

THE  $E_s$  MODEL: A DEFINITION OF SLOPE AND CATCHMENT EROSIONAL  
SUSCEPTIBILITY

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## 1. Introduction

For both academic and practical reasons an increasing number of laboratories and institutions are interested in an adequate prediction (or postdiction) of erosion and denudation rates on slopes and in catchments. A major obstacle to progress in this field is the complex geography of such landscape units. For example in Europe landscape systems in rural areas are often controlled by linear infrastructural elements which generate discontinuous erosion and discrete events, on apparently continuous toposequences (DE PLOEY, 1989). Other difficulties arise from the complex nature of process combinations which act on slopes and in thalwegs. Too often work done by the main geomorphic processes is mapped and discussed as if they were rather independent agents, looping the reality of their integrated impacts. Rill erosion or gullying are generally the result of combined erosion by running water and mass movements on the sidewalls. Dunes in humid and subhumid areas are often shaped by combined wind erosion and sheetwash. Analogous situations can be abundantly exemplified.

Two ways seem to be open to tackle the problem of prediction. One is to design a process-based soil erosion model which integrates (continuity) equations for predicting detachment, transport and deposition processes. Many present-day efforts go in that direction (NEARING et al., 1989).

An alternative may be to find out and to specify the links between a primary input of total available erosional energy and the output of eroded sediments. Such operation, by necessity, implies the introduction of a major black-box coefficient to be defined on an empirical basis. With time such a black-box parameter may shift into the category of grey-box coefficient if future research, including process studies, is able to unravel its complex nature. The present paper is in line with such strategy and it is thought that the proposed model may at least lead to a basic discussion of this approach.

## 2. The $E_s$ model

Basically the model is analogous to the rill or gully headcut retreat model which includes both components of potential and kinetic energy (DE PLOEY 1989):

$$V_T = E_s \cdot M \cdot (g \cdot \bar{R} + \bar{U}^2 / 2) \quad (1)$$

whereby

$V_T$  = total volume eroded within a catchment of size A; erosion is distributed over an area  $A_e \leq A$

M = the total amount of water precipitated on a catchment or slope A during a storm event or during any period of duration t

g = the gravitational acceleration

$\bar{h}$  = the mean elevation head loss. If rill erosion and gullying is predominant,  $\bar{h} = V_{R,G}/A_{R,G}$  whereby  $V_{R,G}$  is the estimated total volume of sediments evacuated in rills and gullies and  $A_{R,G}$  is the estimated total area represented, within A, by rills and gullies. Parameter  $\bar{h}$  would also apply to slides on slopes.

$\bar{u}_o$  = the mean shear velocity of overland flow during the event(s).  
Thereby

$$\bar{u}_o = (g \cdot \bar{R} \cdot \bar{S})^{0.5} \tag{2}$$

with

$\bar{R}$  = the mean hydraulic radius of overland flow

$\bar{S}$  = the mean slope angle within the eroded areas  $A_E$

Fitting (2) in (1) gives

$$E_e = \frac{V_T}{M \cdot g \cdot (h + R \cdot S/2)} \tag{3}$$

Often the mean hydraulic radius of overland flow  $\bar{R}$  will correspond to the mean depth  $\bar{d}$  of the flow. Calculations of  $\bar{R}$  can derive from the Manning equation.

It is proposed to replace  $\bar{S}$  by the estimated value of the mean steepest slope angle within the considered eroded area  $A_E$ .

Basically  $E_e$  relates eroded volume(s)  $V_T$  to an expression of total energy (potential and kinetic energy) which is depending upon the total mass of precipitated water  $M$ . The introduction of the full value  $M$  is justified by the fact that geomorphic work, directly and indirectly, is entirely controlled by  $M$ . Also part of  $M$ , that first infiltrates and thereafter evaporates, has to be considered as a geomorphic agent, for it will have a direct impact on soil creep and, eventually, on other types of subsequent mass movements. Factors  $\bar{h}$ ,  $\bar{R}$ ,  $\bar{S}$  are interdependent parameters of an erosional system, of which mass movements ( $g \cdot \bar{h}$ ) and erosion by flowing water ( $g \cdot \bar{R} \cdot \bar{S}$ ) constitute integrated parts.

