BELL'S FORMULA - A REAPPRAISAL

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RÉSUMÉ

L'un des buts principaux de la régionalisation des données météorologiques et hydrologiques est de permettre l'extension, soit dans l'espace soit dans le temps, des données ponctuelles limitées. Une application spécifique de cette technique a été proposée par Bell (Bell F.C., Generalised rainfall-duration-frequency relationships, Proc ASCE, 95, HY1, 311-327, 1969). Il a fait l'hypothèse que tous les orages de courte durée étaient dus aux cellules convectives dont le caractère était similaire partout dans le monde. Utilisant une base de données tirées de plusieurs pays, il a developpé une formule générale des rapports fréquence/durée d'orages. Cet article étudie l'application de cette formule en utilisant des données pluviométriques de six pays et de trois continents. Il démontre que, d'une façon générale, la formule est de bonne précision mais que des modifications qui se servent des données locales peuvent apporter des améliorations.

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INTRODUCTION

It is a frequent complaint of hydrologists that they do not have enough data, or that the data they do have is not for the site they are interested in. To give some help in these situations various techniques have been developped to enable data from the same or similar regions to be adapted for the purpose in hand. Most countries have standard formula for this which have stood the test of time.

A very common problem is defining the quantity of rainfall to be expected for a given frequency of occurrence. An early attempt to produce a general formula in the United Kingdom was that due to BILHAM (1936) which was of the form :

$$n = 1,25\, t\, (R + 0,1)^{-3.55} \qquad (1)$$

where :

n is the number of events in 10 years,

R is the rainfall depth in inches,

t is the duration in hours.

This equation was valid for periods from 5 minutes to 2 hours. This was based on data from only 12 stations with 10 years record. Using more extensive data a simplified version of the formula was produced by HOLLAND (1967) which was :

$$n = tR^{-3.14} \qquad (2)$$

which was valid for t up to 25 hours.

In the United Kingdom this work has been superseded by the much more detailed studies of the Flood Studies Report (NERC, 1975) which gives design storms of durations from 1 minute to 2 days, for a range of return periods and with appropriate areal reduction factors.

In many parts of the world there is just not enough data available to allow for this sort of detailed exercise so a hydrologist or design engineer has to have recourse to other techniques such as the formula discussed here. In the rest of the paper Bell's formula is presented with comments on its applicability.

BELL'S RAINFALL/DURATION/FREQUENCY RELATIONSHIP

Bell's method was developped after an analysis of rainfall data from the United States, the USSR, Australia and South Africa. His method is based on the assumption that the most intense short duration storms are caused by convective storm cells and that such storms have similar characteristics wherever they occur in the world. For this reason his method is only valid for storms of up to 2 hours duration. In its basic form there are two equations, one of which defines the changes due to different storm durations and the other the changes due to different return periods. The first of these is :

$$R[t,T] = (0.54\, t^{.25} - 0.50)\, R[60,T] \qquad (3)$$

where :
R is the total rainfall in millimetres,
t is storm duration in minutes,
T is return period in years.

The second is :

$$R[t,T] = (0.21\, ln(T) + 0.52)\, R[t,10] \qquad (4)$$

These two equations can be combined to give a generalised formula, which is :

$$R[t,T] = (0.54\, t^{.25} - 0.5\,)\,(0.21\, ln(T) + 0.52)\, R[60,10] \qquad (5)$$

For an evaluation of the formula the following data sets were used :
Medan, Sumatra (Indonesia)
from Wild and Hall (1982) (quoted in SHAW (1988)),
Kumasi, Ghana and Oxford, United Kingdom
both quoted in WILSON (1983),
Maputo, Mozambique
DHV (1981),
Yundum Airport, Banjul, Gambia
from TOWNSEND (1977),
Niamey, Niger
from an unpublished report.

