Estimation of steady water flux density in a porous medium by Fourier analysis of temperature variations in a cyclic heat pulse system

E.F. Sousa[1], M.A. Santolin[2] and F.C. Do[3]

[1]State University Norte Fluminense Darcy Ribeiro, UENF, CCTA/LEAG, Campos dos Goytacazes, RJ, Brazil; [2]Espirito Santo Federal Institute, Alegre, ES, Brazil; [3]IRD, University of Montpellier, Eco&Sols Unit, 34060, France.

Abstract

We tested a novel theoretical model that determines the steady water flux density in a porous medium from Fourier analysis of temperature variations induced by a cyclic heat pulse system. The model depends on the thermal diffusivity of the medium and on the relative spatial variation of the amplitude and phase of the first order sinusoidal component of the heat wave. The model was tested by using a hydraulic column made of a PVC pipe filled with sawdust. The sensor consisted of two hypodermic needles spaced 7 mm apart. One needle contained a heater and a thermocouple while the other contained only a thermocouple. Different combinations of heating and cooling cycles were tested. The flow was controlled by pressure head and volumetrically measured at the outlet of the tube. The experimental results supported the theoretical model. In particular, the convective index defined in terms of the variations of amplitude and phase of the first component of the heat wave was linearly related to the measured flux density, as predicted. The model was independent of the different combinations of heating and cooling cycles. The estimated water flux density was strongly related to the measured flux density (R^2>0.99), having the same slope for the different combinations. The first results of this new approach of cyclic heat pulse system are very promising and suggest further studies and field applications.

Keywords: harmonics series, mathematical model, sap flow measurement

INTRODUCTION

Within the family of heat dissipation methods for sap flow measurement, the cyclic heating system (Do and Rocheteau, 2002; Lubczynski et al., 2012) was first applied in order to reduce the sensitivity to natural thermal gradients found in the constant heating method of Granier (1985). The cyclic heating system was based on intermittent heating of duration between 10 and 45 min, at regular intervals. Compared to the constant heating method, cyclical heating saves energy and can be applied to a single probe (Do et al., 2011). Shortening of the duration of heating has proven feasible (Nhean et al., 2019). However, estimations of sap flux density have so far been based on empirical calibrations, where the uncertainty is difficult to analyze (Flo et al., 2019).

At the other end of intermittent heating systems lies the heat pulse family (Green et al., 2003; Burgess et al., 2001; Vandegehuchte and Steppe, 2012), which uses very short heat pulse durations between 3 and 6 s and at least a triple needle configuration where the heat transmission is analyzed. These methods are largely based on the analytical model provided by Carslaw and Jaeger (1959) for a single pulse and used by Marshall (1958). Carslaw and Jaeger (1959) also presented a deterministic approach of cyclic heat pulse based on Fourier series analysis, but little progress has been made since then.

The objective of this work was to develop and test a deterministic model of steady water flux density estimation in a porous medium based on Fourier harmonic analysis of cyclic heat pulse systems. The first hypothesis was that Fourier analysis provides an efficient model to determine water flux density. The second hypothesis was that the model is independent of the combination of heating and cooling cycles.

MATERIALS AND METHODS

Theory

Following Carslaw and Jaeger (1959), the theory considers first a steady periodic temperature variation in the region x>0 of a moving thermally homogenous medium with flux density J_m, in which the periodic changes of temperature at x=0 is given by $T_o\cos(\omega t + \theta)$, where T_o is the maximum temperature variation, °C; ω and θ are the frequency (rad s^{-1}) and phase (rad) of the temperature oscillation. The steady periodic temperatures T_1 and T_2 at two points x_1 and x_2 can be represented harmonically as a Fourier series,

$$T_1 = \sum_{n=0}^{\infty} B_n cos(n\omega t - \beta_n) \tag{1}$$

$$T_2 = \sum_{n=0}^{\infty} C_n cos(n\omega t - \gamma_n) \tag{2}$$

where B_n and C_n are the amplitudes, and β_n and γ_n are the phases, of the temperature harmonics of order n at x_1 and x_2, respectively.

