

# Estimating the median age at menarche with a Logit model: Application to African DHS surveys

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## Abstract

**Background:** The study investigates the median age at menarche in African countries.

**Data Source and Methods:** DHS surveys provided information on the proportion ever-menstruating among women age 15-19. A linear-logistic regression model was used to derive the median age at menarche (statu quo method). Some 139 DHS surveys were analysed, for a total of 304,826 women.

**Results:** Results show a wide range of variations of the median age at menarche among the 35 countries investigated, from 13.1 to 15.2 years.

**Conclusion:** Country variations were correlated with various variables indicating the level of development (Income per capita; Caloric intake; Urbanization; Child mortality; Fertility; Age at marriage). Two variables were particularly important: food intake and urbanization.

**Keywords:** Age at menarche; Logit model; Socio-economic correlates; DHS surveys; Sub-Saharan Africa.

## Introduction

Menarche is defined as the first menstruation, a dramatic event for adolescent girls. The age at menarche is therefore easy to capture with retrospective questions in demographic surveys, assuming that age is known with accuracy. The age at menarche varies considerably among individuals, from very early onset at age 8 to 9 years to very late onset at age 22 to 23 years. Menarche seems to be the result of complex hormonal changes, which vary greatly among individuals. Many factors of age at menarche have been identified in numerous studies, in particular genetic and hereditary factors (from mother to daughter), nutritional status (height, weight), physical exercise (intense activity, sport), and socio-economic factors (socio-economic status, level of education) [Gray 1983; Tanner 1981; Zacharias & Wurtman 1969].

At national level, the average age at menarche is an indicator of the health status of a population: overall, the healthier the population (as measured by life expectancy for instance), the earlier the average age at menarche. Dramatic changes in the average age at menarche were observed during the health transition

all over the world. In Europe, in the early 19<sup>th</sup> century, when life expectancy ranged from 35 to 45 years, the average age at menarche ranged from 15 to 16 years, whereas at the end of the 20<sup>th</sup> century, when life expectancy ranged from 75 to 85 years the average age at menarche ranged from 12 to 13 years [De Muinck Keizer-Schrama 2001; Wyshak & Frish 1982]. In France, the mean age at menarche was estimated at 15.7 years in the first quarter of the 19<sup>th</sup> century and at 13.1 years in the last quarter of the 20<sup>th</sup> century [Brierre de Boismont 1842; de La Rochebrochard 2000; Ducros & Pasquet 1978; Gaudineau et al. 2010]. Over the world in recent years, the age at menarche was found to be correlated with parameters of the demographic transition (fertility and mortality), and with parameters of economic development (income, urbanization, education, energy intake) [Šaffa et al. 2019].

Age at menarche remains poorly studied in sub-Saharan Africa. Most of the available data come from local studies, based on ad-hoc samples, such as schools, clinics, or demographic surveillance systems

(DSS). Many of these studies are based on small samples, with large confidence intervals, and more important on highly selected samples of healthy girls (elite schools), or privileged urban groups, but rarely on data from deprived rural communities [Pasquet et al. 1999; Rehan 1994; Thomas et al. 2001].

Some of the World Fertility Surveys (WFS) conducted between 1975 and 1984 included a retrospective question on age at menarche. They were recently re-analysed and found to provide consistent results in terms of levels and trends of age at menarche [Garenne 2020a]. Since then, the Demography and Health Surveys (DHS) were conducted in many African countries, but they did not include similar information on age at menarche. However, they included a question on whether the woman had ever-menstruated at time of survey for all women age 15 and above. This information will be used to estimate the median at menarche.

African populations might differ from European populations with respect to age at menarche for a variety of reasons. Firstly, they have specific genetic characteristics, which have a variety of implications for demographic parameters such as the sex-ratio at birth [Garenne 2002; 2004; 2017], anthropometric characteristics [Garenne 2011], and potentially for age at menarche. Secondly, they may differ in numerous correlates of the age at menarche, such as level of economic development, income, wealth, urbanization, food intake, physical exercise, and advances in the health transition. Thirdly, age at menarche has direct implications for early marriage and adolescent fertility, because, in Africa, girl's first marriage often follows shortly the first menstruation [Diop 1994, Gurmu and Etana 2014].

The aim of this study was to develop a mathematical model for estimating the median age at menarche from incomplete data, that is from current menstruating status of women age 15-19. The study is organized in two steps: in a first step the Logit model is tested on historical data, and in the second step it is applied to African DHS surveys.

## Data and methods

### The logit model

The Logit model was used to fit the cumulated distribution of age at menarche because, empirically, it provided the best fit. Other functions were also tried such as Beta distributions, Weibull distributions, and the Pricrate model [Garenne et al. 2011; Matthews & al. 2009], and, but they did not fit as well as the Logit model. The Beta distributions are defined on an age-segment  $[\alpha, \beta]$ , the Pricrate model on an open-ended segment  $[\alpha, +\infty[$ , but despite their nice properties the Logit model based on all real values  $]-\infty, +\infty[$ , performed better in this case.