The data are not presented in a consistent format. In the first four cases they consist of values of intensity for different return periods and different durations. In some cases the shortest duration is 60 minutes and in others it is only 6 minutes. There are also some variations in the case of the return periods used. For Banjul, the maximum storms of different durations over a 20 year period are recorded. For Niamey the data is presented as the number of times when different values were exceeded over a 23 year period. In this case the return periods of the 12 most severe events were assigned using the Weibull formula which is :

$$P = n/(N+1) \qquad (6)$$

where :
P is the probability of excedence,
n is the rank of the event,
N is the total number of years.

Figures 1 to 6 show the results using the formula (equation 5) and the values as presented for the six stations.

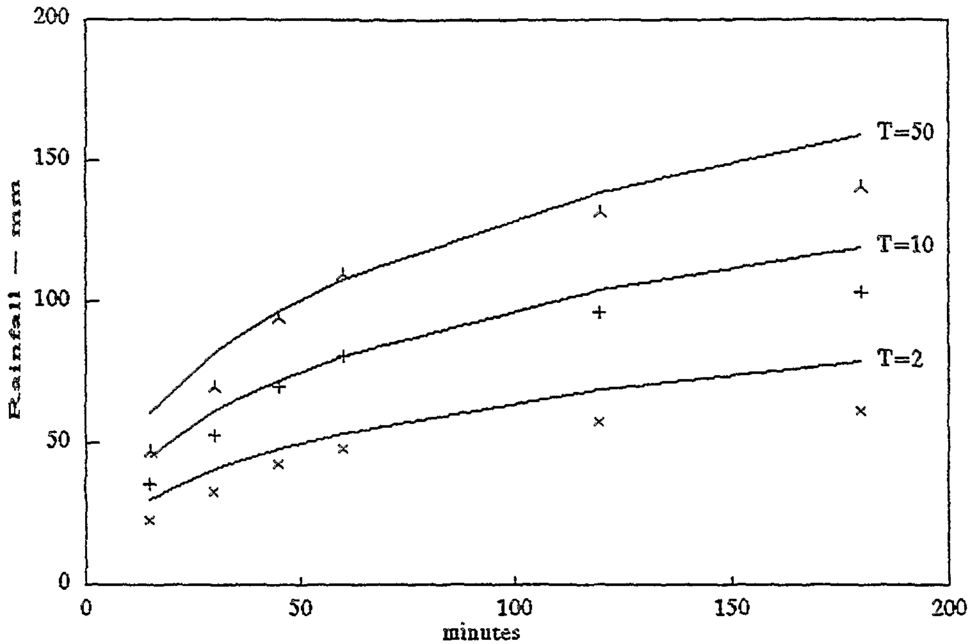

Figure 1

Rainfall / duration / frequency : Maputo - Mozambique

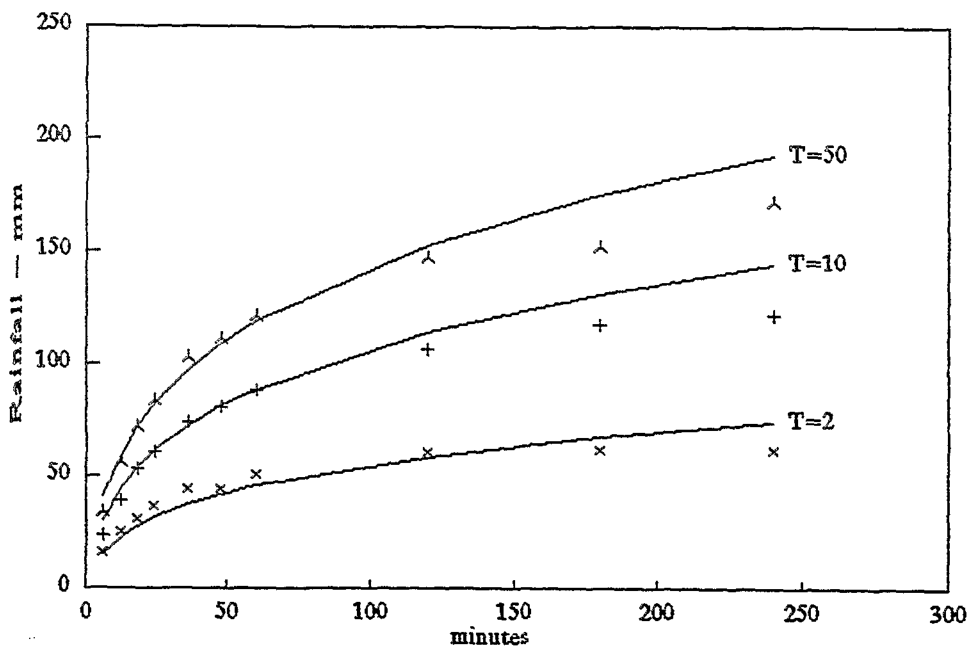

Figure 2

Rainfall / duration / frequency : Kumasi - Ghana

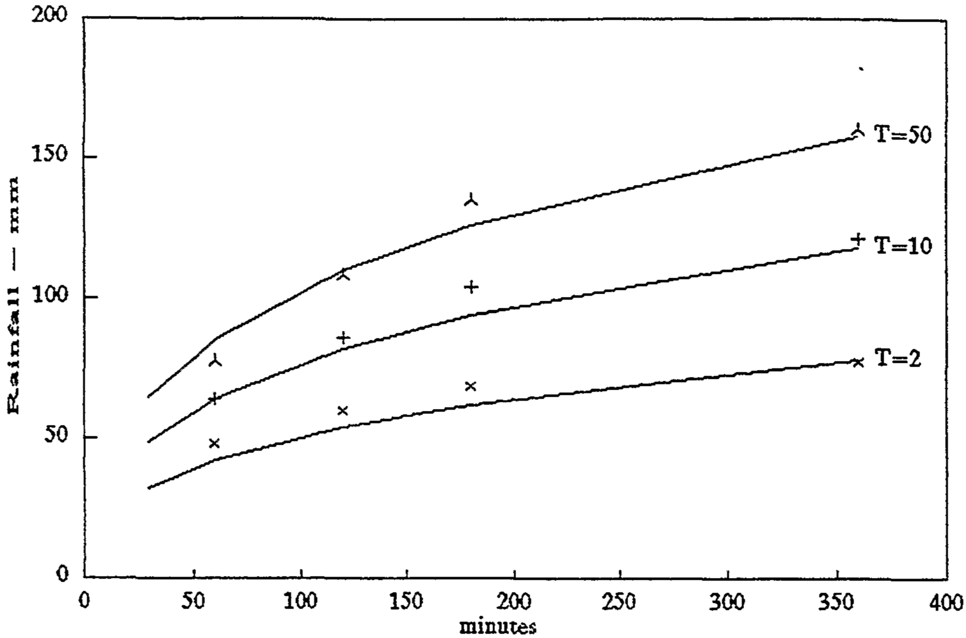

Figure 3
Rainfall / duration / frequency : Banjul - Gambia

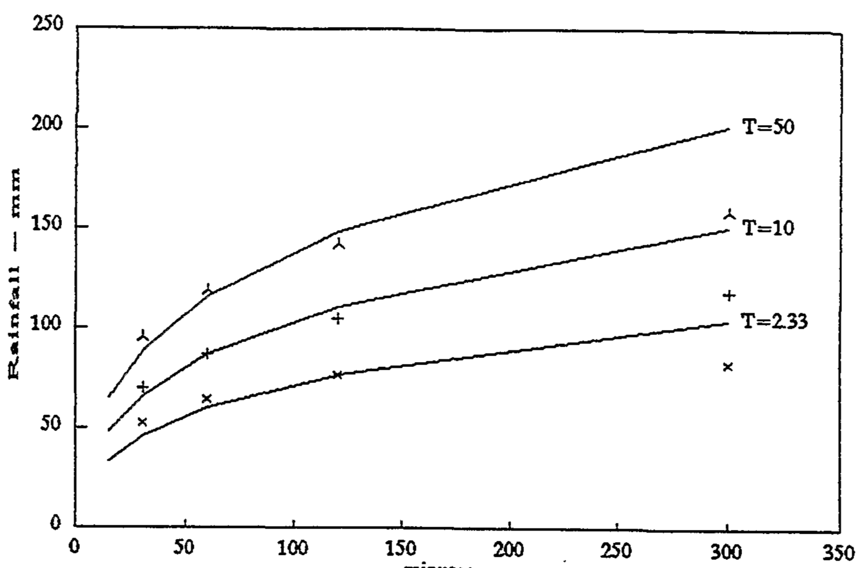

Figure 4
Rainfall / duration / frequency : Medan - Sumatra

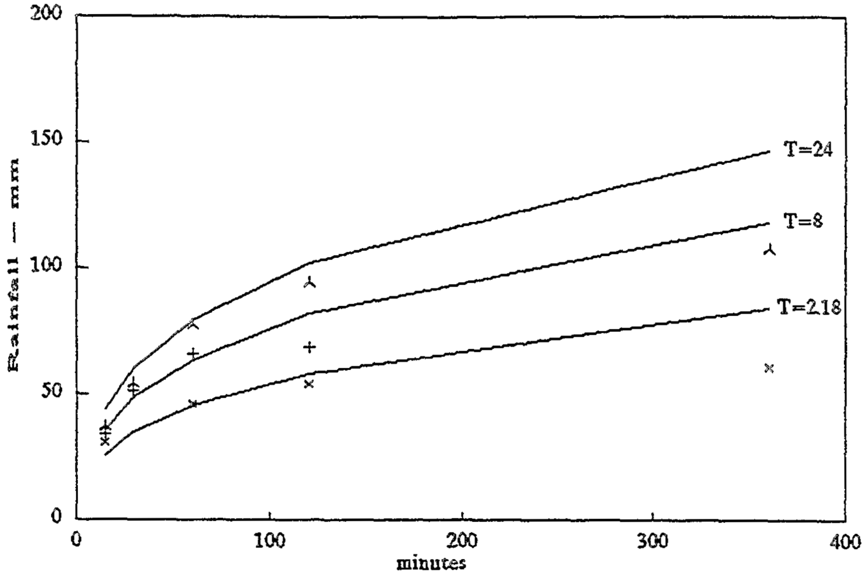

Figure 5
Rainfall / duration / frequency : Niamey - Niger

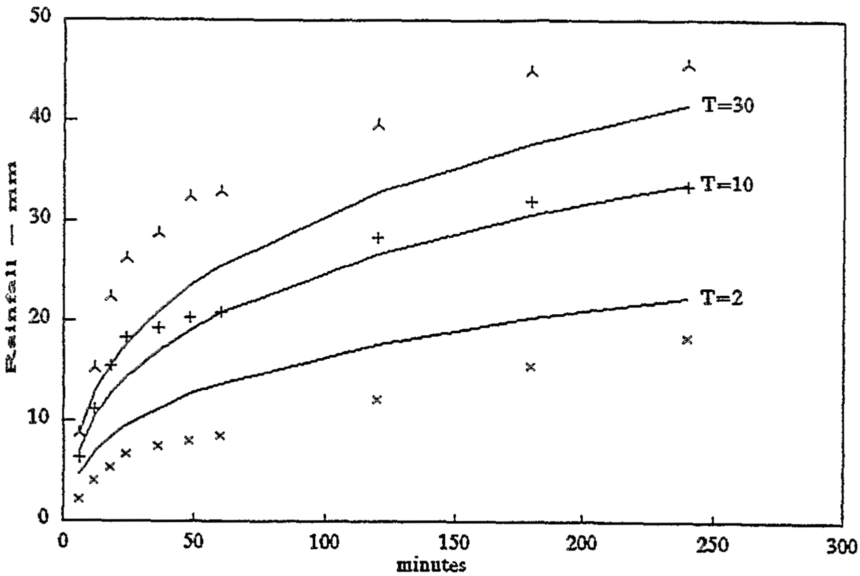

Figure 6
Rainfall / duration / frequency : Oxford - United Kingdom