Denoting the period by T=2π/ω, considering $n=1$ and defining

$$\alpha = \ln\left(\frac{C_1}{B_1}\right) \tag{3}$$

$$\varphi = \gamma_1 - \beta_1 \tag{4}$$

we deduce from Carslaw and Jaeger (1959) that the density flux J_m (m^3 m^{-2} s^{-1}) of the moving medium with thermal diffusivity κ can be written as:

$$J_m = \frac{2\pi}{T}\frac{(x_2 - x_1)}{\varphi} - \frac{2\kappa}{(x_2 - x_1)}\alpha \tag{5}$$

The thermal diffusivity κ can be determined in the zero-flow situation of J_m=0. Defining $\varphi = \varphi_0$ and $\alpha=\alpha_o$ in this situation,

$$\kappa = \frac{\pi}{T}\frac{(x_2 - x_1)^2}{\alpha_o\varphi_o} \tag{6}$$

Substituting (Equation 6) into (Equation 5) and rearranging gives

$$J_m = \frac{2\pi(x_2 - x_1)}{T\,\varphi_o}\left[\frac{\varphi_o}{\varphi} - \frac{\alpha}{\alpha_o}\right] \tag{7}$$

We assume that the theory can be applied to a composite medium as a plant stem or a sawdust column where the sap or the water is moving inside a stationary wood or a porous medium. Hence, we assume that the sap or water flux density J_s should be proportional to the theoretical J_m.

$$J_s = aJ_m \tag{8}$$

Where a is a constant representing the effect of differences between the ideal conditions of the theory and the actual conditions of water or sap flowing in a finite porous medium imperfectly homogenous where probes of finite dimension are inserted with spacing error, flow disrupting effect (wounding) and other possible artifacts (Vandegehuchte and Steppe, 2013) .

The model of Equation 8 estimates the sap flux density flux (J_s, m^3 m^{-2} s^{-1}) in relation to a steady periodic source heat pulse according mainly two terms,

$$\frac{2\pi}{T}\frac{(x_2 - x_1)}{\varphi_o} \tag{9}$$

$$\left[\frac{\varphi_o}{\varphi} - \frac{\alpha}{\alpha_o}\right] \tag{10}$$

The term (9) is the heat wave speed at zero flux density. The term (10) describes the "convective index" which expresses the change induced by J_s on the heat wave speed.

The ratio $\frac{\varphi_o}{\varphi}$ is related to the variation of the heat wave phase, as the ratio $\frac{\alpha}{\alpha_o}$ is related to variation of the heat wave amplitude.

Experimental set up

The model of Equation 7 was tested by using a hydraulic column made of a PVC pipe filled with sawdust. The water flow was controlled by pressure head. The flux density was calculated by measuring the volume of water at the outlet for a given time, divided by the area of the PVC pipe (50 mm diameter). The water flux density (J_w) ranged from 0 (zero) to 293 10^{-6} m^3 m^{-2} s^{-1}.

The sensor consisted of two hypodermic needles, 3 cm long, 1.5 mm in diameter and 7 mm apart. The heated needle contained a heater (constantan wire of 15Ω) and a thermocouple. The downstream needle contained only a thermocouple. The heater probe was taken as the origin (x_1=0) and the other probe was put 7 mm apart (x_2=0.007 m). The heat pulse was obtained by applying a voltage of 5V controlled by a datalogger (CR1000, Campbell, Logan, UT). The measurement of the temperatures took one second of scan time.

Six combinations of time durations of heat pulse and total cycle of heating/cooling (period) were tested, in seconds: 60×120, 15×150, 90×180, 120×240, 30×300, 60×360.