The Logistic function (also called 'sigmoid' function, or the Logit function) was introduced in demography by Pierre-François Verhulst to describe the dynamics of populations towards a maximum size [Verhulst 1845]. The Logistic function was used successfully in USA for fitting the cumulative distribution of age at menarche [MacMahon 1973]. This function has a number of nice properties (see the Appendix for details). The cumulative distribution ranges from 0 (never menstruated) to 1 (all women already menstruated). Its derivative is symmetrical, so that mean = median = mode of the distribution of age at puberty. The associated progression rate,  $dP(x)/(1-P(x))$ , is an increasing function, rising from 0 to a maximum value. So that the process appears to be controlled by this function, which matches well what one could expect from the menstruation process: an hormonal stimulus starting from very low values at an early age (around age 8 years), increasing over the years, soon reaching a maximum (noted  $r_{max}$ ), and inducing the first menstruation within a few years. As a consequence, the stronger the stimulus, the earlier the menarche. The Logit function is also the basis for the Linear-Logistic regression model, which permits multivariate analysis. For this application, the Linear-Logistic model allowed calculating the median age at menarche simply from the slope and intercept of the relationship between the proportion who ever-menstruated and age. In particular, the model can be applied to incomplete datasets, as long as there are enough points for fitting a linear relationship (e.g. age 15-19 in DHS surveys).

### Data

The Logit model was first tested on historical data from France, both on high and low values of the median age at menarche. The first data set was that gathered by Alexandre Brierre de Boimont in the first part of the 19<sup>th</sup> century on a sample of 1200 French women of various socio-economic statuses [Brierre de Boimont 1840]. This set covers a wide range of age at menarche, from 8 to 23 years, with a mean of 15.71 years and a standard deviation of 2.77 years. The second data set was gathered in a survey by INSERM in the last decade of the 20<sup>th</sup> century (*Enquête ACJS*) and studied in a life table framework, leading to a mean of 13.13 years and a standard deviation of 1.29 years [de La Rochebrochard 1999].

The model was then tested on the 1999 Nigeria DHS survey, the only African DHS survey providing complete information on menstruation for girls age 10 years and above. Once the model was validated, it was applied to other African DHS surveys. In these surveys, the current menstruation status was known only for women 15 years and above, and only the 15-19 age group was used in this study for fitting the

logit model, since very few women have their first menstruation above age 20 nowadays, and more importantly because the Logit relationship with age tends to be no longer linear above age 20. A total of 139 DHS surveys were utilized, covering 35 African countries (see list in Table 1).

### Method

The model used for estimating the median age at menarche is straightforward. The current status: (0) if never-menstruated; (1) if ever-menstruated), defines the proportion of women age(x) who ever-menstruated, noted  $P(x)$ , and is matched with age (x). In DHS notations, (0) is defined by  $V215 = 996$ ; (1) by  $V215 = 100$  to  $995$ ; other values  $997-999$  are excluded. Age is expressed in decimal values from 15.0 to 19.99, calculated from age in months and years, using the CMC system (century-month-code). In DHS notations, decimal age is therefore calculated as:  $(V008-V011)/12$ . Note that this provides estimates based on exact age (and not on age at last birthday as in some other studies). The relationship between  $\text{Logit}(P(x))$  and (x) was found linear in most cases, otherwise it was discarded. The model is fully described in the Appendix. When several DHS surveys were available in the same country, the estimates of median age at menarche were standardized for cohort 1990, that is for women born in 1990 who had their first menstruation between year 2000 and year 2009. The relationship was fit by a linear logistic regression model using the statistical software SPSS-17.

### Other data sources

Correlates of the median age at menarche at country level were investigated using various sources, and all standardized for the same cohort (1990) or for the same period (2000-2009). Cohort data were standardized using linear regression on cohort using DHS surveys. Period data were standardized by taking the average values for years 2000 to 2009. Income per capita, expressed at GDP-PPP in constant USD was derived from the OECD database [Maddison 1990]. Caloric intake, expressed in Kcal per person per day was taken from the FAO-stat database [FAO, 2015]. Proportion urban, child mortality, and total fertility were derived from the World Population Prospect database [UNPD 2019]. Age at first marriage and average years of schooling were derived from DHS surveys.

### Results

#### Testing the logit model on historical data from France

Figure 1 displays two contrasted situations, of high and low median age at menarche in France. In both cases, the fit was good on the proportion of women who ever-menstruated by age (x), and the small differences between observed and fitted values could be explained by random fluctuations associated with small sample size in certain age groups.