Since the formula is based on the 60 minute 10-year storm then this value is correct for all the stations. With the possible exception of Banjul, the 60 minute storms for all tropical stations are well represented, suggesting that the part of the formula relating to different return periods is accurate. The 10-year storms for different durations are also well represented. The area where the largest errors, in percentage terms, occur is for short intensity storms for return periods other than ten years. It is also clear that the errors get larger for durations larger than 2 hours, which was the limit put on the formula by Bell. It is interesting that the accuracy of the formula in Medan which in a Asia, a continent not used in the original analysis, should appear similar to that for the African stations. The final figure, number 6, for Oxford in the United Kingdom indicates that the formula may not be applicable in temperate climates.

The question of whether the results of the formula are accurate enough has to be set against other factors which influence the accuracy, and also the possibility of different methods which might give better results.

The estimation of rainfall of short duration is a notably difficult task. A normal autographic rain gauge has a single chart for a twenty four hour period. Reading off values for durations shorter than an hour is hard to do accurately, particularly as during an intense storm the trace may rise rapidly making it almost impossible to identify slight changes in the gradient of the line which in turn represent changes in intensity. A steep gradient also makes it almost impossible to identify the duration of an intense burst of rain. If the total of the values read from the chart is different to those from a check gauge then further errors may be introduced in adjusting rainfall for parts of the day. Other devices, for example tipping-bucket gauges, present similar difficulties.

There is also the question of which probability distribution to use. Most of the above calculations used the Gumbel (type I) distribution but different results would be obtained from different distributions. There is also the question of which of the different distribution formula is most appropriate for the representation of rainfall, particularly where it is necessary to estimate rainfall for a return period significantly greater than the length of the data series.

To test the accuracy of the part of the formula dealing with return periods the values from the formula were compared with those obtained directly from the Gumbel method. Values of 60 minutes rainfall for Banjul were used. The following table shows the results :

Table 1
60 minute rainfall : Banjul - Gambia

	Rainfall (mm)			
Return period (years)	Min95	Central	Max95	Bell
2	43,5	48,2	52,8	42,4
5	49,9	57,7	65,5	54,7
10	53,4	63,9	74,5	64,0
20	56,7	70,0	83,3	73,3
50	61,0	78,1	95,2	85,6
100	64,0	83,8	103,7	95,0

In the above table the « Min_{95} » and « Max_{95} » columns refer to the 95 % confidence limits for the Gumbel distribution. The « Central » column gives the central estimate from the formula and the « Bell » column gives the figure using Bell's formula. With the exception of the 2 year return period estimate all the values from the Bell formula fall within the 95 % limits and the maximum error is around 15 %. In other words, given other errors in calculation, the use of the Bell formula does give substantial errors relative to the use of the Gumbel formula.

To use the Bell formula it is however necessary to have sufficient data to estimate the 1 in 10 year 60 minute rain storm. The question which therefore arises is « would it be better to estimate the mean and the standard deviation and use those to calculate the parameters of Gumbel's distribution ? »

An analysis of the Bell formula shows that it is approximately equivalent to using the Gumbel distribution with a coefficient of variation of 0.35. The rainfall data sets have coefficients of variation ranging from 0.22 to 0.43. It would therefore appear that the accuracy of the Bell formula will depend on how closely a particular data set corresponds to this assumption. To test this, a computer program was written to generate 1 100 rainfall values with a mean of 1.0 and a standard deviation which could be varied. The 1 100 values were used to provide 1 000 overlapping data sets of up to 100 items.

