A GENERALIZED COMBINATION EQUATION DERIVED FROM A MULTI-LAYER MICROMETEOROLOGICAL MODEL

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Abstract. The discrete multi-layer model originally devised by Waggoner and Reifsnyder (1968) is used as a theoretical basis to describe the vegetation-atmosphere interaction. Mathematical development of the basic equations yields Ohm's law-type formulae for sensible and latent heat fluxes from which it is possible to derive a combination equation very close in form to Penman–Monteith's equation. A bulk aerodynamic resistance and a bulk stomatal resistance can be defined and expressed in terms of the elementary resistances of the multi-layer model. This new combination equation offers an alternative to the attempts undertaken by Shuttleworth (1976) to unify multi-layer and single-layer approaches.

1. Introduction

The so-called combination equation is the name generally given to Penman's equation applied to a crop canopy. This equation is derived by assuming that sensible and latent heat exchanges between the canopy and the atmosphere originate from a fictitious plane located within the canopy at a certain height above the ground. First, this plane was thought to be at the same level as the equivalent sink of momentum (Monteith, 1963, 1965) and then, at a lower level (Thom, 1972). The equation has the form:

$$\lambda E = \frac{\Delta(\mathrm{Rn} - S) + \rho c_p D_a / r_A}{\Delta + \gamma(1 + r_S/r_A)} \tag{1}$$

where Rn is the net radiation, S is the soil heat flux, D_a the vapour pressure deficit of the air, Δ the slope of the saturated vapour pressure curve at the temperature of the air, γ the psychrometric constant, c_p the specific heat of air at constant pressure and ρ the mean air density. r_A is the aerodynamic resistance calculated on a conservative path between the plane at which fluxes are supposed to originate and a reference height above the canopy. The physiological control of transpiration, linked with leaf stomatal resistance, is characterized by a canopy bulk stomatal resistance r_s interpreted as the effective resistance of all the leaves acting as resistances in parallel. There has been some controversy regarding the true meaning of r_s and the applicability of the combination equation to a real canopy (Philip, 1966) in spite of a rather good agreement with experimental data.

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From a theoretical standpoint, it would be useful to get a general combination equation which applies to a more realistic canopy in which sources and sinks of sensible and latent heat are distributed throughout the height rather than concentrated at one level within the canopy.

The approach described above and based on Penman's formula is often referred to as single-layer approach. In multi-layer models, the vertical transport of sensible and latent heat is described by considering a continuous or discrete set of horizontal planes, each one exchanging heat and vapour with the air (Waggoner and Reifsnyder, 1968). These models provide an accurate description of transfers within the whole canopy, but unfortunately, they do not yield a simple expression for the total evaporation rate comparable to the combination equation. Nevertheless, Shuttleworth (1976) succeeded in deriving, from a multi-layer-approach, a combination equation. But the relevant resistances are redefined in an uncommon way and contain temperature and humidity profiles within the canopy which are unknown *a priori*. Thus, this equation can not be used as a practical tool in any predictive sense.

The purpose of this paper is to demonstrate that it is mathematically possible to start from a multi-layer description of the vegetation–atmosphere interaction and to derive a general combination equation, similar to that produced by the single-layer approach, where bulk aerodynamic and stomatal resistances are expressed in terms of multi-layer elementary resistances.

2. Multi-Layer Model Basic Equations

The basic equations are those of the model originally devised by Waggoner and Reifsnyder (1968). The crop canopy, considered as horizontally homogeneous, is divided into several parallel layers. Subscript i refers to layer number, counted from 1 to n from the top of the canopy to the soil surface. $\delta \mathrm{LAI}_i$ is the leaf area index of layer i per unit ground surface and $T_{L,i}$ is the mean temperature of the leaves. Vapour pressure inside the substomatal cavity is assumed to be saturated at the temperature of the leaf: $e_s(T_{L,i})$. Air within layer i is specified by its mean temperature $T_{a,i}$ and its mean vapour pressure $e_{a,i}$.

