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Repetitive fishing, local depletion, and fishing efficiencies in the Kerguelen Islands fisheries

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Bez, N., De Oliveira, E., and Duhamel, G. 2006. Repetitive fishing, local depletion, and fishing efficiencies in the Kerguelen Islands fisheries. – ICES Journal of Marine Science, 63: 532–542.

Commercial catch per unit effort (cpue) recorded by observers on board trawlers operating in the French EEZ around the Kerguelen Islands (south Indian Ocean) are used to analyse the impact of repetitive fishing on cpues and to estimate local fishing efficiencies. Series of cpues from the same fishing locations and no more than two days apart are first built, and then the mean depletion effect of repetitive fishing (between 4% and 10% for Patagonian toothfish and around 20% for icefish and marbled notothen) is quantified. Random fluctuations of cpues around an exponential decrease (i.e. the reference model when fishing efficiencies are constant) lead to modelling fishing efficiency as a random variable. Based on the observations of the ratio between two consecutive catches, a method is developed to estimate the probability density function of fishing efficiencies. The significant decrease of average fishing efficiency between the start of the fishery and its full exploitation phase is explained by a change in regulations (from global to individual quotas).

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Keywords: biomass depletion, catchability, commercial cpue, random local fishing efficiency, replicates.

Received 24 May 2004; accepted 11 October 2005.

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Introduction

Commercial catch per unit effort (cpue) is widely used as an index of abundance although many processes are known potentially to break down proportionality between cpue and biomass (Hilborn and Walters, 1987; Fréon and Misund, 1999; Rahikainen and Kuikka, 2002). Among such processes, local fish depletion caused by repetitive fishing (a series of tows at the same location) modifies the local catchability coefficient, i.e. the proportionality between the mean cpue over the series and local fish density. From a management perspective, this is of concern if the proportion of repetitive fishing operations is sufficiently large as to influence the overall cpue statistics. Repetitive fishing is reported in some areas and fisheries, and has led to quantitative analvses on, for instance, rates of depletion (Rijnsdorp et al., 2000). Local fishing efficiency is usually treated as a constant parameter similar to a catchability coefficient, with separate values for each vessel or métier during a given period of time (trip or season). The result of assuming constant fishing efficiencies is exponential local depletion.

Theoretically, fishing efficiencies should be estimated on the basis of cpues realized for the same underlying fish abundance. However, this is never possible in practice, and several methods have been developed to overcome the problem. Multiple regression approaches derived from the work of Robson (1966) account for changes in fish density from quadrat to quadrat (Gavaris, 1980; Richards and Schnute, 1992). Laurec (1977) suggested a geostatistical approach to estimate local fishing power (local fishing efficiency times the capacity to find fish) that accounts for the natural variability between cpues realized some distance apart. Working on fishing overcapacity and fishing efficiencies, such recent approaches as the Data Envelopment Analysis or Stochastic Production Frontiers (Pascoe et al., 2004) are also based on confronting, through regression techniques, the outputs of several production units operating in various input conditions.

We here consider the trawl fishery off the Kerguelen Islands (Figure 1) targeting Patagonian toothfish (*Dissostychus eleginoides*). In the fishery, repetitive fishing operations were common until managers limited the possibility to



Figure 1. The study area, showing the fishing grounds in the EEZ of the Kerguelen Islands. Plus signs represent all tows performed during the two fishing periods (1986/1987 and 1995–1997). Fishing areas are delineated and numbered.

repeat tows (because repetition was *a priori* considered dangerous for the resource). First, we test series of repetitive true cpues provided by observers on board commercial vessels against the exponentially decreasing reference model. Fluctuations around the reference model are interpreted as random fluctuations in fishing efficiency. Ratios of successive cpues are then used to infer the probability distribution of fishing efficiency by fishing period and/or by vessel.

Material

Despite the difficulties in exploiting it, the trawl fishery off the Kerguelen Islands increased until the late 1990s. In the early 1990s, the main targeted species were Patagonian toothfish, a demersal deep-sea predator (Duhamel, 1981, 1992). Its spatial distribution is Subantarctic. It migrates long distances (>2000 km) during its life cycle (Williams *et al.*, 2002), but it is a territorial fish so we assume that it does not move during a short time period (one or two days).

