic coastline (e.g. the South America coastline) cannot be drawn schematically with only one step but, more realistically, with several steps. The case of low frequency waves impinging on a north-south zigzag coast can be deduced in a straightforward way from the results of the previous sections.

If a partial westward boundary is constituted of N steps of north-south extension $a_{i+1} - a_i$, $i = 1, N$, we get for an incoming Kelvin wave the expression :

$$T_i^K = \frac{2^i \prod_{j=1}^{i} \int_{a_j}^{\infty} \psi_o^2(y)\,dy}{\prod_{j=1}^{i} \left\{ 2\int_{a_j}^{\infty} \psi_o^2(y)\,dy + \psi_o(a_j)\int_{a_{j+1}}^{a_j} \psi_o(y)\,dy \right\}}$$

with T_i^K the amplitude of the transmitted Kelvin wave and i the number of steps, $i = 1, N$.

The transport in the west-east boundary current is given by :

$$A_i = T_i^K \int_{a_{i+1}}^{a_i} \psi_o(y)\,dy$$

In the case of an incoming Rossby wave, we get for the amplitude of the reflected Kelvin wave :

$$T_i^R = - \frac{\int_{b_i}^{\infty} (\hat{u} + \hat{h})\psi_o(y)\,dy + \left(\sum_{j=1}^{i-1} T_{j-1}\right)\left(2\int_{a_i}^{\infty} \psi_o^2(y)\,dy + \psi_o(a_i)\int_{a_{i-1}}^{a_i} \psi_o(y)\,dy\right)}{2\int_{a_i}^{\infty} \psi_o^2(y)\,dy + \psi_o(a_i)\int_{a_{i-1}}^{a_i} \psi_o(y)\,dy}$$

$$+ \frac{\psi_o(a_i)B_{i-1} + \psi_o(a_i)\int_{a_{i-1}}^{a_i} \hat{u}\,dy}{2\int_{a_i}^{\infty} \psi_o^2(y)\,dy + \psi_o(a_i)\int_{a_{i-1}}^{a_i} \psi_o(y)\,dy}$$

(in this case, the subscript i counts the number of steps from east to west ; see figure 5).

and the east-west transport at the corner i :

$$B_i = B_{i-1} + \int_{a_{i-1}}^{a_i} \hat{u}dy + (\sum_{j=1}^{i} T_j) \int_{a_{i-1}}^{a_i} \psi_o(y)dy$$

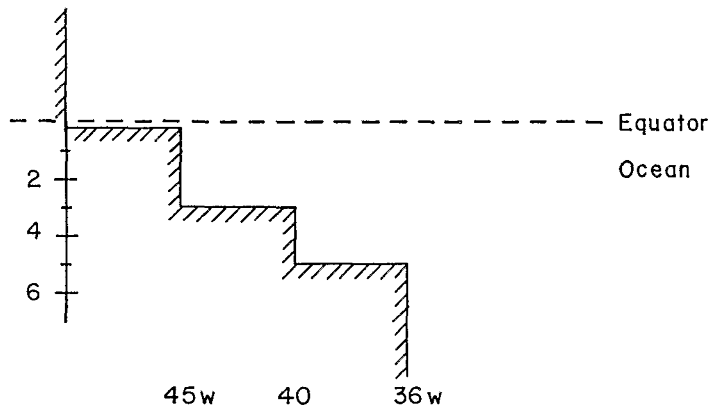

Fig.5 Step structure used to schematize the Brasil coast.

Similar arguments have been used to compute the solution of low frequency waves impinging on a thin island or chain of islands (see Cane and du Penhoat, 1982). We have found a constant depth variation east of the island, two boundary currents on the eastern side of the islands. The solution for an incoming Kelvin wave is sketched in figure 6.