RESULTS

In all combinations of heating and cooling time, the flux density of water altered both the phase and the amplitude of the first harmonic of the heat wave analyzed by Fourier series as shown for 15×150 s in Figure 1. The ratios of amplitude and phase had inverse relationships with the flux density J_w (Figure 2a). As expected by the model, the convective index (Equation 10) had a linear relationship to the flux density J_w (Figure 2b).

The estimations from the model based on the convective index were strongly related to the measured values of water flux density in the sawdust column whatever the heating and cooling cycle used (Figure 3). The convective indexes were calculated from the measured phases shift and amplitudes gap according the theory (Equation 7).

The intercept of the regression was close to the origin (1.55), indicating little systematic error. The common slope of 0.78 differed significantly from 1. This slope reflected the coefficient *a* of Equation 8.

DISCUSSION

The results demonstrated that the harmonic analysis provides an efficient model to determine water flux density. The convective index based on the variations of amplitude and phase of the first component of the heat wave was strongly linearly related to the measured flux density. As expected, the relationship between the flux estimated by the model and the measured water flux density was also strong and linear. The found slope (0.78) represented a calibration coefficient between the ideal system of the theory and the artifacts coming from the practical conditions of measurement. This value will likely change according a variety of conditions (wood type and size, probe type, insertion, etc.). Studies are needed to describe the operational range and clarify the point. However, these artifacts of thermometric methods are quite well known (Vandegehuchte and Steppe, 2013).

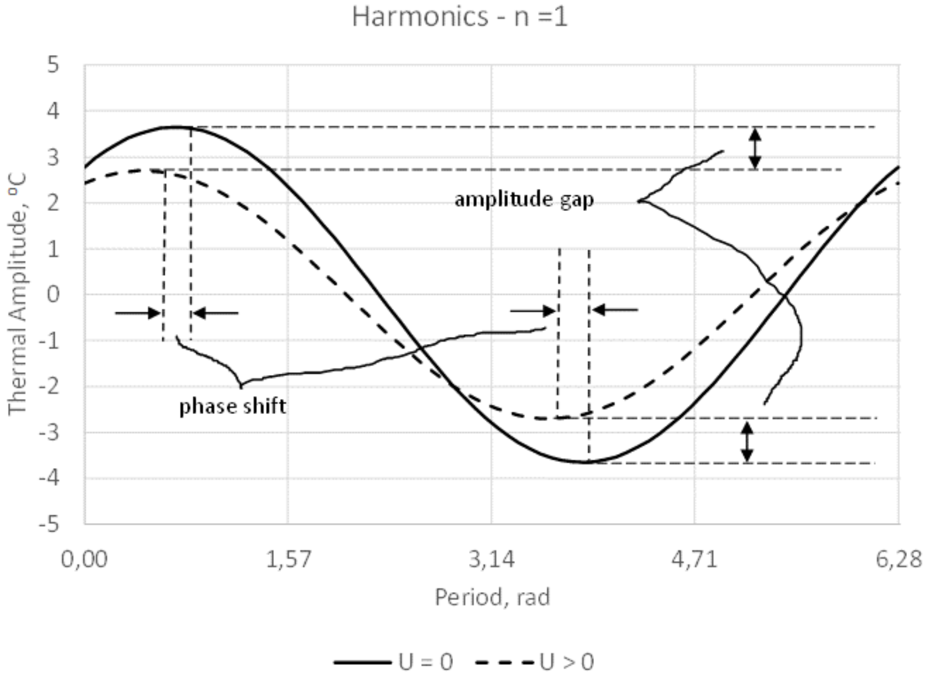

Figure 1. Comparison of the amplitude and phase of the first harmonic of Fourier series obtained from measured temperature values: (solid line) zero flux density ($J = 0$) and (dashed line) high flux density (J = 293 10^{-6} m^3 m^{-2} s^{-1}) at the position $x_1 = 0$, in a sawdust column, combination 15x150 sec.