$E_e$  can be applied to any area A, provided it can be considered as a functional hydrological unit. In fact, hydrology governs erosional systems as far as they depend upon a combination of direct erosion by flowing water and mass movements. In case of massive infiltration  $\bar{h}$  expresses the corresponding mean hydraulic head loss on the slopes, which may or may not cause mass movements ( $V_T > 0$  or  $V_T = 0$ ). But in rills and gullies, both parameters,  $\bar{h}$  and  $\bar{R} \cdot \bar{S}$ , intervene for sidewalls always move by falls, slides and flows.

### 3. Using $E_e$ expressions

#### 3.1. Sheet erosion by water

If no permanent incisions occur ( $\bar{h} = 0$ )

$$E_{e-h=0} = \frac{2 \cdot V_{sh}}{M \cdot g \cdot \bar{R} \cdot \bar{S}} \tag{4}$$

In a first approximation  $\bar{R}$  can be calculated according to the Manning equation.

#### 3.2. Rill erosion and gullying

Linear erosion is supposed to prevail and potential energy is related to mean depths of incision  $\bar{h}$ . This means that often  $\bar{h} > \bar{R} \cdot \bar{S}/2$  and that  $\bar{R} \cdot \bar{S}/2$ , compared to  $\bar{h}$ , becomes negligible:

$$E_{E-R.G} = \frac{V_{R.G}}{M.g.h} = \frac{V_{R.G}}{A.P.g.h} \quad (5)$$

whereby P corresponds to the total precipitation/unit area. The mean depth h of rills and/or gullies can be expressed by

$$h = V_{R.G} / A_{R.G} \quad (6)$$

and introducing (6) in (5) gives:

$$E_{E-R.G} = \frac{A_{R.G}}{A.P.g} \quad (7)$$

$E_E$  is now directly depending upon the total area eroded by rills and/or gullies,  $A_{R.G}$ . This interesting relationship shows that the assessment of  $E_E$  can be related to remote sensing measurements. Moreover, factor  $A_{R.G}$  stresses the impact of lateral erosion and sidewall stability or instability upon  $E_{E-R.G}$  values. The development of flat, wide rills or gullies points to a relative high erosional susceptibility of slopes and catchments; this is a basically sound geomorphological conclusion.

### 3.3. Sheet and rill wash

In this case:

$$V_T = V_R + V_{IR} \quad (8)$$

$V_{IR}$  corresponding to the volumes eroded on interrill areas. If the ratio a :  $V_{IR}/V_R$  is estimated then:

$$E_{E-R,IR} = \frac{V_R (1 + a)}{M.g (V_R/A_R + \bar{R}.S/2)} \quad (9)$$

For the Huldenberg experimental slope, in the Belgian loess area, GOVERS and POESEN (1988) concluded to a factor a = 0.25,  $V_{IR}$  corresponding to about 25 % of the volumes eroded in the rills.

Both factors h ( $V_R/A_R$ ) and  $R.S/2$  are interdependent parameters. They resulted from erosion and generate erosion. It can be indicated to relate R values to estimated peak discharges of flow during extreme events.

Finally the numerator in equations (3), (4), (5) and (9) should be  $(V_T - V_0)$ , related to period  $(t - t_0)$  if an erosional system ( $V_0, A_0$ ) existed already at the beginning of the recorded period  $t_0$ .

### 3.4. Landslides

Massive infiltration of precipitated water relates M to an elevation head loss h and subsequent sliding, when shear stresses exceed shear strength. The  $E_{E-L}$  expression for landslides is comparable to the one for gullies (5), provided that factor  $R.S/2$  is negligible:

$$E_{E-L} = \frac{V_L}{A.P.g.h} \quad (10)$$

and, for  $\bar{h} = V_L/A_L$

$$E_E = \frac{A_L}{A.P.g} \quad (11)$$

Again values of  $E_{E-L}$  depend upon the areal extension of erosion.