The model is based on an electrical analogue where sensible and latent heat fluxes replace current, and corresponding driving potentials are respectively $\rho c_p T$ for sensible heat and $(\rho c_p/\gamma)e$ for latent heat. In Figure 1 the whole stand is depicted as a circuit transmitting sensible and latent heat. In the diffusion processes between the leaves and the air, latent heat experiences two kinds of resistance, stomatal resistance rs_i and boundary-layer resistance rb_i, while sensible heat experiences only a boundary-layer resistance, assumed to be the same for both transfers. Elementary fluxes in each layer are written as:

$$\delta C_i = \rho c_p (T_{L,i} - T_{a,i}) / re_{c,i} \tag{2}$$

$$\delta \lambda E_i = (\rho c_p/\gamma)(e_s(T_{L,i}) - e_{a,i}) / re_{v,i} \tag{3}$$

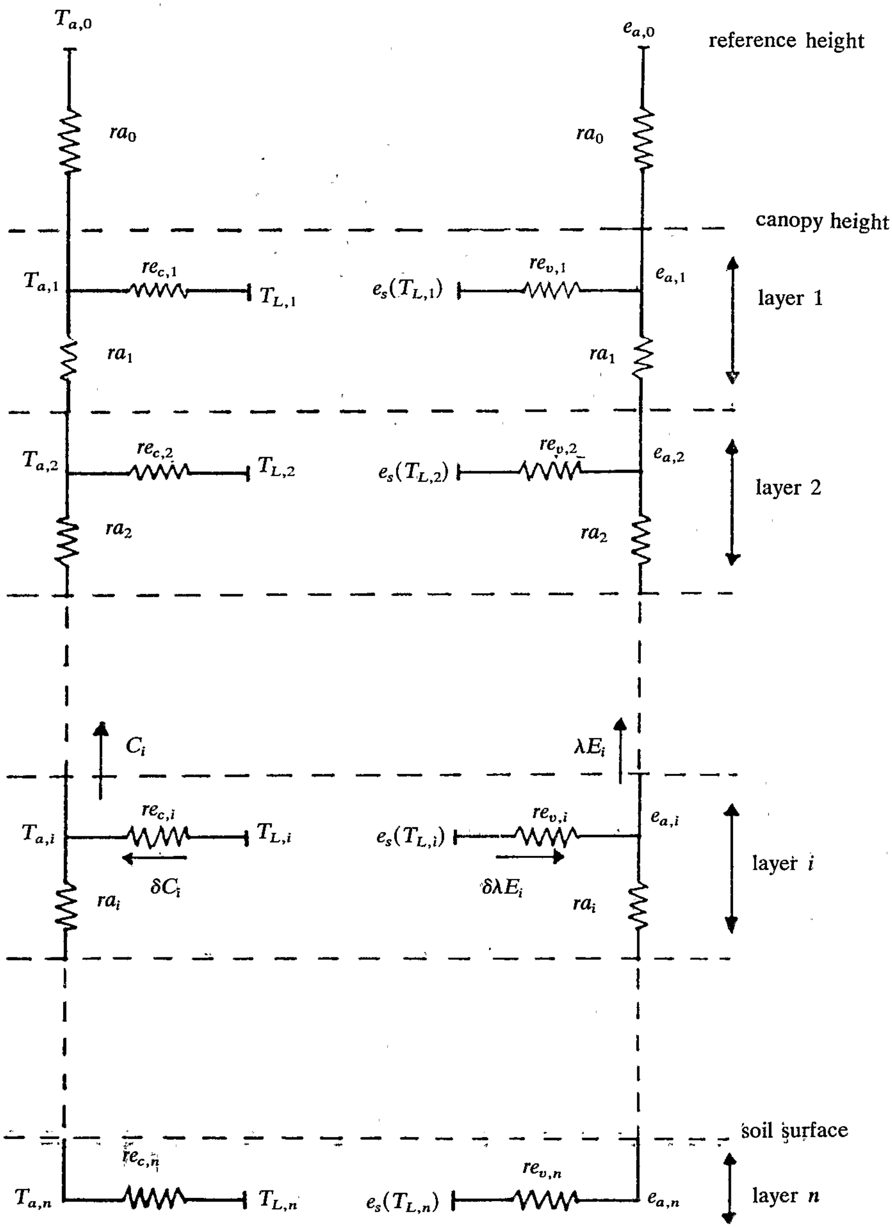

Fig. 1. Electrical analogue of exchange processes within the crop canopy.