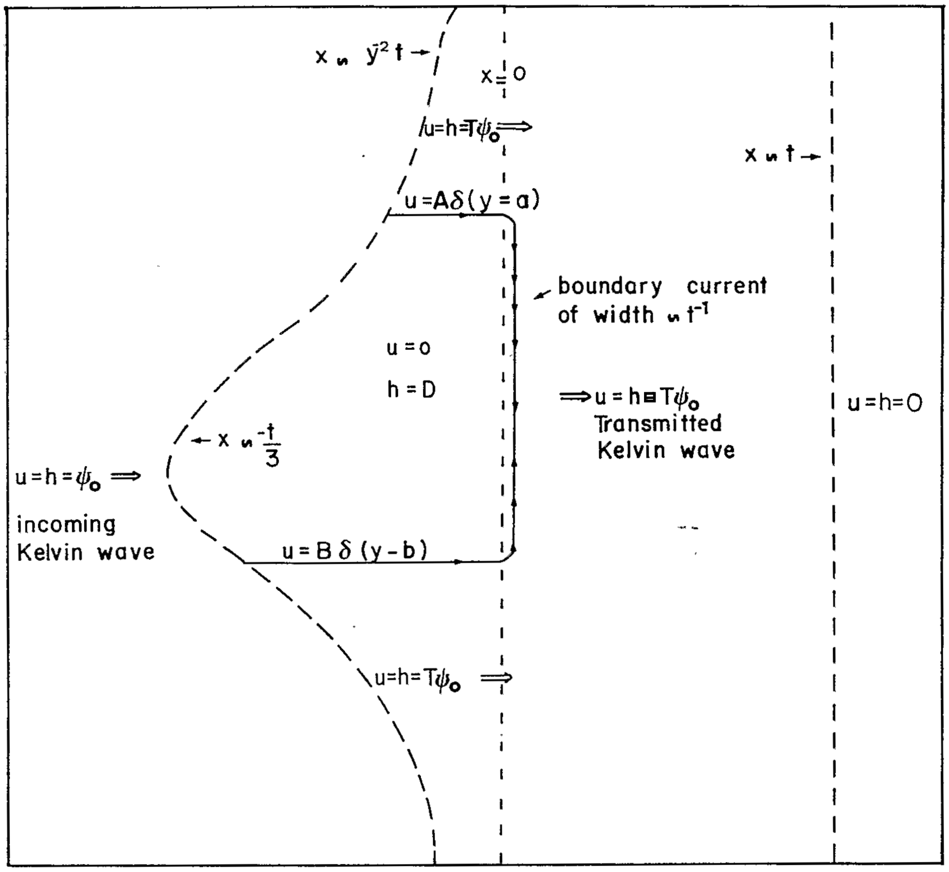

Fig. 6 Sketch of the solution for an incoming Kelvin wave on a thin island. For t>>1, the asymptotic solution holds out to a longitude x~t, the farthest eastward the Kelvin wave travels in time. To the west, it applies behind a front with a form x ~ -1/3t near the equator and $x \sim y^{-2}t$ for high latitude.

If an island is small (i.e., a - b << 1 radius of deformation, a and b latitude of the north and south tip of the island), then low frequency waves pass it almost undisturbed with the mass flux incident on the upstream side flowing around it about equally to the north and south and continuing on downstream in the lee of the island. If the island is large (|a|, |b| ≳ 2 radii of deformation), then the principal response is organized as it would be if the island barrier were meridionally infinite.

DISCUSSION

We now apply our results to the real ocean. In the Atlantic the effect of the coast of the Gulf of Guinea (roughly at 5° latitude north) is very small for either low frequency Rossby waves or the Kelvin wave. (see fig. 2 for $n = 1$, $a \simeq 2$ in non dimensional units, and for higher baroclinic modes $a \geqslant 2$). A constant height is set up on the north-south coast and if we add friction, there is a boundary current along the east-west coast, which widens westward.

In the case of the South America coast, in the Atlantic ocean, represented schematically by 3 steps (fig. 5) : the first corner between 0.20 S and 3 S affects the long equatorial Kelvin wave with an intense westward current at this point (see table 1). For the two other steps, the amplitude of the transmitted Kelvin wave decreases only slightly and results will not change by adding more steps : low frequency waves will not feel a more detailed coast very much. For the $n = 1$ Rossby mode, only approximately 63 % of the energy in the first horizontal mode is transmitted through the 3 steps.

