Rain Measurement by Raingage–Radar Combination: A Geostatistical Approach

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ABSTRACT

A geostatistical approach to areal rainfall estimation using raingage and radar measurements is described. The so-called cokriging method is used to obtain a linear estimator of ground-level rainfall depths by combining gage and radar data under unbiasedness and optimality constraints. The statistical inference of the spatial structure of these two kinds of measurements (required to determine the cokriging system) is discussed in the multi- and single-realization context. A simplified version of the cokriging method is then proposed to obtain a more tractable system for practical applications.

A validation procedure based on (i) the estimation of reference rainfall depths and (ii) the selection of a set of likeness criteria is defined. The reference values are computed in an original way by integrating raingage measurements over radar pixels containing a test raingage using the classical kriging method.

The test case deals with a set of 11 daily rainfall events observed in the Paris region by the 10 cm "Melodi" weather radar system. The available raingage network includes 98 stations spread over 20 000 km^2: 69 stations have been used for validation purposes and the remaining 29 for the simplified cokriging operating method.

The available radar dataset presents severe limitations for hydrological applications mainly in relation to ground echo effects within a 52 km radius of the radar site. In spite of these unfavorable conditions, the proposed combination method appears to improve slightly the performance of the raw radar data and to exceed that of the classical uniform calibration method.

Further application of this method using a more appropriate dataset is necessary to confirm these initial results.

1. Introduction

Accurate knowledge of rainfall patterns is needed both in hydrologic and atmospheric sciences. An important application is the calculation of areal rainfall estimates, constituting one of the main inputs to watersheds models (cf. Hudlow, 1983) and a valuable validation criterion for mesoscale atmospheric model outputs (cf. Medal et al., 1984).

Raingage networks of varying complexity were once the only available source of rain measurements. The main drawback of this kind of measurement is its point nature, requiring the use of simple or sophisticated interpolation techniques for extension to the whole space (e.g., Hall and Barclay, 1975; Thorpe et al., 1979; Creutin and Obled, 1982). In addition, real time applications are strongly limited by the heavy costs of telemetered data collection.

Over the last two decades, the development of weather radar technology, adding the advantages of digital processing to the intrinsic properties of radar measurements (high spatial resolution and real time availability), have opened very promising possibilities for rainfall estimation. Numerous studies comparing gage and radar measurements have carefully explored these possibilities (a summary of these studies can be found in Wilson and Brandes, 1979). One of their main conclusions points to the high variability in space and time of the relationship between reflectivity factor Z and rainfall rate R; this is partly due to the variations of the raindrop-size distribution $N(D)$ both within and between clouds, but also due to vertical air movements, incomplete beam filling, attenuation and beam blockage, among many other reasons. (An extensive review of the relative importance of these discrepancy factors can be found in Zawadzki, 1984.) To reduce the uncertainty of the Z–R relationship, without making coarse assumptions on $N(D)$, two kinds of solutions have been proposed (see Doviak, 1983). The more promising is probably the development of dual polarization radar, but it could be a long time before such devices become available on an operational basis. The alternative answer, more immediately applicable, is the raingage–conventional radar combination.

Various methods for radar calibration by gages have been proposed from the simplest, relying on the identification of a constant multiplicative calibration factor (e.g., Barnston and Thomas, 1983; Harrold et al., 1974) to more sophisticated approaches aimed at regionalizing the Z–R fluctuations (such as Brandes, 1975; Hildebrand et al., 1979) or taking the rainfall type into

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account (see Collier et al., 1983). All these procedures are based on a correction of the multiplicative factor of the Marshall–Palmer Z–R relationship.