with:

$$re_{c,i} = rb_i/2\text{LAI}_i \tag{4}$$

$$re_{v,i} = (rs_i + rb_i)/2\text{LAI}_i\,. \tag{5}$$

The vertical fluxes denoted by C_i and λE_i experience a diffusive resistance when crossing layer i. This resistance is calculated as the reciprocal of eddy diffusivity $K(z)$ integrated through layer i:

$$ra_i = \int_{z_i}^{z_{i-1}} (1/K(z))\,\mathrm{d}z \approx \delta z_i/K_i \tag{6}$$

where z_i is the height of the layer i, δz_i is the layer thickness and K_i is the mean diffusivity for this layer. The vertical fluxes are written as:

$$C_i = \rho c_p(T_{a,i} - T_{a,i-1})/ra_{i-1} \tag{7}$$

$$\lambda E_i = (\rho c_p/\gamma)(e_{a,i} - e_{a,i-1})/ra_{i-1}\,. \tag{8}$$

The ground surface is considered as the last layer denoted by subscript n. The boundary-layer resistance of the soil surface is denoted by rb_n and a surface resistance to water vapour transfer rs_n, similar to stomatal resistance, is defined for the soil surface.

The total fluxes of sensible and latent heat at the top of the canopy can be expressed as the algebraic sum of elementary fluxes emanating from each layer:

$$C_0 = \sum_{i=1}^{n} \delta C_i \quad \text{and} \quad \lambda E_0 = \sum_{i=1}^{n} \delta\lambda E_i\,. \tag{9}$$

3. Recurrent Expressions for Elementary Fluxes δC_i and $\delta\lambda E_i$

Climate parameters, leaf temperatures and the whole set of elementary resistances are assumed to be known. The problem consists in determining air temperature $T_{a,i}$ and vapour pressure $e_{a,i}$ at the circuit nodes in terms of known parameters. For each node, assuming no energy storage, the following conservation equations can be written:

$$C_i - C_{i+1} = \delta C_i \tag{10}$$

$$\lambda E_i - \lambda E_{i+1} = \delta\lambda E_i \tag{11}$$

where C_i and λE_i are the upper vertical fluxes; C_{i+1} and λE_{i+1} are the lower vertical fluxes; δC_i and $\delta\lambda E_i$ are the lateral fluxes. Expressing these fluxes in terms of driving potentials and resistances, Equations (10) and (11) become:

$$(T_{a,i-1} - T_{a,i})/ra_{i-1} - (T_{a,i} - T_{a,i+1})/ra_i = (T_{L,i} - T_{a,i})/re_{c,i} \tag{12}$$

$$(e_{a,i-1} - e_{a,i})/ra_{i-1} - (e_{a,i} - e_{a,i+1})/ra_i = (e_s(T_{L,i}) - e_{a,i})/re_{v,i}\,. \tag{13}$$

These relations can be rewritten as:

$$T_{a,i+1} = a_{c,i}T_{a,i} + b_{c,i}T_{a,i-1} + c_{c,i}T_{L,i} \tag{14}$$

$$e_{a,i+1} = a_{v,i}e_{a,i} + b_{v,i}e_{a,i-1} + c_{v,i}e_s(T_{L,i}) \tag{15}$$

with subscript x replacing c or v in each formula:

$$\begin{aligned} a_{x,i} &= 1 - b_{x,i} - c_{x,i} \\ b_{x,i} &= -\, ra_i/ra_{i-1} \\ c_{x,i} &= -\, ra_i/re_{x,i}\,. \end{aligned} \tag{16}$$

Air temperature $T_{a,1}$ and vapour pressure $e_{a,1}$ at the top of the canopy are linked with the same entities at the reference height $T_{a,0}$ and $e_{a,0}$ by the following equations:

$$T_{a,1} = T_{a,0} + C_0 ra_0/\rho c_p \tag{17}$$

$$e_{a,1} = e_{a,0} + \lambda E_0 ra_0/(\rho c_p/\gamma) \tag{18}$$

where ra_0 is the aerodynamic resistance calculated between the top of the canopy and the reference height. $T_{a,2}$ and $e_{a,2}$ are easily calculated from equations (10) and (11) with $i = 1$:

$$T_{a,2} = (1 - c_{c,1})T_{a,1} + c_{c,1}T_{L,1} + ra_1 C_0/\rho c_p \tag{19}$$

$$e_{a,2} = (1 - c_{v,1})e_{a,1} + c_{v,1}e_s(T_{L,1}) + ra_1\lambda E_0/(\rho c_p/\gamma)\,. \tag{20}$$