Observers on board each vessel collect accurate and precise information on tow location (precise to one second of longitude), tow duration, and catch. Cpues are then precisely geo-referenced and expressed in hundreds of kg per hour. Tows shorter than 15 min were removed from the data set (short tow duration is usually indicative of a problem in the behaviour of the trawl).

We selected two contrasting periods of the Patagonian toothfish trawl fishery for the study: the start of exploitation (1986/1987), under a global quota, and what is now considered full exploitation (1995-1997), under individual quotas. The fishery experienced massive IUU (illegal, unregulated, and unreported) fishing activity, but only licensed vessels are considered in this study. During the first period, 12 vessels were operating (mainly in fishing area 1; Figure 1). Three species were caught: Patagonian toothfish, icefish (Champsocephalus gunnari), and marbled notothen (Notothenia rossii). Being marginal to this study, marbled notothen and icefish are treated together. During the second period, only two vessels were fishing, switching between fishing zones 2 and 3 (Figure 1). The marbled notothen fishery was closed and the icefish stock had collapsed, so only Patagonian toothfish were caught then.

Spatial distributions of each species are largely distinct and do not overlap. Each species is caught in a specific area (Figure 2), so we consider each fishery to be monospecific. The fishery is not organized by trips, but we defined some trips on the basis of periods of continuous fishing in a given area. For Patagonian toothfish, the proportion of tows located on an already fished point during a trip (same coordinates ± 0.5 mile) ranges from 0% to 80% and averages 42% for the first period and 58% for the second. This quantifies the importance of repetitive fishing in this particular fishery.

Methods

For each vessel, a series of replicated tows is defined so that (i) each series comprises at least three non-zero cpues, (ii) the maximum distance between all cpues of a series is one nautical mile, and (iii) the maximum time-lag between two consecutive cpues is two days. We denote $U_{i,j}$ as the j(th) cpue of the i(th) series, with $j \in [1,L_i]$, where L_i is the length of the i(th) series ($L_i \ge 3$). Any single series corresponds to a vessel (1–12), a period (1 or 2), and a target species (Patagonian toothfish or icefish and marbled notothen).

Cpue series were pooled across vessels by period and target species. To prevent series with large cpues driving the overall average, all series were standardized by their average:

$$m_i = \frac{1}{L_i} \sum_{j=1}^{L_i} U_{i,j}$$

$$U_{i,j}^r = \frac{O_{i,j}}{m_i}.$$

For each group, average profiles of the standardized series were then calculated as follows:



Figure 2. Average cpue (in 10^2 kg h⁻¹) by fishing area and species (Patagonian toothfish, icefish, and marbled notothen). Fishing zone 6 includes cpues outside the five defined zones (see Figure 1).

$$\overline{U}_{j}^{r} = \frac{\sum_{i=1}^{N_{j}} U_{i,j}^{r}}{N_{i}},$$

where N_j is the number of available observations at rank j, i.e. the number of series with at least j observations, where j > 3 and $N_1 = N_2 = N_3 > N_{j_{>3}}$. Local depletion rates were estimated by linear regression of the output profiles.

Estimating local fishing efficiency

In the simple case where fishing efficiency is constant, depletion would be anticipated to be exponential. Departure from this simple model led us to consider fishing efficiency as a random variable Φ , changing from one tow to the next.

