## Modeling surfaces using quadratic surface patches

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#### Abstract

In this paper, we show how to extract reliable informations about the shape of 3D objects, obtained from volume medical images. We present an optimal region-growing algorithm, that makes use of the differential characteristics of the object surface, and achieves a stable segmentation into a set of patches of quadratic surfaces. We show how this segmentation can be used to recognize and locate a target sub-structure on a global anatomic structure.

keywords : Shape and object representation, Segmentation and perceptual grouping

### I Introduction

For a computer vision system, performing tasks of recognition and locating is a major challenge. Achieving these tasks automatically may help in a great number of applications, such as computer-assisted surgery. In the work presented here, we focused on the processing of 3D anatomic structures : we show how to extract reliable informations about their shape, and how these informations can be used to recognize and locate a particular anatomic piece on a bigger structure. These informations consist of a set of patches of quadratic surfaces, that locally approximate relevant parts of the object studied. The main problem is to ensure the stability of the segmentation in quadratic patches. For this purpose, we present a region-growing algorithm that uses the differential characteristics of the object surface. This work is more precisely described in the reference [7].

## II Method

The data involved in this work are volumic images obtained from scanner medical systems. According to the methodology we have developped, the extraction of relevant quadratic patches requires the steps described in the following paragraphs.

#### **II.1** Edge detection and differential computations

We used algorithms described in [3] and [4] to perform the edge detection, and the computation of 2 differential features on the object surface : the *gaussian curvature* and the *gaussian extremality*. The last one has been introduced in [6]; gaussian extremality contains information about extrema of the curvature, for instance about crest lines. These differential features have a major interesting property : they are characteristic of the object shape, and invariant through any rigid displacement.

#### **II.2** Initial segmentation

The initial segmentation of the object surface consists in finding homogeneous patches in term of differential features. Thus, the object is first divided into maximal connected patches such that signs of gaussian curvature and extremality are constant. As a consequence, boundaries between patches correspond to parabolic lines and lines of extremal curvature. Besides, it is necessary to improve this initial segmentation thanks to a few mathematical morphology operators : in particular, an *opening* disconnects parts where the diameter is below the one of the structuring element used.

#### **II.3** Region growing

The segmentation then go on through a region-growing process. The algorithm used is described in [2]; the principle is to merge patches according to quality criteria. This particular algorithm is optimal, in the fact that each step chooses the best possible pair of patches to merge. This is done quickly, thanks to the use of adapted data structures : a dynamic graph of neighbourhood and a priority heap. The quality criteria is obtained from an error computation, during the approximation of the patch by a quadratic surface. The approximation method used is presented in [1]; given the points of a particular patch, we use a least mean square estimation of the best quadratic fitting surface. The search for stability of the region-growing process led us to implement a specific strategy, involving the following features :

- We introduced 2 quality criteria, associated to 2 thresholds allowing a merging :
  - 1. The *continuity criterion*, for a pair of patches, estimates how well one patch extends the other. To do so, we have chosen to compute the error when approximating one patch by the quadratic surface associated to the other, and vice versa.
  - 2. The boundary criterion estimates the error of approximation by the quadratic surface, for the points located at the boundary of a pair of patches. Thus we can control the quality of the approximation along characteristic parts of the object, such as crest and parabolic lines.
- Thanks to the computation of reference errors, we can specify relative thresholds (that is to say, percentage of tolerance) : we authorize the merging only if the increase of the error after merging is less than a given ratio of a local reference error. Thus we avoid the need for absolute threshold values, that would be dependent of each particular application, in particular when the noise on the data is not homogeneous.

- We divided the region-growing process into various steps, beginning with restrictive thresholds, then allowing more mergings.
- When merging 2 patches of different gaussian nature (an elliptic and a hyperbolic patch), we introduced more restrictive criteria.

Using 3D rendering algorithms [8], we display some examples of quadric patches provided by our algorithm. We have chosen the biggest regions in terms of number of voxels. We notice that these quadric patches have often some anatomical meaning (eye cavity...).

In these 3D displays (figures 1 to 3) voxels belonging to the particular patch shown are in red, and the corresponding quadric surface is in blue.

In Figure 1 we see that the main part of the eye orbit corresponds to a quadric patch (ellipsoid) extracted automatically by our algorithm. Figure 2 presents the same kind of results but for the forehead.

Figure 3 shows the result of the algorithm on two data corresponding to the same upper jaw but taken at two different positions. We notice the very good stability of our method both for the labelling and the surface primitives.



Figure 1: Right : Red points forming a single extracted patch ; left : Ellipsoid provided by our algorithm corresponding to this patch



Figure 2: Right : Red points forming a single extracted patch ; left : Ellipsoid provided by our algorithm corresponding to this patch



Figure 3: Top : Different perspective views of the ellipsoid corresponding to the red extracted patch for the first position of the skull; bottom : same but for the second position of the skull, the corresponding patch is also in red.