Figure 1

Fitting age at menarche with a Logit model, France 1840 & 1994

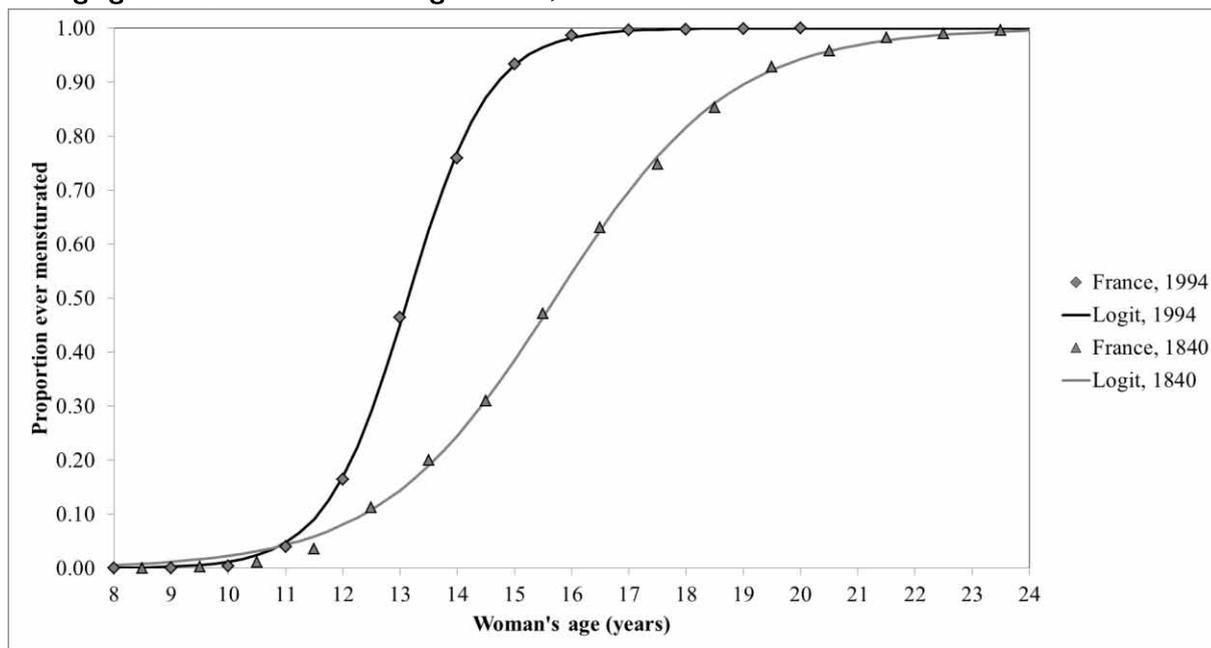
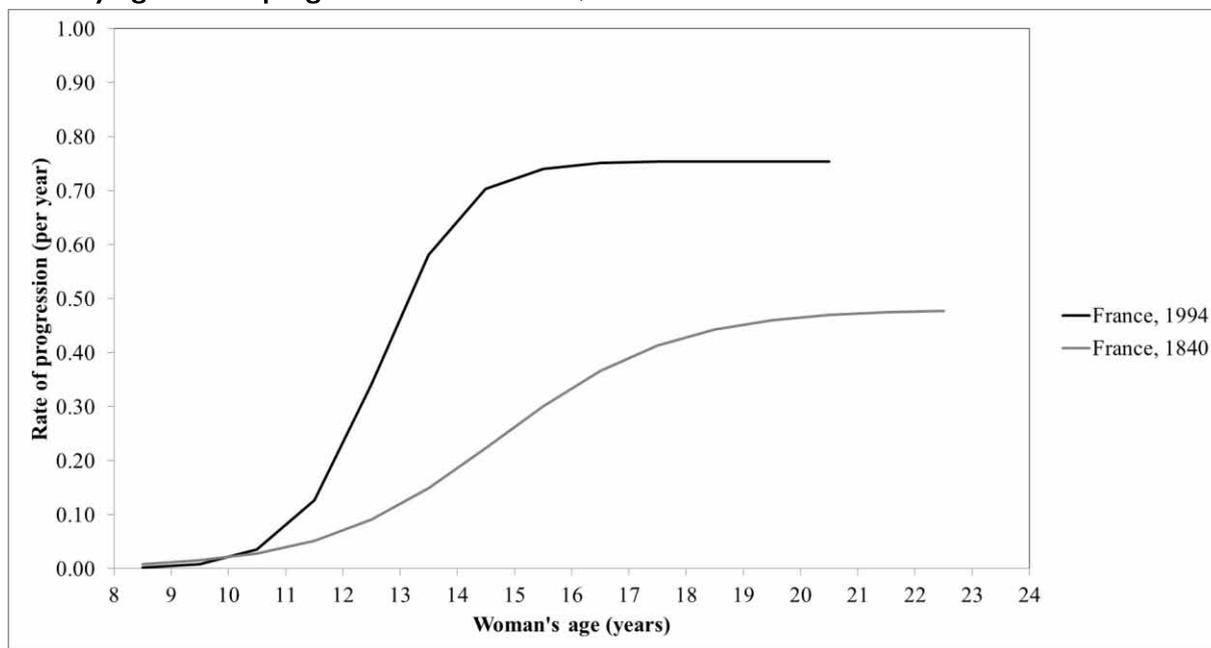


Figure 2 displays the underlying force of progression to menarche in both situations. It starts from values close to zero at age 8, then increases slowly in the first case (high median age) and rapidly in the second

case (low median age), to reach a maximum, lower in the first case ( $r_{max} = 0.480$ ) than in the second case ( $r_{max} = 0.755$ ).