We assumed that the swept area was constant within series and that there was no migration of fish in or out of the swept area during a series. Under such assumptions, if we denote z_i as the fish density at the beginning of each series and $\Phi_{i,j}$ as the fishing efficiency of each tow, the first two cpues of a series are

$$U_{i,1} = \Phi_{i,1} z_i$$

$$U_{i,2} = \Phi_{i,2}(z_i - \Phi_{i,1}z_i) = \Phi_{i,2}(1 - \Phi_{i,1})z_i$$

so after *j* repetitions,

$$U_{i,j} = \Phi_{i,j} \prod_{k=1}^{j-1} (1 - \Phi_{i,k}) z_k$$

The ratio $R_{j+1,j}$ between two successive cpues of a series is then

$$R_{j+1,j} = \frac{U_{i,j+1}}{U_{i,j}} = \Phi_{i,j+1} \frac{1 - \Phi_{i,j}}{\Phi_{i,j}} = \Phi_{i,j+1} \Psi_{i,j}, \text{ with } \\ \Psi_{i,j} = \frac{1 - \Phi_{i,j}}{\Phi_{i,j}}.$$

In practice, we do not observe realizations of Φ , but rather realizations of a random variable *R*, a function of Φ . The objective is therefore to determine the characteristics of Φ from those of *R*.

We chose a type I Beta law for the random fishing efficiency variable Φ . There are several reasons for this choice: (i) this probability law is defined by just two parameters, *n* and *p*; (ii) it is defined on the interval [0,1]; (iii) it accommodates a very large variety of possible shapes from left skewed to right skewed. By definition, $1 - \Phi$ follows a type I Beta law with parameters *p* and *n*, and one can demonstrate (see Appendix) that *f*(*r*), the probability density function (pdf) of *R*, is

$$f(r) = r^{p-1} \left(\frac{\Gamma(n+p)}{\Gamma(n)\Gamma(p)}\right)^2 \int_0^1 \frac{t^{2n-1}(1-t)^{p-1}}{(t+r)^{n+p}} \mathrm{d}t,$$

where $\Gamma(n)$ represents the Gamma function. As neither f(r) nor its log-likelihood is linear with regards to parameters n and p, parameter estimation is iteratively based on the minimization of the sum of squared residuals.

Model fitting was only carried out for Patagonian toothfish cpue (standardized or not, because this is equivalent when taking the ratio) either by fishing periods for all vessels pooled or for individual vessels.

Results

During time period 1, 48 series were identified for Patagonian toothfish and 33 for icefish plus marbled notothen. In time period 2, 98 series were identified for Patagonian toothfish, with 13 observations for the longest series. Of these 179 series, not all presented a decreasing profile (Figure 3). Some did not show any trend, and some even increased. However, standardized series exhibited, on average, a mean depletion effect over repetitive trawls (Figure 4). Average rates of depletion for Patagonian toothfish were 7% and 4% for the first and second fishing periods, respectively, and 18% when icefish or marbled notothen were targeted (which happened only during the first fishing period). Strong fluctuations exist around the trends, especially for toothfish. In particular, for the first period, the fourth point of the mean profile departed clearly from the overall trend, even though it was based on 20 series (Figure 4a). The average rate of depletion without this point is 10%, rather than the 7% calculated with it.

Toothfish cpues are used to estimate distribution laws of fishing efficiency for each of the two time periods. Trawl durations were remarkably stable over repetitions, not showing any trend (Figure 5). Hence, we assume that swept area was constant within series. Empirical histograms of the ratio of two consecutive cpues $R_{j+1,j}$ differed between the



Figure 3. Three examples of standardized series of co-located cpues.

two time periods, inducing differences in the estimated fishing efficiency pdf (Figure 6). At the start of the fisheries, fishing efficiencies ranged from 0 to 0.9, with an average of 0.28. After 10 years of exploitation, fishing efficiencies were smaller and more similar, ranging from 0 to 0.4, with an average of 0.11.

The same procedure was applied to individual vessels with the objective of distinguishing between highly efficient and poorly efficient boats. This was possible for vessels 10 (first and second periods), 11 (first period), 1 (first period), and 5 (second period). Vessel 10 appeared to be representative of the mean behaviour of the fisheries, because its fishing efficiency resembled the average efficiency described above estimated using data for all fishing vessels (Figure 7). Vessels 11, 1, and 5 had the same mode for fishing efficiency distribution (i.e. same maximum pdf), but with different dispersal and therefore different average fishing efficiencies (Figure 8).