And, for any subscript i, it can be proven that the following relations hold:

$$T_{a,i} = \alpha_{c,i}T_{a,1} + \beta_{c,i}[ra_1 C_0/\rho c_p] + \sum_{j=1}^{i-1} \epsilon_{c,i}T_{L,j} \tag{21}$$

$$e_{a,i} = \alpha_{v,i}e_{a,1} + \beta_{v,i}[ra_1\lambda E_0/(\rho c_p/\gamma)] + \sum_{j=1}^{i-1} \epsilon_{v,i}e_s(T_{L,j})\,. \tag{22}$$

Coefficients α, β and ϵ are calculated by means of the following recurrent formulae (subscript x replacing c or v):

$$\begin{aligned} \alpha_{x,i+1} &= a_{x,i}\alpha_{x,i} + b_{x,i}\alpha_{x,i-1} \\ \beta_{x,i+1} &= a_{x,i}\beta_{x,i} + b_{x,i}\beta_{x,i-1} \\ \epsilon_{x,i+1}^{j<i-1} &= a_{x,i}\epsilon_{x,i}^{j} + b_{x,i}\epsilon_{x,i-1}^{j} \\ \epsilon_{x,i+1}^{i-1} &= a_{x,i}\epsilon_{x,i}^{i-1} = a_{x,i}c_{x,i-1} \\ \epsilon_{x,i+1}^{i} &= c_{x,i}\,, \end{aligned} \tag{23}$$

the first coefficients being defined as:

$$\alpha_{x,1} = 1 \quad \beta_{x,1} = 0 \quad \alpha_{x,2} = 1 - c_{x,1} \quad \beta_{x,2} = 1 \quad \epsilon^1_{x,2} = c_{x,1} . \tag{24}$$

Substituting relations (21) and (22) respectively into relations (2) and (3) and defining:

$$\epsilon^i_{x,i} = -1 , \tag{25}$$

we obtain:

$$-\delta C_i/\rho c_p = \frac{\alpha_{c,i}}{re_{c,i}} T_{a,1} + \frac{\beta_{c,i}}{re_{c,i}} [ra_1 C_0/\rho c_p] + \sum_{j=1}^{i} \epsilon^j_{c,i} T_{L,j} \tag{26}$$

$$-\delta \lambda E_i/(\rho c_p/\gamma) = \frac{\alpha_{v,i}}{re_{v,i}} e_{a,1} + \frac{\beta_{v,i}}{re_{v,i}} [ra_1 \lambda E_0/(\rho c_p/\gamma)] + \sum_{j=1}^{i} \epsilon^j_{v,i} e_s(T_{L,j}) . \tag{27}$$

4. Ohm's Law-Type Formulae for Total Fluxes

Introducing relations (26) and (27) into relations (9) and defining ($x = c$ or v):

$$A_x = \sum_{i=1}^{n} \alpha_{x,i}/re_{x,i} \tag{28}$$

$$B_x = \sum_{i=1}^{n} \beta_{x,i}/re_{x,i} , \tag{29}$$

we obtain after rearranging:

$$-C_0(1 + ra_1 B_c)/\rho c_p = A_c T_{a,1} + \sum_{i=1}^{n} \sum_{j=1}^{i} (\epsilon^j_{c,i}/re_{c,i}) T_{L,j} \tag{30}$$

$$-\lambda E_0(1 + ra_1 B_v)/(\rho c_p/\gamma) = A_v e_{a,1} + \sum_{i=1}^{n} \sum_{j=1}^{i} (\epsilon^j_{v,i}/re_{v,i}) e_s(T_{L,j}) . \tag{31}$$

Noticing the following formal identity between coefficients ϵ:

$$\sum_{i=1}^{n} \sum_{j=1}^{i} (\epsilon^j_{x,i}/re_{x,i}) X_j = \sum_{i=1}^{n} \sum_{j=i}^{n} (\epsilon^i_{x,j}/re_{x,j}) X_i , \tag{32}$$

where X replaces T_L when $x = c$ and $e_s(T_L)$ when $x = v$, and defining:

$$E_{x,i} = -\sum_{j=i}^{n} \epsilon^i_{x,j}/re_{x,j} , \tag{33}$$

the total fluxes C_0 and λE_0 at the top of the canopy can be rewritten as:

$$C_0(1 + ra_1 B_c) = \rho c_p \left(\sum_{i=1}^{n} E_{c,i} T_{L,i} - A_c T_{a,1} \right) \tag{34}$$

$$\lambda E_0(1 + ra_1 B_v) = (\rho c_p/\gamma)\left(\sum_{i=1}^{n} E_{v,i} e_s(T_{L,i}) - A_v e_{a,1}\right). \tag{35}$$