Figure 2

Underlying force of progression to menarche, France 1840 & 1994



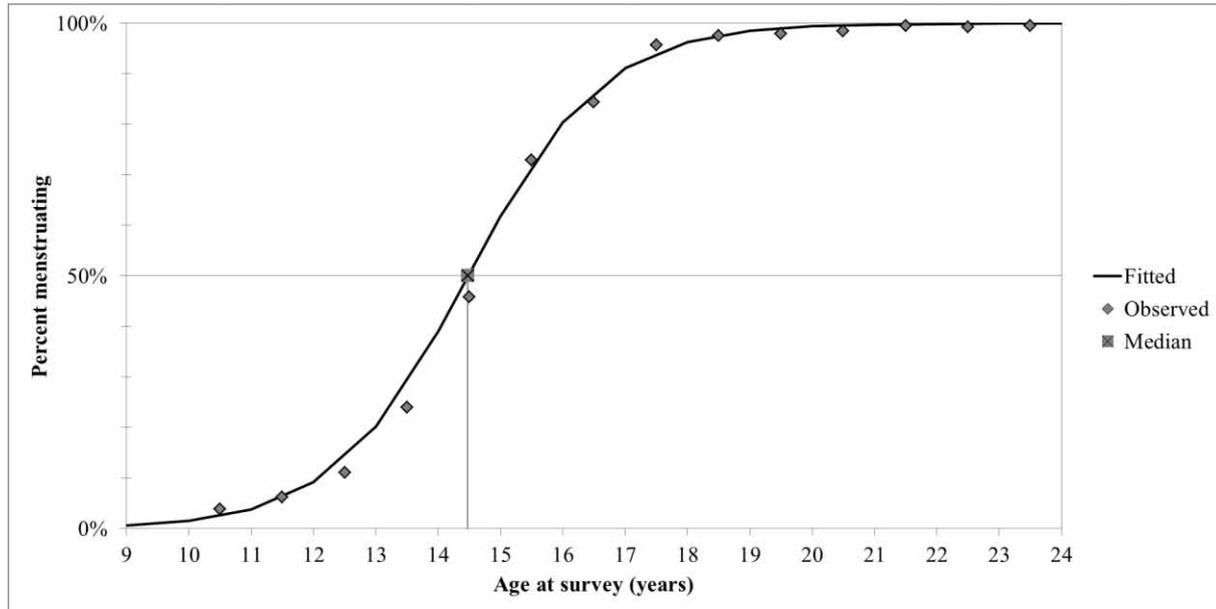
Testing the Logit model on complete African data: Nigeria 1999

A similar exercise was conducted on data from the 1999 Nigeria DHS survey, the only African survey providing information for all women age 10 to 20 years. Here again, the goodness of fit was

appropriate, given the sample size and the probable inaccuracy in age reporting (Figure 3). The median age at menarche was 14.48 and the standard deviation was 1.96. Note that the age at menarche has been undergoing a dramatic decline in Nigeria since the 1950's [Garenne 2020b].

Figure 3

Fitting with a Logit model, women age 10-24, Nigeria, 1999 DHS survey (complete data)



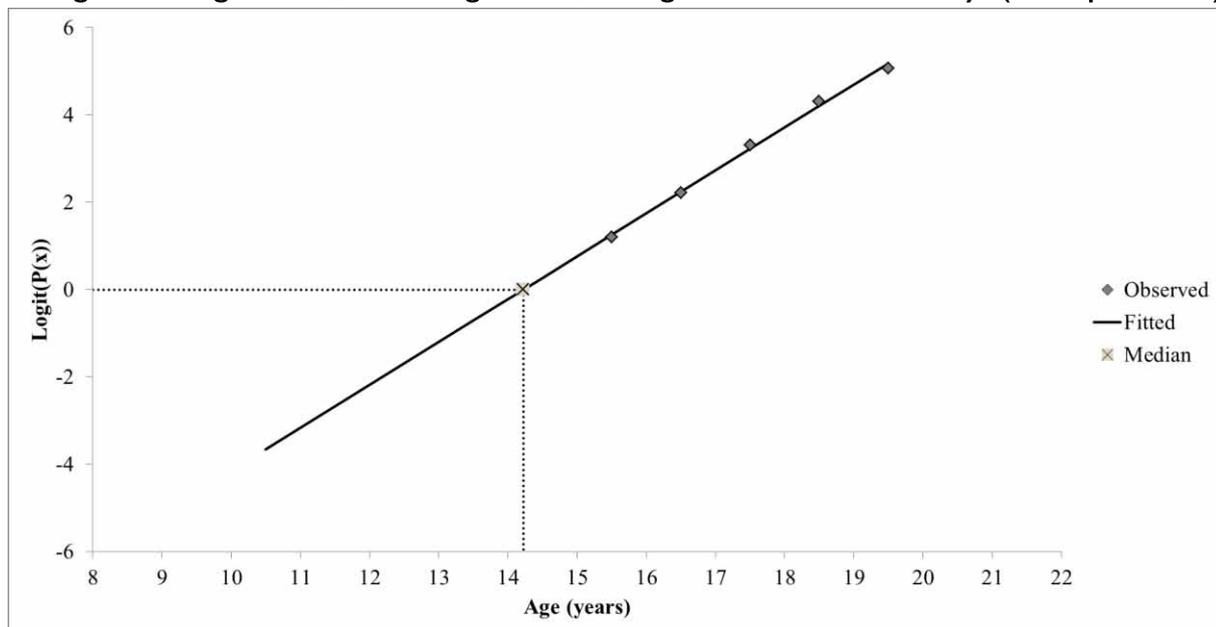
**Average African pattern**

The model was then applied to all DHS surveys conducted in sub-Saharan Africa with relevant information on menstruation status for women age 15-19, an age group where the relationship between  $\text{Logit}(P(x))$  and age ( $x$ ) was linear. Only a few surveys

were discarded, mostly because of erratic data or small sample size (Angola, Comoro islands). All the relevant DHS surveys were merged together to create an 'average African pattern', which is displayed in Figure 4.