Discussion

In this study, as in some recent publications (e.g. Trenkel and Skaug, 2005), fishing efficiency is interpreted as a random variable. The rationale behind this choice is that turning an encounter between a fish and a gear into a capture depends on many factors that are mainly uncontrolled or unknown. Consequently, fishing efficiency is expected to vary randomly. The method developed in this study makes it possible to estimate the pdf of the fishing efficiency rather than to estimate the fishing efficiency for a single tow. Based on methods developed by Carle and Strub (1978), we could have used the series of cpues to estimate the fish density at the beginning of each series. However, this would have required that individual series consistently showed a depletion effect, which was not the case.

Sensitivity of estimation of parameters n and p

For time periods 1 and 2, we looked at both the log-likelihood and the sum of squares functions for all possible sets of (n,p) values (log-likelihood being the natural equivalent of the sum of squares). Most of the time, the functions are flat in the areas of interest, i.e. the areas of, respectively, maximum and low values (Figure 9). For each of the two algorithms considered (log-likelihood and sum of squares), we selected the areas corresponding to the 20% most favourable cases (i.e. 0.8 quantile for the log-likelihood, 0.2 quantile for the sum of squares), mapped it in an $n \times p$ coordinate system, and superimposed onto it the iso-mean and iso-variance line of a type I Beta distribution (Figure 10). Several important considerations can be derived from comparing the results for time periods 1 and 2.

First, the conditions, in terms of (n,p) values, for maximum likelihood and minimum sum of square estimation are very similar. Nevertheless, the areas of low sum of



Figure 4. Mean standardized cpue series at the same rank for Patagonian toothfish during (a) time period 1, (b) time period 2, and (c) for icefish and marbled notothen during time period 1. The numbers of cpues available at each rank for the average are indicated. The linear regression line is also shown in each panel.



Figure 5. Mean relative tow duration as a function of the rank in the series. The numbers of observations available for each rank are represented by symbol size (max = 98, min = 1).

squares shift slightly towards lower n values for time period 1 and towards larger p values for time period 2. These areas of favourable conditions are large, especially in terms of p values. This indicates that the precision of the estimations is low and that one should not put too much confidence in the point estimates of parameters n and p. This also indicates that, relative to the intrinsic uncertainty of the estimates, both algorithms for estimating n and p parameters provided similar results (but not exactly the same point estimates).

Second, the consequence of the low precision of the estimates in terms of mean fishing efficiency (and variance) differs for time periods 1 and 2. For time period 1, the area of large log-likelihood (respectively, low sum of squares) corresponds to an area where small changes in *n* and *p* values induce large differences in terms of mean fishing efficiency (and variance). There is great uncertainty around the fishing efficiency pdf for time period 1 where the mean fishing efficiency is likely to range between 0.1 and 0.5 (with a variance between 0.005 and 0.04). The precision was stronger for time period 2, where the mean fishing efficiency likely ranged between 0.1 and 0.2 (with a variance between <0.005 and 0.01).



Figure 6. Ratio between two consecutive and co-located cpues of Patagonian toothfish. Empirical histogram and fitted model. Values for parameters n and p are indicated. (a) Time period 1, (b) time period 2, (c) estimated fishing efficiency probability density function (pdf) for time periods 1 (continuous line) and 2 (dashed line).

Finally, the difference between time periods 1 and 2 is larger than the uncertainties around individual estimates, irrespective of the algorithm selected. This indicates that the differences between the distributions of fishing efficiencies for the two time periods are sufficiently strong to be indicative of real change. This change will be further interpreted in terms of fishing practices and regulations.

Sensitivity to the spatio-temporal window used to define a series

When increasing the spatio-temporal window used to build series, we noted that the depletion effect was less and less clear. For instance, when including tows more than two days apart and/or 5 miles apart, instead of the 2 days \times 1 mile window used in this study, the mean depletion effect was no longer visible. Together with the fact that not all series showed any depletion effect, and that some even showed opposite tendencies, this is an indication that the depletion effect was weak. To the best of our knowledge, however, this is one of the few cases where this concept has evidence in real field data. Rijnsdorp *et al.* (2000), working on the results of beam trawlers in the North Sea, found a 10% decrease over 48-h periods for flatfish fisheries. In other studies, this phenomenon is postulated *a priori* (e.g. Maury and Gascuel, 2001).