A **b**

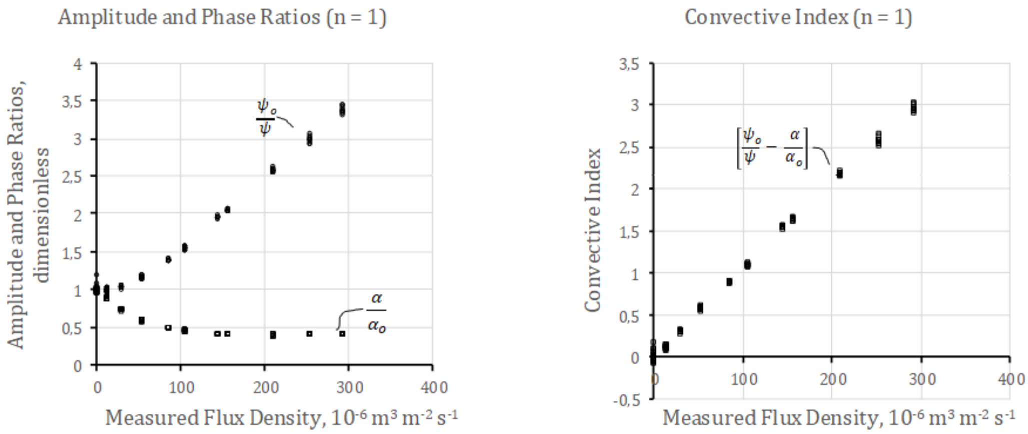

Figure 2. Relationships of measured flux density with (a) the ratios of thermal amplitude (α/α_o) and difference of phase (φ_o/φ) and (b) the convective index. The temperatures were measured at sawdust column at the position x=0 mm and x=7 mm; heat pulse of 15 s with a period (T) of 150 s.

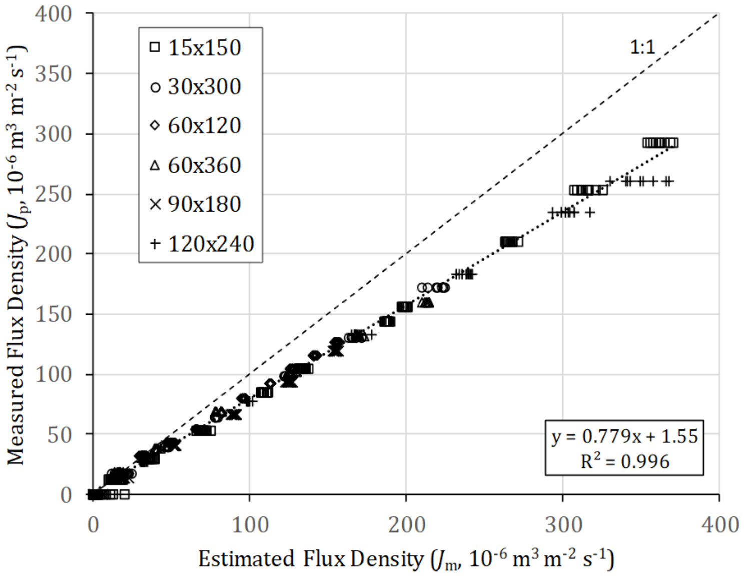

Figure 3. Measured water flux density versus flux density estimated by the model based on the harmonic analysis of the heat wave (phase shift and amplitude gap of the first harmonic), for six combinations of pulse width and time period of heating/cooling cycle in a sawdust column.

The results also demonstrated the generality of the model, which was independent of the combination of heating and cooling cycles in the sawdust column. This point outlines the performance and versatility of the model. However, it is likely that non steady state conditions of flow will impact the model response according the timing of heating and cooling cycles. Short cycles should be more adapted to quickly changing conditions. For application, a short heating time and an extended period between heat pulses can be used to reduce energy consumption.

This new approach also faces classical limitations of the majority of methods of sap flow measurement (Vandegehuchte and Steppe, 2013). The knowledge of zero flux conditions are necessary to apply the model. In vivo, such a determination is generally supported by environmental conditions inducing zero flux, as nighttime, low VPD and closed stomata (Ward et al., 2017).