Parameters A, B and E are functions only of elementary resistances (ra_i, $re_{c,i}$, $re_{v,i}$) and they have the dimensions of a conductance (reciprocal of a resistance). At this stage it is convenient to define the following parameters:

$$rc_c = (1 + ra_1 B_c)/A_c \tag{36}$$

$$rc_v = (1 + ra_1 B_v)/A_v \tag{37}$$

which have the dimensions of a resistance and will be called canopy resistances, respectively, to sensible and latent heat transfer; rc_c is a purely aerodynamic resistance while rc_v includes aerodynamic and stomatal resistances. It is also convenient to introduce what we shall call canopy equivalent temperatures for sensible heat transfer Te_c and for latent heat transfer Te_v. They are defined as:

$$\mathrm{Te}_c = \sum_{i=1}^{n} E_{c,i} T_{L,i}/A_c \tag{38}$$

$$e_s(\mathrm{Te}_v) = \sum_{i=1}^{n} E_{v,i} e_s(T_{L,i})/A_v\,. \tag{39}$$

These two temperatures represent weighted means of surface temperatures (leaves and soil) because:

$$A_c = \sum_{i=1}^{n} E_{c,i} \quad \text{and} \quad A_v = \sum_{i=1}^{n} E_{v,i}\,. \tag{40}$$

These relations can be easily proven by giving the same value to all the driving potentials in relations (34) and (35). In that case, total fluxes C_0 and λE_0 are equal to zero and relations (40) must be verified. Te_c and Te_v are two different temperatures because of their weighting system. Te_c takes only account of aerodynamic resistances within the canopy while Te_v takes account of aerodynamic and stomatal resistances. Equations (34) and (35) become:

$$C_0 = \rho c_p(\mathrm{Te}_c - T_{a,1})/rc_c \tag{41}$$

$$\lambda E_0 = (\rho c_p/\gamma)(e_s(\mathrm{Te}_v) - e_{a,1})/rc_v\,. \tag{42}$$

These equations constitute Ohm's law-type formulae for total fluxes. The canopy can be considered as a system exchanging sensible and latent heat with the atmosphere from two sources or sinks at two different temperatures Te_c and Te_v. Taking into account relations (17) and (18), the total fluxes can be rewritten as a function of air characteristics $T_{a,0}$ and $e_{a,0}$ at the reference height:

$$C_0 = \rho c_p(\mathrm{Te}_c - T_{a,0})/(ra_0 + rc_c) \tag{43}$$

$$\lambda E_0 = (\rho c_p/\gamma)(e_s(\mathrm{Te}_v) - e_{a,0})/(ra_0 + rc_v)\,. \tag{44}$$

For a canopy consisting of only one layer, Te_c and Te_v are equal to $T_{L,1}$ and

$rc_c = re_{c,1}$, $rc_v = re_{v,1}$. For a canopy divided into two layers, we have:

$$\mathrm{Te}_c = \frac{(ra_1 + re_{c,2})T_{L,1} + re_{c,1}T_{L,2}}{ra_1 + re_{c,1} + re_{c,2}} \tag{45}$$

$$e_s(\mathrm{Te}_v) = \frac{(ra_1 + re_{v,2})e_s(T_{L,1}) + re_{v,1}e_s(T_{L,2})}{ra_1 + re_{v,1} + re_{v,2}} \tag{46}$$

$$rc_c = \frac{re_{c,1}(ra_1 + re_{c,2})}{ra_1 + re_{c,1} + re_{c,2}} \tag{47}$$

$$rc_v = \frac{re_{v,1}(ra_1 + re_{v,2})}{ra_1 + re_{v,1} + re_{v,2}}. \tag{48}$$

The temperatures characterizing sensible and latent heat sources and the bulk resistances opposed to each transfer by the canopy are perfectly determined by relations (36) and (39). They are mathematically expressed as a function of multi-layer model parameters.

5. Derivation of a Combination Equation

The familiar combination equation is derived by eliminating the surface temperature between the flux equations and the energy balance equation after linearizing the saturated vapour pressure curve. In our case, we have two surface temperatures Te_c and Te_v. Then it is convenient to put:

$$\delta\mathrm{Te} = \mathrm{Te}_v - \mathrm{Te}_c \tag{49}$$

and to rewrite sensible heat flux in the form:

$$C_0 = \rho c_p(\mathrm{Te}_v - T_{a,0} - \delta Te)/(ra_0 + rc_c) \tag{50}$$

in order that the same surface temperature appears in both flux equations.