TABLE 1

Parameter values for the coast of South America (a) for an incident Kelvin wave, (b) for $n = 1$ Rossby wave. The equatorial length scale R_1 et R_2 are from Cane and Moore (1981).

(a)

	y	T^K	B^K
first baroclinic mode	-0.113	0.727	0.412
Equatorial length scale	-1.022	0.693	0.142
R_1 = 326 km	-1.704	0.687	0.057
Second baroclinic mode	-0.149	0.704	0.465
Equatorial length scale	-1.344	0.688	0.099
R_2 = 248 km	-2.240	0.687	0.016

(b)

n = 1 Rossby wave	y	T^R	B^R	γ
First baroclinic mode	-1.704	0.001	0.09	0.95
R_1 = 326 km	-1.022	0.01	0.111	0.79
	-0.1136	0.06	-0.078	0.63
Second baroclinic mode	-2.24	0.0001	0.043	0.99
R_2 = 248 km	-1.344	0.004	0.114	0.88
	-0.149	0.061	-0.064	0.63

We have computed the different coefficients for the coast of New Guinea (Pacific ocean) schematically represented with 6 steps (table 2). For an incident Kelvin wave, the amplitude of the transmitted wave is greater than 0.73 for the first and second baroclinic modes and more steps will not change it significantly. For the n = 1 Rossby wave, 64 % of the energy is transmitted in this mode pass the last step. For higher horizontal mode (not shown), this value decreases.

TABLE 2

Parameter values for the coast of New Guinea (a) for an incident Kelvin wave, (b) for n = 1 Rossby wave.

(a)

	y	T^K	B^K
First baroclinic mode	-0.155	0.877	0.193
	-0.465	0.780	0.214
R_1 = 357 km	-0.933	0.754	0.097
	-1.245	0.735	0.104
	-1.867	0.733	0.041
	-3.112	0.733	0.001

(b)

n = 1 Rossby wave	y	T^R	B^R	γ
First baroclinic mode	-3.112	~0	0.005	0.999
	-1.867	$7x10^{-4}$	0.076	0.971
R_1 = 357 km	-1.245	0.006	0.117	0.860
	-0.933	0.009	0.104	0.773
	-0.465	0.027	0.018	0.671
	-0.155	0.031	0.075	0.643

New Guinea is situated at the connection between the Indian and Pacific oceans and the flow through this connection could turn out to have considerable significance in the exhange between the two oceans and its possible relations to El Ninõ. A crude calculation, using Cane and du Penhoat's (1982) results and a rough schematization of the area shows that for a n = 1 incident Rossby wave, only less than 9 % of the incoming energy is transmitted pass Borneo island and that Java-Sumatra islands act almost as an infinite barrier for low frequency equatorial Rossby waves.

Calculations carried out for equatorial islands show that no island, in the world ocean, influences low frequency equatorial waves very much, because their north-south extension is too small compared to the equatorial radius of deformation (see Cane and du Penhoat, 1982). West of the island, the sea level signal is slightly enhanced (and constant at the coast) and the thermocline is thickened (assuming it to be described by the second baroclinic mode). East of the island, the thickness decreases toward the equator due to the presence of short Rossby waves. Even for the Galapagos archipelago, our theory predicts a transmission coefficient over 0.98 for the first and second baroclinic mode Kelvin waves, so that the islands do not affect the propagation of these waves. This result agrees with Yoon's (1981) numerical calculations. An incident Rossby wave is transmitted without major loss of energy and there is only a weak reflected Kelvin wave. In fact, its propagation will be more severely influenced by the mean current system (Philander, 1978). Calculations carried out for the Maldives islands show that they do not act as a significant barrier, because, although they have a greater latitudinal extension, the islands close to the equator are small.

We conclude that islands in the real equatorial oceans will not affect the propagation of low frequency waves significantly and that perturbations will occur only in their vicinity. The irregularities in the Brazilian coast have a noticeable effect but little is gained representing them by more than a single step. The Gulf of Guinea has little effect on incoming Kelvin waves (though the response along its coast is of course of interest for its own sake) and the complex, ragged boundary in the western Pacific is an effective boundary for such low frequency waves.

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