Finally, one key result from the harmonic analysis, both theoretical and experimental, is that the flow changes both the velocity and the form of the heat wave propagating in the medium. These effects are not taken into consideration together in heat pulse methods and we believe this can contribute to estimation error and particularly underestimation as often quoted (Flo et al., 2019).

CONCLUSIONS

We proposed and tested a new approach of cyclic heat pulse based on Fourier harmonic analysis to determine water density flux in porous medium. The results in a sawdust column with different combination of heating and cooling cycles were successful. They stimulate further studies and field application.

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Literature cited

Burgess, S.S., Adams, M., Turner, N.C., Beverly, C.R., Ong, C.K., Khan, A.A., and Bleby, T.M. (2001). An improved heat pulse method to measure low and reverse rates of sap flow in woody plants. Tree Physiol. *21*, 589–598.

Carslaw, H.S., and Jaeger, J.C. (1959). Conduction of heat in solids (Oxford, UK: Clarendon Press).

Do, F., and Rocheteau, A. (2002). Influence of natural temperature gradients on measurements of xylem sap flow with thermal dissipation probes. 2. Advantages and calibration of a noncontinuous heating system. Tree Physiol. *22* (*9*), 649–654. PubMed

Do, F.C., Isarangkool Na Ayutthaya, S., and Rocheteau, A. (2011). Transient thermal dissipation method for xylem sap flow measurement: implementation with a single probe. Tree Physiol. *31* (*4*), 369–380. PubMed

Flo V., J. Martinez-Vilalta, K. Steppe, Bernard Schuldt R. Poyatos. (2019). A synthesis of bias and uncertainty in sap flow methods. Agric. For. Meteorol. *271*, 362–374.

Granier, A. (1985). Une nouvelle method pour la measure du flux de seve brute dans le tronc des arbres. Ann. For. Sci. *42* (*2*), 193–200 https://doi.org/10.1051/forest:19850204.

Green, S., Clothier, B., and Jardine, B. (2003). Theory and practical application of heat pulse to measure sap flow. Agron. J. *95* (*6*), 1371–1379 https://doi.org/10.2134/agronj2003.1371.

Lubczynski, M.W., Chavarro-Rincon, D., and Roy, J. (2012). Novel, cyclic heat dissipation method for the correction of natural temperature gradients in sap flow measurements. Part 1. Theory and application. Tree Physiol. *32* (*7*), 894–912 https://doi.org/10.1093/treephys/tps030. PubMed

Marshall, D.C. (1958). Measurement of sap flow in conifers by heat transport. Plant Physiol. *33* (*6*), 385–396 https://doi.org/10.1104/pp.33.6.385. PubMed

Nhean, S., Isarangkool Na Ayutthaya, S., Rocheteau, A., and Do, F.C. (2019). Multi-species test and calibration of an improved transient thermal dissipation system of sap flow measurement with a single probe. Tree Physiol. *39* (*6*), 1061–1070 https://doi.org/10.1093/treephys/tpz017. PubMed

Vandegehuchte, M.W., and Steppe, K. (2012). Sapflow+: a four-needle heat-pulse sap flow sensor enabling nonempirical sap flux density and water content measurements. New Phytol. *196* (*1*), 306–317 https://doi.org/10.1111/j.1469-8137.2012.04237.x. PubMed

Vandegehuchte, M.W., and Steppe, K. (2013). Sap-flux density measurement methods: working principles and applicability. Funct. Plant Biol. *40* (*3*), 213–223 https://doi.org/10.1071/FP12233. PubMed

Ward, E.J., Domec, J.C., King, J., Sun, G., McNulty, S., and Noormets, A. (2017). TRACC: an open source software for processing sap flux data for thermal dissipation probes. Trees (Berl.) *31* (*5*), 1737–1742 https://doi.org/10.1007/s00468-017-1556-0.