### 3.5 Wind erosion

The general expression of the  $E_s$  model (3) is also adaptable to wind erosion:

$$E_{s-w} = \frac{V_w}{A \cdot u_o \cdot t \cdot g (h + u_o^2/g)} \quad (12)$$

whereby,

$V_w$  : the total volume(s) eroded by wind, both by sheet erosion and/or concentrated deflation (factor h)

A = the considered area over which the wind has blown during a period t, with a mean or representative shear velocity  $u_o$

h = the elevation head loss of the wind in depressions (e.g. Blowouts) or behind barriers. Therefore h may correspond to  $V_c/A_c$ , the ratio between the volume of sediments evacuated by concentrated erosion on sites with a total area  $A_c$ .

In case of sheet erosion  $h = 0$  and  $E_{s-w}$  is inversely proportional to  $u_o^3/g$ , which expresses the transporting capacity of the wind (BAGNOLD, 1941):

$$E_{s-w,sh} = \frac{V_{sh}}{A \cdot t \cdot u_o^3} \quad (13)$$

### 4. $E_s$ and the efficiency EF of processes

ES is the reversal of the resistance of catchments to erosion. Dimensionally  $E_s$  corresponds to  $T^2/L^2$  ( $s^2/m^2$ ) and the efficiency of processes or process combinations EF is proportional to  $E_s$ :

$$EF = E_s = \frac{1}{1/E_s} = \frac{1 \text{ kg}}{1 \text{ kg} \cdot m^2/s^2} = \frac{1 \text{ kg}}{\text{Joule}} \quad (14)$$

EF or  $E_s$  express the amount of geomorphic work (Joule)/kg removed sediment. This makes clear that really  $E_s$  considers the erosional susceptibility of catchments in terms of energy.

### 5. Results and comments

The model has been applied to a limited number of representative slopes and catchments, on the basis of some data obtained from the literature and from the Leuven laboratory. They are listed in an order of decreasing  $E_s$  values.

1. Sheetwash on the windward side of a crescent dune at Kalmthout (Belgium, period 1973-1977, data collected by DE PLOEY:  $a = 10^4$   $m^2$ ;  $V_{sh} = 60$   $m^3$ ; P (annual precipitation) = 0.7 m; S = 0.17; R between 0.001 and 0.005 m. Application of formula (4):  $E_s$  between 2-10  $s^2/m^2$ .
2. Sheetwash on Rwaza hill, section 2A (Rwanda, period 1978-1982), MOEYERSONS, 1989): A = 1  $m^2$ ;  $V_{sh} = 0.00463$   $m^3/m^2$ ; P = 4m; S = 0.27 - 0.32; R between 0.002 and 0.003 m.  $E_s$  between 1 and 10  $s^2/m^2$ .
3. Planar landslides around Caraguatatuba (Brazil, Serra do Mar, extreme event 17-18/3/1967, CRUZ, 1974):  $A_L/A = 0.5-0.7$ ; P = 0.7 m; steep slopes 20°-30°. Application of formula (11):  $E_s$  between 10<sup>-1</sup> and 5 x 10<sup>-2</sup>  $s^2/m^2$ .
4. Slides in the Mgeta area (Tanzania, Western Uluguru Mts, TEMPLE and RAPP, 1972):  $A_L/A$  between 1/35 and 1/10; P between 0.1 m and 0.185 m; steep slopes 20°-40°.  $E_s$  between 10<sup>-1</sup> and 10<sup>-2</sup>  $s^2/m^2$ .

