Optimal Thresholding for Image Segmentation

Djemel ZIOU
Département de mathématiques et d'informatique
Université de Sherbrooke
Sherbrooke, Canada, J1K 2R1

Abstract: *This paper presents an optimal multithreshold selection algorithm for segmentation of grey level images when objects can be distinguished by their grey level values. We define performance measures and we compare our algorithm to others.*

1. Introduction

In computer vision, thresholding is a fundamental tool for segmentation of grey level images when objects and background pixels can be distinguished by their grey level values. The histogram of such an image is formed by M distinguishable populations. By selecting adequate thresholds T_i, the original image $I(x,y)$ can be transformed into another image $R(x,y)$ using the rule: $\forall\, i \in [1,M]$, $R(x, y) = M\text{-}i$ if $T_{M-i} < I(x, y) \leq T_{M-i+1}$. Many threshold selection methods have been proposed and are summarized in [3]. We use the maximum likelihood estimate and minimum classification error to compute thresholds. The algorithm is summarized in three steps (for more details see [4]): 1) the grey level histogram is approximated by a linear combination of Gaussians, 2) thresholds are computed using minimum classification error, 3) edges and regions are labeled using the rule defined above. This last step is easy to do and is not described in this paper. The first two steps of the algorithm are presented in the next two sections. Furthermore, we describe performance measures and compare our algorithm to others.

2. Histogram Approximation

The histogram $h(x)$ of a grey level image can be viewed as an estimate of the probability density function $p(X/\Theta)$ of populations comprising grey levels of objects and background. The random vector X equals to $(x_1,\ldots,x_n)$, where the random variable x_i is the abscissa of the histogram. It is often realistic to assume that each population $p(x/i)$ of $p(X/\Theta)$ is distributed normally with distinct parameters. Under this assumption, thresholds can be computed using the Bayes rule defined in terms of the mean $\mu(i)$, the standard deviation $\sigma(i)$ and *a priori* probability $P(i)$. We use the maximum likelihood estimate of the parameter vector $\Theta = (\theta_1,\ldots,\theta_M)$, where $\theta_i = (\mu(i), \sigma(i), P(i))$. That is, we compute Θ such that: $\max_\Theta p(X / \Theta)$, subject to constraints: $\forall i \in [1, M]$ $P(i) \geq 0$), and $\Sigma\, P(i) = 1$. The analytical resolution of this problem leads to an iterative algorithm, where the parameters are:

$$\mu(i) = \frac{\Sigma_{k=1}^{N} h(x_k)p(i/x_k)x_k}{\Sigma_{k=1}^{N} h(x_k)p(i/x_k)}, \quad \sigma^2(i) = \frac{\Sigma_{k=1}^{N} h(x_k)p(i/x_k)(x_k - \mu(i))^2}{\Sigma_{k=1}^{N} h(x_k)p(i/x_k)}, \quad P(i) = \frac{\Sigma_{k=1}^{N} h(x_k)p(i/x_k)}{\Sigma_{k=1}^{N} h(x_k)},$$

where $h(x_k) = \Sigma_{i=1}^{N} P(i)p(x_k/i) = \Sigma_{i=1}^{N} \frac{P(i)}{\sqrt{2\pi}\,\sigma(i)} e^{-\frac{(x_k-\mu(i))^2}{2\sigma^2(i)}}$ and $p(i/x) = \frac{p(x/i)P(i)}{\Sigma_{k=1}^{M} p(x/k)P(k)}$.

A good initial estimate of parameters to ensure faster convergence to the correct solution is: $\mu^{(0)}(i)=\frac{\sum_{k=n_1(i)}^{n_2(i)} x_k h(x_k)}{\sum_{k=n_1(i)}^{n_2(i)} h(x_k)}$, $\sigma^{2(0)}(i)=\frac{\sum_{k=v_1(i)}^{v_2(i)}(x_k-\mu(i)^{(0)})^2 h(x_k)}{\sum_{k=v_1(i)}^{v_2(i)} h(x_k)}$, $P^{(0)}(i)=\frac{\sum_{k=v_1(i)}^{v_2(i)} h(x_k)}{\sum_{j=0}^{N} h(x_k)}$,

where $n_1(i)$ and $n_2(i)$ are the two inflexion points of the i^{th} mode and $v_1(i)$ and $v_2(i)$ are means of the two valleys of the i^{th} mode. The number of modes M of h(x) is equal to half the number of the inflexion points of h(x). Consequently, to form an initial estimate of populations and therefore to compute the number M, the first order derivative of h(x) is computed and is scanned, each maximum (resp. minimum) is grouped with the consecutive minimum (resp. maximum).