Linearizing the saturated vapour pressure curve by its slope Δ, determined at the temperature of the air at the reference height $T_{a,0}$, yields:

$$\Delta = [e_s(\mathrm{Te}_v) - e_s(T_{a,0})]/(\mathrm{Te}_v - T_{a,0}) \tag{51}$$

and introducing the saturation deficit of the air:

$$D_{a,0} = e_s(T_{a,0}) - e_{a,0}, \tag{52}$$

Equation (44) giving the evaporation rate can be rewritten as:

$$\lambda E_0 = (\rho c_p/\gamma)\Delta(\mathrm{Te}_v - T_{a,0} + D_{a,0}/\Delta)/(ra_0 + rc_v). \tag{53}$$

Eliminating $\mathrm{Te}_v - T_{a,0}$ between equations (50) and (53) and combining with the energy balance equation:

$$C_0 + \lambda E_0 = \mathrm{Rn}_0 - S \tag{54}$$

where Rn_0 is the net radiation of the canopy and S the soil heat flux, we obtain the following combination equation:

$$\lambda E_0 = \frac{\Delta(\mathrm{Rn}_0 - S) + \rho c_p (D_{a,0} + \Delta \delta\, \mathrm{Te})/(ra_0 + rc_c)}{\Delta + \gamma[1 + (rc_v - rc_c)/(ra_0 + rc_c)]} . \tag{55}$$

The bulk stomatal resistance of the canopy and the bulk aerodynamic resistance can be defined respectively by:

$$rc_s = rc_v - rc_c \tag{56}$$

$$rc_a = ra_0 + rc_c . \tag{57}$$

When the canopy is completely wet, all exchange surfaces are saturated. Therefore, leaf stomatal resistances are equal to zero and bulk resistances rc_v and rc_c are identical. Assuming function $e_s(T)$ to have linear properties over the small interval determined by leaf temperatures, Te_v and Te_c are identical and $\delta\mathrm{Te}$ equals zero. The general formula giving the evaporation rate simplifies to:

$$\lambda E_0 = \frac{\Delta(\mathrm{Rn}_0 - S) + \rho c_p D_{a,0}/rc_a}{\Delta + \gamma} . \tag{58}$$

This equation has the same form as the classical Penman formula (1948) established for a saturated surface.

In the case of a single-layer model, $\delta\mathrm{Te}$ equals zero and the bulk canopy resistances are written as:

$$rc_c = rb/2\mathrm{LAI} \tag{59}$$

$$rc_v = (rs + rb)/2\mathrm{LAI} \tag{60}$$

$$rc_s = rs/2\mathrm{LAI} , \tag{61}$$

rb and rs being the mean values of leaf boundary-layer and stomatal resistances, respectively.

When the number of layers is greater than one, the difference $\delta\mathrm{Te}$ between the two equivalent temperatures is not zero because the weighting coefficients are different. Te_c and Te_v can never be equal except in the case of a completely wet canopy. But for the practical use of Equation (55), it is important to know in which cases $\delta\mathrm{Te}$ is sufficiently small to be disregarded, because surface temperatures are generally unknown and $\delta\mathrm{Te}$ can not be easily determined. To study the behaviour of $\delta\mathrm{Te}$, a numerical simulation has been carried out.

6. Numerical Simulation

The surface temperature profile, needed to determine $\delta\mathrm{Te}$, is calculated by using the following procedure. All elementary resistances (stomatal, boundary-layer, diffusive) are assumed to be known as also are the profile of net radiation, the soil heat flux, the temperature and the vapour pressure of the air at a reference height. The total fluxes of sensible and latent heat C_0 and λE_0 at the top of the canopy are calculated by means of general equations given by Lhomme (1988). Hence, the temperature and the vapour pressure of the air at the top of the canopy (first layer) are calculated from Equations (17) and (18). Then the following recurrent process is used. Knowing air temperature $T_{a,i}$ and vapour pressure $e_{a,i}$ in layer i, leaf temperature $T_{L,i}$ is determined by solving iteratively the energy balance equation:

$$\delta \mathrm{Rn}_i = \rho c_p (T_{L,i} - T_{a,i})/re_{c,i} + (\rho c_p/\gamma)(e_s(T_{L,i}) - e_{a,i})/re_{v,i} \tag{62}$$