Figure 4

Fitting with a Logit model, women age 15-19, average of African DHS surveys (incomplete data)

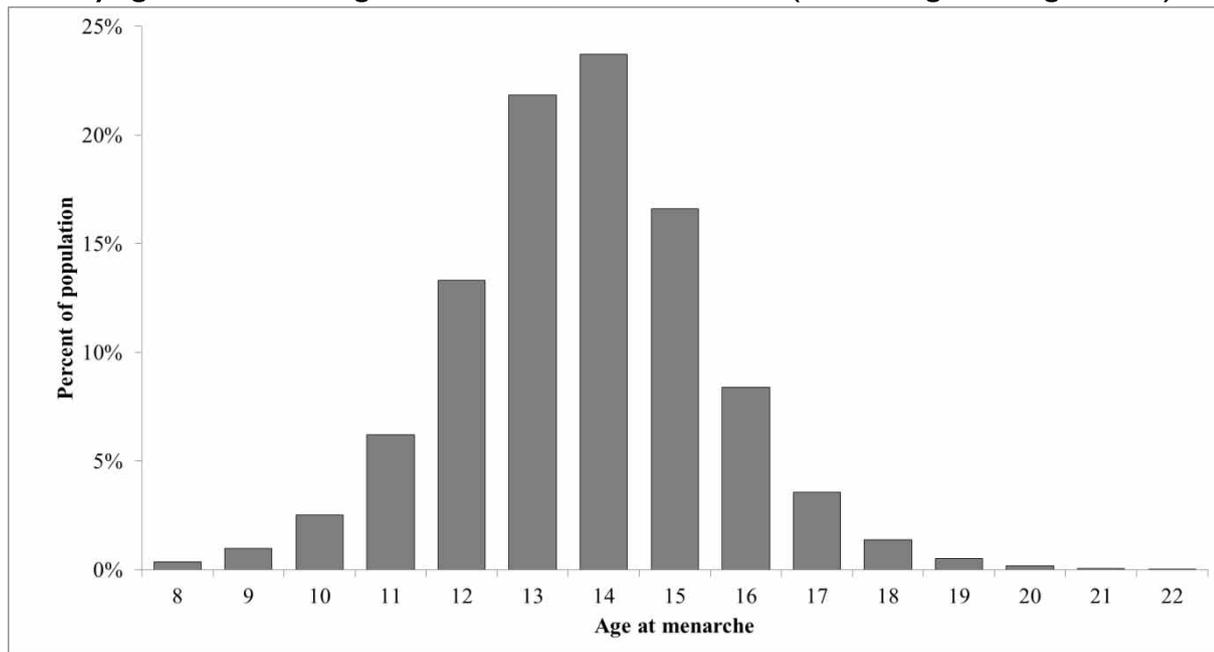


Due to the large sample size, the linear-logistic relationship was stable, and one could derive an underlying distribution of age at menarche (Figure 5). The median value of age at menarche was 14.18 years and the standard deviation was 1.83 years,

which are realistic values locating Africa in the middle of the transition between high and low values. The maximum annual force of progression to menarche was  $r_{\max} = 0.63$ .

Figure 5

Underlying distribution of age at menarche, African countries (after fitting with Logit model)



### Country variations

The same exercise was repeated for 35 African countries with DHS surveys, standardizing for cohort 1990, who had menarche in 2000-2009. Results show a wide range of median age at menarche, from 13.1 (Gabon, Sao-Tome & Principe) to 15.2 (Central African Republic), covering a large chunk of the range of variations identified in the European experience.

Countries with the highest values (> 14.7 years) were also the poorest countries in the continent (Central African Republic, Burkina Faso, Chad, Rwanda, Burundi, Ethiopia), whereas countries with the lowest values (< 13.5 years) were among the wealthiest and most advanced countries (South-Africa, Gabon, Sao-Tome & Principe).

Table 1

Estimates of the median age at menarche from the Logit model, 35 African countries, women age 15-19.

Country	Nb. of women	Nb. of surveys	Period covered		Age at menarche		St. error median
			First	Last	Median	St. dev.	
Benin	11963	6	1982	2018	14.30	1.58	0.009
Burkina Faso	9045	4	1993	2010	15.10	1.53	0.007
Burundi	7077	3	1987	2016	14.78	1.62	0.009
Cameroon	9967	5	1978	2011	14.09	1.39	0.008
Central African Rep.	1286	1	1994	1994	15.22	1.77	0.020
Chad	7041	3	1997	2015	14.73	1.57	0.009
Congo (Kinshasa)	6058	2	2007	2013	14.18	1.98	0.014
Congo (Brazza)	3711	2	2005	2012	13.78	1.74	0.021
Cote d'Ivoire	6100	4	1980	2012	13.74	1.55	0.015
Ethiopia	14112	4	2000	2016	14.89	2.21	0.008
Gabon	3386	2	2000	2012	13.09	1.55	0.028
Gambia	2418	1	2013	2013	13.57	1.66	0.027
Ghana	7774	7	1979	2014	14.11	1.73	0.011
Guinea	7436	4	1999	2018	13.33	1.58	0.017
Kenya	13333	7	1978	2014	14.43	1.71	0.007
Lesotho	5106	3	2004	2014	13.98	1.46	0.016
Liberia	3214	2	2007	2013	14.59	1.37	0.015