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#### **II.4** Registration

We now consider 2 images where we can find the same structure, although not in the same position. The segmentation into quadratic surfaces, presented above, helps us to achieve the matching process. We developped a registration algorithm according to the prediction-verification paradigm, with the following main features :

- From a set of matched patches, the displacement (represented by a quaternion) is estimated by superimposing the centroid of a patch and the centroid of the matched patch. This estimation uses a least mean square estimation of the best fitting quaternion (see [1]).
- Once a displacement has been estimated, we try to match a new pair of patches. We search for the pair of patches that are the nearest, according the following vicinity criterion : we compute the error when approximating the first patch by the quadratic surface of the second patch, after displacement; and we compare this error with the error when approximating the first patch by its own quadratic surface. Thus, this criterion is much alike the continuity criterion introduced above. This criterion makes use of the quadratic segmentation information, and is able to discriminate with a great accuracy among the possible matches, according to the position and shape of the different patches.
- For the initial estimation of a displacement, 3 matches are needed. They are guided by the similarity of parameters (eigenvalues and eigenvectors of the quadratic surfaces) for the possible patches.

Here we do *not* tackle the problem of a precise registration. Our aim is to rapidly recognize and locate (approximatively) a particular structure, despite of the following difficulties :

• we want to recognize and locate a particular substructure on a bigger structure; both can have thousands of edge points.

• we have no a-priori information about where the structure should be found.

A more precise registration can follow the one we present; for example thanks to an iterative method using splines (see [5]). This type of methods requires a good enough initial estimate, in order to avoid local minima; thus registration from quadratic segmentation can complete and extend application field for iterative methods.

# III Experimental results : Recognition and locating

We focus on the problem of recognizing and locating a target substructure on a global anatomic structure. The practical application field could be the computer-assisted guidance of a surgical act. Here, a volumic scanner im-



Figure 4: Eye - 2 sections from the target substructure.

age including 28 successive sections defines the target substructure (the left eye). Two sections are reproduced figure 4. Using our quadratic patches extraction algorithm, we obtained 5 patches of important size. Our algorithm is then able to match each of the 5 patches in the substructure with the



Figure 5: Grey points : sections from global structure — Black points : sections from target substructure (eye) after recognition and locating.

corresponding patch in the global structure. The locating result is shown in figure 5. More experimental results can be found in [7].

## IV Conclusion

This work is the natural continuation of the ones described in the references [3],[4]. The goal of these works is to define from original data (volume images) a hierarchical sequence of shape representations that may be used at each level to solve a given task such as registration, localization, recognition... In this paper we have adressed the problem set by the determination of intrinsic parametric surface models using the differential characteristics. We used the key idea of reference [1], that is to approximate a surface using quadratic patches thanks to a graph based region growing strategy. Our contributions with respect to [1] are : - We take into account the differential characteristics of the studied object (gaussian curvature and extremality), for the initial segmentation and during the mergings. - We implement, thanks to the use of an adapted data structure, a fast optimal region-growing algorithm. - We introduce merging criteria that are specific (continuity, boundary) and flexible (relative thresholds).

The relevance and stability of the quadratic segmentation obtained is, as far as we know, without equivalent in previous works. The use of this quadratic information has allowed us to achieve efficient recognition and locating of real anatomic structures.

#### References

- O.D. Faugeras, M. Hebert.: The representation, recognition and locating of 3-D objects. The International Journal of Robotics Research, vol 5 no 3, 1986.
- [2] O. Monga.: An optimal region growing algorithm for image segmentation. International Journal of Pattern Recognition and Artificial Intelligence, vol 1, 1987.
- [3] O. Monga, R. Deriche, G. Malandain et J.P. Cocquerez.: Recursive filtering and edge tracking: two primary tools for 3D edge detection. Image and vision computing, vol 9 no 4 août 91.
- [4] O. Monga, R. Lengagne, R. Deriche.: Extraction of the zero-crossing of the curvature derivative in volumic 3D medical images: a multi-scale approach. IEEE conference in Computer Vision and Pattern Recognition, Seattle (USA), June 1994.
- [5] R. Szeliski, S. Lavallée.: Matching 3D anatomical surfaces with nonrigid deformations using octree-splines. IEEE workshop on Biomedical image analysis, Seattle (USA), June 1994.
- J.P. Thirion.: The extremal mesh and the understanding of 3D surfaces. IEEE workshop on Biomedical image analysis, Seattle(USA), June 1994.
- [7] I. Bricault, O. Monga.: From volume medical images to quadratic surface patches. INRIA Research Report number 2380, September 1994.
- [8] F. Neyret, A General and Multiscale Method for Volumetric Textures. Graphics Interface '95 Proceedings, May 1995.