The rate of depletion was smaller for Patagonian toothfish (by 5-10%) than for the group containing icefish and marbled notothen (around 20%), indicating lesser vulnerability to repetitive fishing for Patagonian toothfish. This is consistent with the history of the fishery in this area, because icefish were swiftly overexploited while Patagonian toothfish proved more resistant to exploitation (Anon., 1997). Icefish being more spatially aggregated than toothfish, they are more sensitive to local overfishing.

If data had been aggregated to statistical square, this phenomenon would have been missed. Relaxing the spatiotemporal window amounts to the inclusion of short time scale spatio-temporal variability which is, in the present study, large compared with the strength of the signal and therefore would hide it. This reinforces the idea that Patagonian toothfish exhibit a territorial behaviour that is



Figure 7. Ratio between two consecutive and co-located cpues of Patagonian toothfish for vessel 10. Empirical histogram and fitted model. Values for parameters n and p are indicated. (a) Time period 1, (b) time period 2, (c) estimated fishing efficiency probability density function (pdf) for time periods 1 (continuous line) and 2 (dashed line).

responsible for great small-scale heterogeneous spatial distribution; it also reinforces belief in the usefulness of precise and fine observations on fisheries activities (e.g. observer data, VMS).

Basic assumptions and simplifications

We assumed that between two successive catches of a particular series, there was no emigration or immigration of fish from or into the fishing area, i.e. Patagonian toothfish do not move during a period of two days. Therefore, all fluctuations observed around the exponential reference model were interpreted as being a consequence of the variability of fishing efficiency. This simplistic assumption made it possible to proceed, but it meant that an increase in cpue from one particular tow to the next was interpreted as an increase in fishing efficiency rather than an immigration of newcomers into an area. Obviously, part of the observed variability can be attributed to fish movement. However, no direct field experiment was available to help quantify this effect. While in the present study all variability is attributed to fishing efficiency, other studies (e.g. Trenkel and Skaug, 2005) do postulate a random behaviour of the fishing efficiency to distinguish between the effects of efficiency and biomass on catch fluctuations.

Fishing efficiency, fishing regulation, and fishing tactics

On average, fishing efficiency halved between 1986/1987 and 1995–1997. Improvements in fishers' knowledge of Patagonian toothfish and in fishing technology (e.g. GPS) likely induced an increase in fishing efficiency. On the other hand, decrease of Patagonian toothfish biomass through exploitation (Anon., 1997) was expected to reduce fishing efficiency. Many studies show that catchability of a stock is



Figure 8. Ratio between two consecutive and co-located cpues of Patagonian toothfish for (a) vessel 11 in time period 1, (b) vessel 1 in time period 1, and (c) vessel 5 in time period 2. Empirical histogram and fitted model. Values for model parameters n and p are indicated. (d) Estimated fishing efficiency probability density function (pdf) for vessel 11 (continuous line), vessel 1 (dashed line), and vessel 5 (dotted line).



Figure 9. Log-likelihood (a) and sum of squares (b) functions for all possible set of (n,p) values for time period 1.



Figure 10. Analysis of the sensitivity of the results. Comparison of a log-likelihood algorithm (left panels) and a sum of squares approach (right panels) for time periods 1 and 2. Areas corresponding to the 20% of most favourable cases (i.e. 0.8 quantile for log-likelihood and 0.2 quantile for sum of squares) are represented by the iso-mean and iso-variance lines of a type I Beta distribution (superimposed).

negatively correlated with its abundance (MacCall, 1976; Pitcher, 1995; Mackinson *et al.*, 1997).

Fishing rules are also responsible for the evolution of fishing efficiencies. During the first fishing period, the fleet was regulated by a global quota (a total allowable catch, TAC). Fishers then sought the largest catch in order to gain the maximum possible proportion of the TAC. During the second fishing period, individual quotas were implemented so that each fisher was attributed a capital of allowed catches. This change in fishing regulation induced a more even exploitation of the fishing grounds: fishers took less risk and favoured small (i.e. less efficient) but repetitive tows in order to provide the crew with more regular catches, while maintaining the same total catch. Vessel 10, which operated in both periods, is an instructive example in this regard. Its technical characteristics and crew did not change between the two periods. However, its fishing efficiency became smaller and more homogeneous.