Madagascar	8541	4	1992	2008	13.83	1.81	0.012
Malawi	16694	5	1992	2016	14.13	1.63	0.008
Mali	12146	6	1987	2018	14.10	1.69	0.009
Mozambique	7535	3	1997	2011	14.04	1.47	0.011
Namibia	6758	4	1992	2013	14.64	1.45	0.009
Niger	6932	4	1992	2012	14.54	1.70	0.009
Nigeria	31629	7	1982	2018	14.11	1.68	0.004
Rwanda	13879	6	1983	2015	15.04	1.72	0.006
Sao-Tome & Principe	539	1	2009	2009	13.07	1.49	0.076
Senegal	20636	8	1993	2017	13.93	1.74	0.008
Sierra Leone	5255	2	2008	2013	13.45	1.76	0.021
South Africa	3849	2	1998	2016	13.38	1.62	0.024
Swaziland	1261	1	2006	2006	14.29	1.23	0.024
Tanzania	12295	6	1991	2016	14.55	1.72	0.007
Togo	4209	3	1988	2014	14.15	2.00	0.017
Uganda	12519	6	1988	2016	14.26	1.67	0.009
Zambia	10997	5	1992	2013	14.38	1.49	0.008
Zimbabwe	10125	6	1988	2015	14.30	1.41	0.009

NB. Estimates of median age standardized for the 1990 cohort (women who had menarche in 2000-2009), except for countries with only one survey.

**Socio-economic correlates of age at menarche**

The socio-economic correlates of the median age at menarche were further investigated at aggregate country level. Several variables were considered: income per capita (GDP-PPP in constant USD), average caloric intake (in kcal per person per day), urbanization (proportion urban), under-five mortality (deaths per 1000 births), total fertility rate (children per women), female level of education (average number of years of schooling), and age at first marriage (median age). Results show correlations in

the expected direction (negative or positive) given the level of development: lower median age at menarche associated with higher income per capita, higher caloric intake, more urbanization, higher level of education, higher age at marriage; and higher median age at menarche associated with higher child mortality and higher total fertility. In multivariate analysis, only caloric intake and urbanization remained statistically significant and of any importance for the median age at menarche (Table 2).

**Table 2**  
**Correlations of the mean age at menarche with socio-economic variables, 35 African countries**

Correlate	Univariate correlation	Multivariate correlation		
		Net effect (1 St. dev.)	P-value	Signif.
Caloric intake	-0.526	-0.19	0.039	*
Urbanization	-0.651	-0.26	0.014	*
Income per capita	-0.338	+0.05	0.715	ns
Years of schooling	-0.359	-0.17	0.141	ns
Age at marriage	-0.116	+0.08	0.544	ns
Child mortality	+0.243	+0.01	0.914	ns
Fertility	+0.287	-0.04	0.764	ns
Baseline		14.17		

NB: Correlations at country level, standardized over the 2000-2009 period. Sources of data: Caloric intake: FAO statistics; Income per capita: Maddison 2010; Urbanization, Child mortality, Fertility: World Population Prospects 2019; Age at marriage, Years of schooling: DHS surveys.

**Correlation between slope and intercept in the Logit model**

The Logit model was further analysed among the African countries investigated. In fact, there was a

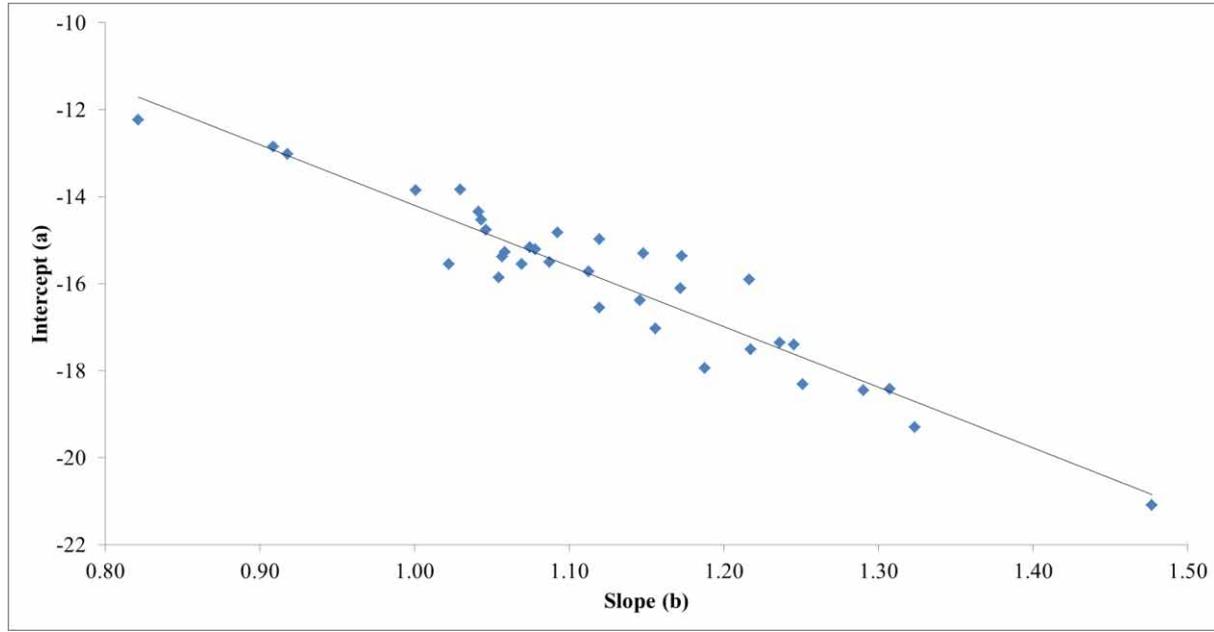
strong correlation ( $\rho = 0.947$ ) between the slope (determining the variance) and the intercept (determining the onset and therefore the mean). This correlation matches observations made in Europe: at