5. Thalweg gullying in the Nono Valley basin (North Central Nigeria, erosion in clayey-sandy deposits, data collected by DE PLOEY, 1970-1972).  $A = 350 \text{ km}^2$ ;  $A_0 = 1-5 \text{ km}^2$ ;  $P = 17.5 \text{ m}$  for a 25 years period; mean  $S$  of the order of 0.10;  $h$  of the order of 2-3 m. Application of formula (7):  $E_s$  between  $10^{-6}$  and  $8 \times 10^{-6} \text{ s}^2/\text{m}^2$ .
6. Sheet erosion by wind on the windward side of the crescent dune of Kalmthout (Belgium, period 1973-1977, data collected by DE PLOEY):  $A = 20,000 \text{ m}^2$ ;  $V_{wh} = 480 \text{ m}^3$ ;  $t = 360 \text{ hrs}$  of deflation during a 4 yrs period;  $u_0$  between 0.4 m/s and 0.6 m/s. Application of formula (13):  $E_{s-w,wh}$  between  $10^{-7}$  and  $3 \times 10^{-7} \text{ s}^2/\text{m}^2$ .  
A set of 3 comparable results is available from silty loess loam areas.
7. Sheet and rill erosion on the experimental slope of Huldenberg (Belgium, data collected by GOVERS, 1986):  $A = 7,500 \text{ m}^2$ ;  $V_T = 121 \text{ m}^3$  ( $97 \text{ m}^3 = V_{RS}$  rill erosion and  $24 \text{ m}^3 = V_{IR}$  interrill erosion);  $a = 0.25$ ;  $A_R = 680 \text{ m}^2$ ;  $P = 0.7 \text{ m}$ , in 1 yr. Application of formula (9):  $E_s$  between  $10^{-2}$  and  $2 \times 10^{-2} \text{ s}^2/\text{m}^2$ . Factor  $R.S/2$  was negligible compared to  $h = V_R/A_R$ .
8. The Huldenberg site belongs to the Central Belgian loess area with its hilly topography. The maximal slope angle of this site is  $11^\circ$  ( $S = 0.20$ ). In the same belt, near Leuven, heavy erosion occurred on 17-18th June 1986 ( $P$  of the order of 0.04 m) in catchments with a mean  $S = 0.04-0.08$ . The main features were rills and flat, undep (up to 30 cm) thalweg gullies of which  $A_{R,0}$  was estimated on aerial photographs by VANDAELE. For 3 monitored catchments with sizes  $A$ , between 1 and  $3.8 \text{ km}^2$ ,  $E_{s-R,wh}$  of the rill-and-sheetwash system amounted to  $10^{-3}$ , up to  $7 \times 10^{-3}$ .
9. Gullies and badlands on steep valley slopes of the deeply entrenched Chinese Loess Plateau (N. China, Xingzihe River Basin, TANG KELI et al., 1987):  $A = 1,486 \text{ km}^2$ ;  $A_0/A = 0.55$ ;  $P$  (annual) = 0.51 m; annual soil loss =  $V_0 - V_1 = 1.5 \times 10^7 \text{ m}^3$ ;  $h$ , between 200 m and 300 m. Application of formula (7) gives:  $E_{s-0}$  between  $10^{-6}$  and  $5 \times 10^{-6} \text{ s}^2/\text{m}^2$ .

This first analysis reveals a remarkable sequence of  $E_s$  values, ranging between  $10$  and  $10^{-7} \text{ s}^2/\text{m}^2$ , or an efficiency  $EF$  of processes between 0.1 Joule and  $10^7$  Joule, for 1 kg of sediment removed. The most efficient process is sheetwash, whereby flows are in closest contact with the eroded material. On Rwaza hill there is a tendency to plunge-pool effects which means that more energy is dissipated in non-erosional effects. Therefore  $E_{s-wh}$  is inferior to the value obtained on the Kalmthout dune sands.