The first test carried out was to generate data sets of 10 to 50 items in steps of 10. The 1 in 10 year value was taken as the highest for the 10 year data set, the second highest for the 20 year data set, and so on. For each of the five test periods and each data set three values of rainfall for three different return periods were calculated. The first was the « true » value using the mean and standard of the 1 100 data items and the appropriate Gumbel factors ; the second was to calculate the mean and standard deviation from the 10 to 50 items of the data set ; the third was to use the Bell formula using the 1 in 10 year values calculated as described above. The return periods tested were 2, 10 and 50 years. The test showed that for a coefficient of variation of 0.35, the same as that implicit in the Bell formula, there was little to choose between using the formula or the data with the Gumbel distribution. If the coefficient of variation was 0.22, the lowest in the data sets described above, then it was better to use the mean and standard deviation calculated from even 10 years of data only. With 10 years of data the formula gave a worse accuracy for around 70 % of the trials and with 50 years of data it was worse in 95 % of the trials. If the coefficient of variation was 0.43 there was a slight advantage in using the data rather than the formula but not as much as in the previous test — it was worse for lower periods of return but gave similar answers for return periods of 10 and 50 years. For the test with the coefficient of variation of 0.35 the average absolute errors were around 6 to 7 % for all ranges and both methods. For a coefficient of variation of 0.22 the errors with 10-year data sets were 6 % to 7 % using the Gumbel distribution but up to 17 % using the formula. For the highest coefficient of variation the errors started off at around 10 % for a 10 year return period and dropped to around 5 % for 50 years but with the errors for a 1 in 2 year storm being almost twice as high using the formula.

The basic version of the Bell formula uses the 10-year 60-minute storm as the basis for extrapolation. Bell however proposed another formula which was based on the 2-year 60-minute storm. This is :

$$R[t,T] = (0.35\ ln(T) + 0.76)(0.54\ t^{0.25} - 0.50)\ R[60,2] \qquad (7)$$

This formula was considered less reliable than the formula using the ten year storm. It does however have the advantage that if the data is normally distributed then the 2-year storm is the average. For the Gumbel distribution the 2.33 year storm is the average. By adjusting the parameters of the above equation a different version of the formula for the 2.33 storm was produced which is :

$$R[t,T] = (0.34\ ln(T) + 0.712)(0.54\ t^{0.25} - 0.50)\ R[60,2.33] \qquad (8)$$

The second test using the trial data set was to use data for from 2 to 100 years length to estimate the mean and to use that value as the 2.33 year return period storm in the above formula. For a coefficient of variation 0.35 there was little difference in accuracy between the methods with a slight improvement from using the Gumbel distribution for more than 20 year of data, but even so the difference was between 7 % for the Gumbel formula and 8 % for equation 8. For a coefficient of variation of 0.22 similar results to those in the first series of test were obtained, with the Gumbel distribution giving better results after 3 years' data was available. In this case the error for a 1 in 50 storm was always around 30 %. For the 0.43

coefficient of variation, the use of the Gumbel distribution was better after 10 years but the errors were of the order of 10 % with little variation in accuracy between the two methods.

CONCLUSIONS

From a visual inspection of the results presented on the 6 figures the broad conclusions are that the Bell formula gives usable results for tropical countries but the results are worst for the one rain gauge in a temperate climate. Further analysis demonstrates that a critical factor is the coefficient of variation of the data set being studied. If the coefficient of variation of low, around 0.22, then it is better to use even a few years of data to calculate the parameters for the Gumbel distribution. On the other hand if the coefficient of variation is close to the implicit value in the Bell formula, 0.35, or even higher then some 10 to 20 years of data is needed before the standard deviation can be estimated with sufficient accuracy for the Gumbel distribution to give better accuracy. However, from only a few years' data it will not be possible to know with any accuracy whether the coefficient of variation is low or not. The suggested approach therefore is :

- — if there are more than 20 years data, calculate the mean and standard deviation and use the Gumbel formula ;
- — if there are less than 10 years data use the Bell formula as modified in equation 8 ;
- — if there are between 10 and 20 years of data then use the Bell formula (equation 5) if the coefficient is above 0.3, otherwise use the Gumbel distribution.

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