higher levels of the mean age at menarche the variance is larger. This is the result of the process of progression to menarche: the lower the force of progression to menarche, the higher will be the mean and the larger the variance. The intercept also takes into account the fact that whatever the force of

progression to menarche, the earliest age is still around age 8 years, the age of awakening of sexual hormones. This effect was verified empirically on the 35 countries: the correlation between slope and intercept was high, showing a narrow range of possible variations around the linear trend (Figure 6).

**Figure 6**

**Correlation between slope and intercept of the Logit model, 35 African countries**



### Estimating the standard error of the median age at menarche

Estimating the standard error of median age at menarche is a complex exercise, because the estimates are obtained from two regression coefficients, and not from direct calculations. Estimating the standard errors using a bootstrap method would be exceedingly cumbersome in this case. Another simpler approach was followed: considering that the median age is estimated by a ratio  $(-a/b)$ , and that there was a linear relationship between  $(-a)$  and  $(b)$ , one could derive an estimate of the standard error of the median from the standard error on  $(b)$  provided by the regression model. Calculations are shown in the Appendix. Results show that precise estimates of the average median age at menarche were obtained:  $14.24 \pm 0.01$  for Africa as a whole ( $N = 304,826$  women). Most country estimates were also precise to the first decimal (average  $\pm 0.03$ ), with the exception of countries based on small samples, such as Sao-Tome & Principe ( $N = 539$  women), Swaziland ( $N = 1261$  women), and Central African Republic ( $N = 2404$  women), where confidence intervals were larger (See Table I, Column 8). These estimates of the standard errors were consistent with a rough estimate

calculated as if it were a mean (standard deviation/ $\sqrt{N}$ ), although somewhat smaller in case of large sample size and somewhat bigger in case of small sample size.

### Discussion

This study showed that the median age at menarche could be estimated at national level from incomplete data, that is by using an age group where more than half of women had already menstruated. This was possible because of the linear relationship between  $\text{Logit}(P(x))$  and  $(x)$ , and the use of decimal ages. In a few cases the method did not work, because of small sample size, erratic data or age misreporting. In the few countries where both retrospective WFS data and current status DHS data were available, levels and trends in median age at menarche were found consistent, which cross-validates both approaches [see Garenne 2020a,b for details].

Country estimates revealed a wide variety of situations across Africa, with the median age at menarche ranging from 13 years to 15 years at national level. One more time, African countries appear at different levels of development, ranging from poor to wealthy, from poorly fed to well fed, from low to high urbanization, from poorly educated

to high level of education, and from high to low levels of mortality and fertility. The correlations between indicators of development and age at menarche were reassuring, and gave one more piece of evidence of the reliability of the estimates.

Among the development indicators, two appeared more important: food intake and urbanization. Urbanization is associated with more diverse foods and less physical exercise, as compared with rural areas, where young girls work hard and eat less. At individual level anthropometry (fat mass in particular) and physical exercise are among the most important determinants of age at menarche. So, it is reassuring to find out that countries with higher food intake and higher level of urbanization also have lower mean age at menarche.

Across the 35 countries investigated, some appear at outliers given their level of development, with either lower- or higher- average age at menarche than expected given their socio-economic characteristics. In particular, a cluster of low values appears in West Africa (Guinea, Liberia, Cote d'Ivoire), and a cluster of high values in East Africa, in a large band going from Kenya to Namibia. These cases suggest the impact of genetic factors, which need to be further explored. In particular, the East-African cluster seems to be located along the lines of major Bantu migrations of the past centuries. These populations might have had somewhat higher age at menarche in the past, still visible nowadays.

This study did not address the issue of women who declare to never have had their periods at older ages. The proportion of such women aged 22-24 years was 0.23% in DHS surveys. This point seems to never have been properly investigated and deserves further attention. Is this situation real? Or due to misreporting? In any case, this seems to be a minor issue at population level, without importance for the estimation of the median age at menarche.

This analysis stopped at age 20, because the relationship between  $\text{Logit}(P(x))$  and  $(x)$  was no longer linear above this age. This could be due to heterogeneity. In fact, the force of progression to menstruation seems to be declining after age 20, as if the sample of women remaining without menstruation at this age had a lower hormonal profile. This could have been anticipated, as heterogeneity is common in this type of situation. The model used in this study is based on homogeneity (same risk for all women at a given age), and therefore could somewhat underestimate the true value of the standard deviation of age at menarche. However, this should not affect the median estimate.