Planar slides too are efficient agents (Caraguatatuba).  $E_{s-L}$  values for the Mgeta sheet slides are lower than in Caraguatatuba for, there, large part of the slides seem to be limited to thalweg areas, where water had to concentrate before mass movements could start. According to the definition of  $E_s$  concentrated flows, in rills and gullies, are definitely less efficient than sheetwash. In such flows a relatively larger part of total available energy is non-erosional, although erosional forms may be rather spectacular. A system of limited soil erosion on slopes, combined with thalweg gullying (Nono Valley), in fact points to a relatively high resistance of the catchment to erosion, which cannot start until flows reach a maximal concentration in thalwegs. For similar reasons erosion, by combined runoff and mass movements, is less efficient in the spectacular canyons of the Chinese Loess Plateau than on the rolling loess slopes of Brabant, in Belgium.

Among the least efficient processes certainly is wind erosion, as illustrated by the  $10^{-7}$   $E_s$  values obtained from the Kalmthout dune. This is imputed to the very nature of such flows which are characterized by a high degree of turbulence, with much dissipation of energy.

## References

- BAGNOLD R.A., (1941): The Physics of Blown Sand and Desert Dunes, Methuen, London.
- CAINE N., (1976): A uniform measure of subaerial erosion. Geological Society of America Bulletin 87, 137-140.
- CRUZ O., (1974): A Serra do Mar e o Litoral na area DE Caragiatauba. Inst. Geogr. Univ. Sao Paulo, Serie Teses e Monogr., 11.
- DE PLOEY J. and POESEN J., (1987): Some reflections on modelling hillslope processes. Catena Supplement 10, 67-72.
- DE PLOEY J., (1989): A model for headcut retreat in rills and gullies. Catena Supplement 14, 81-86.
- DE PLOEY J., (1989): Erosional systems and perspectives for erosion control in European loess areas. Soil Technology Series 1, 3-102.
- DE PLOEY J., (1990): Modelling the erosional susceptibility of catchments in terms of energy. Catena 17, 175-183.
- GOVERS G., (1986): Mechanismen van akkererosie op lemige gronden. Ph. D thesis Univ. Leuven.
- GOVERS G., (1987): Spatial and temporal variability in rill development processes at the Huldenberg experimental site. Catena Supplement 8, 17-34.
- GOVERS G. and POESEN J., (1988): Assessment of the interrill and rill contributions to total soil loss from an upland field plot. Geomorphology 1, 343-354.
- MOEYERSONS J., (1989): La nature de l'érosion des versants au Rwanda. Ann. Museum Midden-Afrika Tervuren 19, 379 p.
- NEARING M.A., FOSTER G.R., LANE L.J. and FINKNER S.C., (1989): A process-based soil erosion model for USDA-water erosion prediction project technology. Trans. ASAE, 32, 5, 1587-1593.
- SAVAT J., (1977): The hydraulics of sheet flow on a smooth surface and the effect of simulated rainfall. Earth Surface Processes, 2, 125-140.
- SAVAT J., (1976): Discharge velocities and total erosion of a calcareous loess: a comparison between pluvial and terminal runoff. Revue de Géomorphologie dynamique 24, 113-122.
- POESEN J. and GOVERS G., (1990): Gully erosion in the loam belt of Belgium: typology and control measures. In: J. Boardman, J.A. Dearing & L.D. Foster (Eds.); Soil Erosion on Agricultural Land, 515-529. Wiley, Chichester.
- TANG KELI, XI DAOGING and ZHANG PINCANG, (1987): The main types of soil erosion related to the characteristics of loess distribution: a representative basin of Xingzihe River in "Aspects of loess Research" (Ed. Liu Tungsheng). China Ocean Press Beijing, 437-445.
- TEMPLE P.H. and RAPP A., (1972): Landslides in the Mgeta area, Western Uluguru Mountains, Tanzania. in "Studies of soil erosion and sedimentation in Tanzania". Geografiska Ann. 54A, 3-4, 157-193.
- TIJSKENS E., (1988): Experimenten betreffende een model over de terugschrijding van geultrappen. M. Sc. thesis, K.U.Leuven.