where $\delta\mathrm{Rn}_i$ is the net radiation absorbed by layer i. $T_{L,i}$ being determined, the elementary fluxes δC_i and $\delta\lambda E_i$ emanating from layer i can be calculated from Equations (2) and (3). Knowing the vertical fluxes C_i and λE_i at the upper boundary of layer i and the horizontal fluxes in the same layer, the vertical fluxes at the lower boundary are calculated by means of conservation Equations (10) and (11). Also air characteristics in layer $i+1$ are determined from Equations (7) and (8). The same recurrent process is used for layer $i+1$ and so on down to the soil surface. Once the profile of surface temperature is determined, it is possible to calculate $\delta\mathrm{Te}$ and the evaporation rate by means of Equation (55).

We have simulated the microclimate of a standard canopy like a maize crop, whose characteristics are the following: canopy height: 1.5 m; number of layers in the vegetation: 5; layer thickness: 0.3 m per layer; leaf area profile: constant (0.6 per layer). The climatic characteristics at the reference height of 3 m are: air temperature: 25 °C; vapour pressure: 2000 Pa; wind velocity: 3 ms^{-1}; global radiation: 800 Wm^{-2}. To describe the stomatal resistance profile, a simple parameterization as a function of global radiation has been used:

$$rs(z) = k_0/\mathrm{Rg}(z) \tag{63}$$

where $\mathrm{Rg}(z)$ is the short-wave radiation profile and k_0 is a parameter which varies as a function of water status. Rg is expressed in Wm^{-2}, rs in sm^{-1} and in our simulation, k_0 varies from 0 for a completely wet canopy up to 9×10^5 for a canopy with important water stress. The profiles obtained in this way are in good agreement with experimental data measured in a maize crop. The range defined by $2 \times 10^5 < k_0 < 3 \times 10^5$ corresponds approximately to a well watered crop. The other profiles involved as input to the simulation program are detailed in the appendix.

The magnitude of $\delta\mathrm{Te}$ and its importance in calculating the evaporation rate by means of Equation (55) have been analysed as a function of the stomatal resistance profile specified by the value of k_0. The exact value of the evaporation rate is denoted by λE_0. An approximate value, denoted by λE_A, is determined by using Equation (55) with $\delta\mathrm{Te} = 0$. In Table I, the soil surface is considered as wet ($rs_n = 0$). In Table II, the soil surface is considered as dry, rs_n being set equal to the stomatal resistance of the last vegetation layer. And Table III shows, for a

TABLE I

For a wet soil surface ($rs_n = 0$), variation in the difference δTe(°C) between the two equivalent temperatures and in the evapotranspiration rates as a function of the stomatal resistance profile specified by the value of k_0. λE_0(Wm^{-2}) is the exact value of the evapotranspiration rate. λE_A(Wm^{-2}) is an approximate value calculated from Equation (55) with $\delta\mathrm{Te} = 0$.

$k_0(10^5)$	0	1	2	3	4	5	6	7	8	9
λE_0	564	409	330	281	249	226	208	194	183	174
δTe	0.00	−0.47	−1.21	−1.90	−2.49	−3.00	−3.44	−3.82	−4.15	−4.44
λE_A	564	423	362	328	306	291	280	271	264	259

TABLE II

For a dry soil surface, same variations as in Table I. The soil surface resistance rs_n is taken equal to the stomatal resistance of the last vegetation layer.

$k_0(10^5)$	0	1	2	3	4	5	6	7	8	9
λE_0	564	391	300	243	204	176	155	138	125	114
δTe	0.00	−0.01	−0.01	−0.01	−0.01	−0.01	0.00	0.00	0.00	0.00
λE_A	564	391	300	243	204	176	155	138	125	114

TABLE III

Variations in exact and approximated evapotranspiration rates (Wm^{-2}) and in δTe(°C) as a function of the value of the soil surface resistance rs_n (sm^{-1}) for a given stomatal profile corresponding to $k_0 = 4$ (the stomatal resistance of the last vegetation layer is 2700 sm^{-1}).