The median age at menarche estimated from demographic surveys compares with values found in

epidemiological studies, based on smaller samples [see review by Thomas et al. 2010]. Overall, among the 11 African countries for which comparable estimates were available, the average median age at menarche was similar in both cases (14.11 years in demographic surveys vs 14.18 years in epidemiological surveys), even though the time frame was not identical. When studied country by country, there was an average 0.75 year absolute difference between the two sources. In two cases there were large inconsistencies due to very strong biases in the second samples (+2.2 years difference in Niakhar, rural Senegal compared with the national average; and -1.8 years difference in Brazzaville, urban Congo compared with the national average). Outside these extreme cases, the differences were smaller and could be explained by sample size and sample bias.

More can be learned on levels, trends, determinants and differentials of age at menarche from DHS surveys, as shown by a case study in Nigeria [Garenne 2020b]. In particular, some complex patterns seem to be emerging in Africa, with divergent trends in adult height, body composition (fat mass), and age at menarche, as shown in a recent case study in urban and rural South Africa [Said-Mohamed et al. 2018].

## Conclusion

This study shows the great value of African DHS surveys for investigating a delicate issue such as the median age at menarche, which has a small range of variations among populations, despite a wide range of variations among individual women. DHS surveys have large sample size, are representative of the national populations, and are conducted in a standardized way which allows for comparisons in time and space. In addition they provide insights on possible correlates, such as socio-economic status and anthropometry. These surveys also allow for further investigations at country level about trends, socio-economic differentials, ethnic differences and other factors of the median age at menarche.

This preliminary investigation confirms earlier findings that African populations have menarche characteristics similar to those of European populations, and seem to be in the middle of the transition from high to low median age. However, further investigations could reveal somewhat different patterns, at regional or ethnic level. In particular the case of Eastern Africa, displaying higher median age at menarche than expected from socio-economic characteristics, deserves further analysis.

## Appendix: the logit model applied to age at menarche

Let  $x$  = age at which women first menstruated, such as  $\alpha \leq x \leq \beta$ , and  $P(x)$  = Proportion of women age ( $x$ ) who already menstruated. With  $\alpha = 8.0$ ;  $\beta = 25.0$ ;  $P(\alpha) \approx 0$ ;  $P(\beta) \approx 1$ . Formulae displayed here are similar to life table formulae, but applied with a Logit model.

### Basic model

$$\text{Logit}(P(x)) = \ln(P(x)/(1-P(x))) = a + b \times x$$

With Intercept =  $a < 0$ ; Slope =  $b > 0$ ;  $a$  and  $b$  can be estimated by linear regression from empirical data.

The distribution of women by age at first menstruation ( $x$ ) is the derivative of the cumulative function =  $dP(x)$

Median( $x$ ) is obtained for  $P(x) = 0.5$ , that is  $\text{Logit}(P(x)) = 0$ , therefore:

$$\text{Mean}(x) = \text{Median}(x) = \text{Mode}(x) = -a/b$$

Since  $\text{Var}(\text{Logit}(P(x))) = b^2 \times \text{Var}(x)$ , then:

$$\text{Variance}(x) = (\pi^2/3 \times b^2) ; \text{Standard Deviation}(x) \approx 1.8138/b$$

The rate of progression to first menstruation at age ( $x$ ) is given by the Log-derivative, that is new cases / susceptibles:  $r(x) = dP(x)/(1-P(x))$

A property of the Logit function is that  $r(x) \rightarrow r_{\max}$  when  $x \rightarrow \infty$ . This is shown in Figure 2.

### Estimating trends in age at menarche

The basic model can be extended with a time trend:

$$\text{Logit}[P(x)] = a + b \times \text{Age}(x) + c \times \text{Cohort}(t)$$

Where  $t=0$  can be set at a fixed point, such as women born in year 1990.

Coefficients  $a$ ,  $b$ ,  $c$  can be estimated by linear regression.

The effect of time ( $t$ ) on Median( $x$ ) can be obtained by adding the cohort effect to the constant, that is:  $-(a + c \times t)/b$

### Introducing covariates

The basic model can be extended with covariates:

$$\text{Logit}[P(x)] = a + b \times \text{Age}(x) + c \times \text{Cohort}(t) + \sum_i Z_i$$

The effect of covariates on mean age at menarche can be obtained similarly by adding its effect on the constant. An example of the full model can be found in another paper [Garenne, 2020b].

### Calculating confidence intervals for median

$$\text{Median} = -a/b$$

Linear relationship between  $a$  and  $b$ :

$$-a = k + m \times b$$

$$\text{Empirically: } k = +0.255; m = +13.942$$

$$\text{Then: } -a/b = k/b + m$$

$$\text{Var}(-a/b) = \text{Var}(k/b) = k^2 \times \text{Var}(1/b)$$

$$\text{Since } \text{Var}(1/b) \approx \text{Var}(b) / (b^4)$$

$$\text{Var}(-a/b) \approx k^2 \times \text{Var}(b) / b^4$$

$$\text{St. Error}(\text{Median}) \approx \text{St. Error}(b) \times k/b^2$$

Confidence intervals can then be calculated as:  $\text{Median} \pm 1.96 \times \text{St. Error}(\text{Median})$ .

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