3. Thresholds Selection

We select threshold T_i using Bayes minimum error rule defined in terms of parameters $\mu(i)$, $\sigma(i)$, and $P(i)$; that is $P(i)\, p(x / i) \geq P(i+1)\, p(x / i+1)$ if $x \in [T_{i-1}+1, T_i]$. It is possible to show that this rule minimizes the classification error. By replacing p(x/i) by $\frac{1}{\sqrt{2\pi}\,\sigma(i)} \exp\left(\frac{-(x-\mu(i))^2}{2\sigma^2(i)}\right)$ and by taking the logarithm of each member, the Bayes minimum error rule becomes: $\frac{1}{2}\left(\frac{1}{\sigma(i)^2}-\frac{1}{\sigma(i+1)^2}\right)T_i^2+\left(\frac{\mu(i+1)}{\sigma(i+1)^2}-\frac{\mu(i)}{\sigma(i)^2}\right)T_i+\frac{\mu(i)}{2\sigma(i)^2}$ $-\frac{\mu(i+1)^2}{2\sigma(i+1)^2}-\ln\left(\frac{P(i)\sigma(i+1)}{P(i+1)\sigma(i)}\right)=0.$ No threshold can be selected between modes i and i+1 if there are no real solution of this second degree equation. It should be recalled that our algorithm is devoted to images where modes of their histograms are distinguishable (i.e, multimodal) and if this assumption is not fulfilled thresholds selected are wrong. We assign to a given T_i a modality merit which includes three factors: 1) distance between two consecutive modes: $E(T_i)=\frac{\mu(i+1)-\mu(i)}{g_{max}-g_{min}}$, where g_{max} (resp., g_{min}) is the maximum (resp., minimum) grey level value of these modes, 2) the ratio of their amplitudes $R(T_i)=\frac{\min_{x\in\{\mu(i),\mu(i+1)\}}p(x/\Theta)}{\max_{x\in\{\mu(i),\mu(i+1)\}}p(x/\Theta)}$, 3) the ratio of the valley amplitude to the mode amplitude $V(T_i)=\frac{\min_{x\in\{\mu(i),\mu(i+1),\ldots,\mu(i+1)\}}p(x/\Theta)}{\min_{x\in\{\mu(i),\mu(i+1)\}}p(x/\Theta)}$. The modality merit of two consecutive populations is $B(T_i) = E(T_i) * R(T_i) * (1 - V(T_i))$. $B(T_i)$ is in [0,1] and is near 0 when the two populations are not distinguishable.

4. Performance Evaluation

We have experimented with our algorithm on a varied artificial and natural images of grey levels. Figure [1] shows a natural image of 256x256 pixels of 256 grey levels with its histogram and edges obtained by our algorithm. We define the properties desired for the segmentation algorithm that works well in general contexts. They are based on the uniformity of regions and the accuracy in the location of their border edges. For a bimodal histogram, we define performance measures of a thresholding algorithm by P(T) = U(T) S(T). U(T) is an uniformity measure of the two regions depending on the summation of their standard deviations. S(T) is their shape measure depending on the contrast of their border edges. Table 1 summarizes the performance evaluation of algorithm of Kittler and al.[1], Pun's algorithm [2], our algorithm, and the optimal values. In addition to U(T) , S(T), P(T), and T this table gives the execution time and the modality merit B(T) computed at the optimal threshold. U(T) = 0.928 (74) means that 74 is the threshold T for each U(T) is maximal and is equal to 0.928. It should be noted that the performance of Kittler's algorithm are more sensitive to B(T). Pun's algorithm is the faster. Our algorithm provides homogeneous regions and accurate localization of border edges.

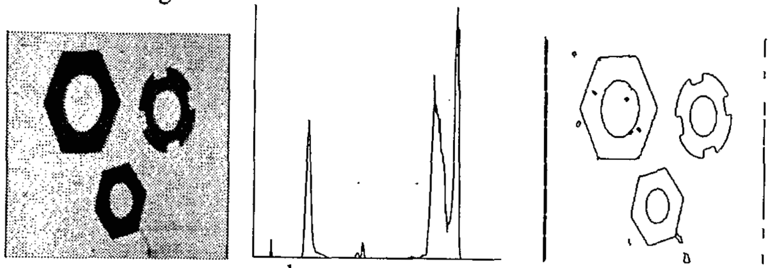

Figure 1: *a) Image, b) Histogram, c) Edges results from our algorithm.*

	Kittler and al.		*Pun*		*Ziou*		*Optimal*	
Images	*cam*	*geol*	*cam*	*geol*	*cam*	*geol*	*cam*	*geol*
T	224	151	189	130	142	118	123	122
U(T)	0.701	0.941	0.728	0.946	0.887	0.928	0.928 (74)	0.948 (137)
S(T)	0.300	0.134	0.546	0.577	0.784	0.653	0.803 (130)	0.656 (119)
P(T)	0.211	0.126	0.398	0.545	0.696	0.606	0.731 (123)	0.612 (122)
B(T)	0.001	0.040	0.001	0.040	0.001	0.040	0.001	0.040
time	0:30	0:01	0:01	0:01	0:19	0:09		

Table 1: *Evaluation of the three algorithms. We used two images of 256 grey levels: popular cameramen image* (cam) *and slate image* (geol).

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