The consequences of a reduction of fishing efficiency on mean cpue can be sketched easily. If the change in fishing tactics induced by a shift in fishing regulations consists of spreading the same total catches over several tows, and if the fishing effort needed to get the same quantity of fish is smaller when doing one tow instead of several, then the mean cpue would decrease even if biomass does not change. Unfortunately, this hypothesis cannot be tested as the fishery switched to longlining in the late 1990s, putting an end to the series of trawl cpue.

This study also made it possible to revisit the criteria used to select a reference fishery or a reference vessel when standardizing cpues. When data permit, i.e. when pdf of fishing efficiencies can be computed reliably for different métiers or vessels, the reference vessels could be selected among those with more stable fishing efficiencies, regardless of any technological improvements that may have taken place in the meantime.

Acknowledgements

We emphasize the key role and the importance of the work done by observers, on whom the quality of the data used in this study is completely dependent, despite difficult and sometimes extreme weather conditions. We therefore understand their importance and express our gratitude to all of them. We also acknowledge the Centre de Géostatistique, Fontainebleau, France, for whom the first two authors of this paper worked when this study first took place.

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Appendix

We assume that fishing efficiencies φ , viewed as outcomes of a random variable Φ , follow a type I Beta distribution with parameters *n* and *p*:

$$b_1(\varphi) = \frac{\Gamma(n+p)}{\Gamma(n)\Gamma(p)} \varphi^{n-1} (1-\varphi)^{p-1} \quad n,p > 0 \text{ and } \varphi \in [0,1],$$

where $\Gamma(n)$ is the Gamma function with parameter *n*:

$$\Gamma(n) = \int_0^\infty \mathrm{e}^{-t} t^{n-1} \mathrm{d}t = (n-1)!$$

By definition, $1 - \Phi$ also follows a type I Beta distribution but with parameters *p* and *n*, and $\Psi = (1 - \Phi)/\Phi$ follows a type II Beta distribution, noted *b*₂, with parameters *p* and *n* (Saporta, 1990, p. 41):

$$b_2(\psi) = \frac{\Gamma(n+p)}{\Gamma(n)\Gamma(p)} \frac{\psi^{p-1}}{(1+\psi)^{n+p}} \quad n, p > 0 \text{ and } \psi \in [0, \infty[.$$

Assuming that fishing efficiency is independent between tows amounts to considering that the two consecutive ratios $\Phi_{i,j}$ and $\Phi_{i,j+1}$ of the *i*(th) series are independent and that Φ and Ψ are independent. The expected value of the random variable $R = \Phi \Psi$ is then

$$D_{\Phi} = [0, 1] \text{ and } D_{\Psi} = [0, \infty].$$

If we note $\varphi = t$ and $\psi = r/t$, then

$$E(R) = \int_{D_R} \int_{D_T} rb_1(t) b_2\left(\frac{r}{t}\right) \left|\frac{D(x,y)}{D(r,t)}\right| dr dt, \text{ with}$$
$$D_R = [0, \infty [\text{ and } D_T = [0,1]$$

$$E(R) = \int_{D_R} r \mathrm{d}r \int_{D_T} b_1(t) b_2\left(\frac{r}{t}\right) \frac{\mathrm{d}t}{t}.$$

By definition,

$$E(R) = \int_{D_R} rf(r) \mathrm{d}r.$$

The probability density function f(r) of R is then

$$f(r) = \int_0^1 b_1(t) b_2\left(\frac{r}{t}\right) \frac{dt}{t}$$

= $r^{p-1} \left(\frac{\Gamma(n+p)}{\Gamma(n)\Gamma(p)}\right)^2 \int_0^1 \frac{t^{2n-1}(1-t)^{p-1}}{(t+r)^{n+p}} dt.$