rs_n	0	500	1000	1500	2000	2500	3000	3500	4000	4500	600	7000
λE_0	249	225	216	210	207	205	203	202	201	200	199	198
δTe	−2.49	−0.24	−0.01	0.03	0.02	0.00	−0.02	−0.03	−0.05	−0.06	−0.010	−0.11
λE_A	306	229	216	210	207	205	204	203	202	201	200	200

given stomatal profile corresponding to $k_0 = 4$, the variations of λE_0, δTe and λE_A as a function of the value of the soil surface resistance rs_n. From these numerical results, it is clear that when the foliage is dry ($k_0 > 0$) and the soil surface is wet ($rs_n = 0$), the parameter δTe is not negligible and the difference between λE_0 and λE_A is significant, whereas when both the foliage and the soil surface are dry (or wet), δTe and the difference between λE_0 and λE_A are small, whatever the stomatal resistance profile may be. As is shown in Table III, the difference δTe and the error committed in the evaporation rate by disregarding it in Equation (55) increase with the difference existing between the stomatal resistance of the last vegetation layer and the resistance of the soil surface, the smallest difference coinciding with the case when the two resistances are very similar.

7. Conclusion

It has been proven that a combination equation similar to that produced by the single-layer approach can be derived from a multi-layer approach. It is written as:

$$\lambda E_0 = \frac{\Delta(\mathrm{Rn}_0 - S) + \rho c_p D_{a,0}/rc_a}{\Delta + \gamma(1 + rc_s/rc_a)} \tag{64}$$

where rc_a and rc_s are respectively the bulk aerodynamic resistance and the bulk stomatal resistance of the canopy; rc_a is the sum of two resistances: ra_0 and rc_c; ra_0 is the familiar aerodynamic resistance of the air stream above the canopy, calculated between the canopy height and the reference height (where saturation deficit $D_{a,0}$ is measured); rc_c is the proper aerodynamic resistance of the canopy and represents a combination of diffusive resistances ra_i and boundary-layer resistances rb_i (Equation 36); rc_s is the bulk stomatal resistance defined as the difference between the bulk resistances opposed by the canopy respectively to water vapour transfer rc_v and to sensible heat transfer rc_c. Equation (64) is only an approximate version of a more general Equation (55) and does not hold every time the soil surface resistance is much smaller than the stomatal resistance of the lowest layers of vegetation, e.g., a dry canopy with a wet soil surface. These results provide further evidence that a major factor governing whether the evaporation of a dry canopy can be adequately described by a simple combination equation, is the presence of significant below-canopy evaporation (Lindroth and Halldin, 1986).

As a concluding remark, and at the risk of denegrating the generality of this analysis, it seems worthwhile pointing out that the theoretical formalism presented in this paper rests on K-theory (diffusive resistances ra_i are directly related to turbulent diffusivity) and depends ultimately on the the validity of the diffusion equation. At present, everybody in the micrometeorological community knows it is somewhat difficult to relate fluxes at a specific level, within the canopy, to a transfer coefficient and a concentration gradient. Turbulent diffusivity must be handled with care (Waggoner and Turner, 1972; Legg and Monteith, 1975). But it is very difficult to quantify the errors introduced into the model by using this assumption.

Appendix

The profiles of wind velocity $u(z)$, turbulent diffusivity $K(z)$ and boundary-layer resistance $rb(z)$ are taken from Perrier (1967, 1976):

$$u(z) = u(z_h)\exp(-B_0 L(z)) \tag{A1}$$

$$K(z) = [A_0/l(z)^2]\,\mathrm{d}u(z)/\mathrm{d}z \tag{A2}$$

$$rb(z) = rb_0 u(z)^a, \tag{A3}$$

$L(z)$ is the downward cumulative leaf area index related to leaf area density $l(z)$ by:

$$L(z) = \int_z^{z_h} l(z)\,\mathrm{d}z\,; \tag{A4}$$

z_h specifies the canopy height. Coefficients A_0 and B_0 whose values are respectively 0.4 and 0.6 are theoretically derived; rb_0 is equal to 50 and a to -0.5.

Global radiation Rg and net radiation Rn are assumed to decrease as exponential functions of cumulative leaf area index $L(z)$:

$$\mathrm{Rg}(z) = \mathrm{Rg}(z_h)\exp(-\alpha_0 L(z)) \tag{A5}$$

$$\mathrm{Rn}(z) = \mathrm{Rn}(z_h)\exp(-\alpha_0 L(z))\,; \tag{A6}$$

attenuation coefficient α_0 is considered the same for both profiles and is taken to be equal to 0.6.

In addition, the net radiation above the canopy is calculated as 60% of global radiation and soil heat flux as half the net radiation at the soil surface.

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