Following a rather different approach, this paper uses a statistical combination procedure considering ground and radar measurements as sampling devices of two distinct but correlated signals. This procedure aims at a linear estimation of rainfall depths at ground level merging gage and radar data under unbiasedness and optimality constraints. In its complete form, this method is known in atmospheric sciences as multivariate objective analysis (see Gandin and Kagan, 1974; Schlatter, 1975) and in earth sciences as cokriging (see Journel and Huijbrechts, 1978). The only difference between these two approaches is that the first requires a multirealization context while the second can be applied to a single realization context. For application to remote sensed data, certain assumptions have been made to reduce the computational burden (see Delhomme, 1979).

After a detailed discussion of the proposed combination method, the dataset used in the case study is briefly described. Then the various practical steps in combination processing are presented. Finally, a comparison is made between the results obtained and other calibration or gage interpolation results.

2. Proposed combination method

a. Mathematical formulation of cokriging

General descriptions of multivariate objective analysis (MOA) and cokriging systems are available in the literature (see Thiebaux, 1973; Myers, 1982). Their similarity lies in the aim to satisfy, basically, the same statistical criterion (minimum mean square error of estimate). To deal with the single realization context, cokriging uses an additional unbiasedness constraint.

The description here is devoted to the case of two variables in a single realization context, corresponding to the radar–raingage combination need when extensive records are not available (a dataset including only a few realizations does not allow a multirealization approach).

Let $z_R(x, \omega)$ and $z_G(x, \omega)$ denote the radar and ground values, respectively, of the rainfall depth at a given geographic point x and for a given event ω. These two functions are measured over a radar grid $\{x_\alpha{}^R, \alpha = 1, N\}$ and a ground network $\{x_i{}^G, i = 1, n\}$, respectively.

As a basic assumption, the different functions $z_R(x, \omega)$ and $z_G(x, \omega)$ obtained as ω varies can be considered as independent realizations of random functions $z_R(x)$ and $z_G(x)$. Statistical criteria may then be applied to these random functions to obtain the so-called best linear unbiased estimator of the phenomenon at each point x and for all events ω.

A **linear estimator** of ground rainfall depth at a given point x_0 can be expressed by the following combination (the star denotes an estimated value):

$$z_G^*(x_0) = \sum_{\alpha=1}^{N} \lambda^\alpha z_R(x_\alpha{}^R) + \sum_{i=1}^{n} \lambda^i z_G(x_i{}^G). \qquad (1)$$

If this estimator is required to be **unbiased,** its expectation must equal that of the unknown true value of the ground rainfall depth at the ungaged point x_0:

$$Ez_G^*(x_0) = Ez_G(x_0). \qquad (2)$$

Since the expectation is linear and z_G^* is given by (1), we obtain

$$\sum_{\alpha=1}^{N} \lambda^\alpha E z_R(x_\alpha{}^R) + \sum_{i=1}^{n} \lambda^i E z_G(x_i{}^G) = E z_G(x_0). \qquad (3)$$

In (3), the weights λ^i and λ^α give an exact interpolation of the expectation of the ground rainfall depth from both the gage and radar measurement expectations. This is questionable, however, since radar measurements may be subject to major systematic over or underestimation. So, if $Ez_G(x)$ is assumed to be different from $Ez_R(x)$, (3) can be split up as follows:

$$\sum_{\alpha=1}^{N} \lambda^\alpha E z_R(x_\alpha{}^R) = 0$$

$$\sum_{i=1}^{n} \lambda^i E z_G(x_i{}^G) = E z_G(x_0). \qquad (4)$$

Here the weights λ^i are used to interpolate the expectation of the ground rainfall depth using the gage measurement expectations, while the weights λ^α filter the radar measurement expectations. We assume here that the gage measurements are free of systematic error or, at least, that such errors are negligible. Note that the validity of this solution may be doubtful for very low network densities.

If the estimator (1) is also required to be **optimal,** the weights must minimize the following mean square error:

$$\begin{aligned} E[z_G^*(x_0) - z_G(x_0)]^2 &= Ez_G^*(x_0)^2 + Ez_G(x_0)^2 - 2Ez_G^*(x_0)\cdot z_G(x_0) \\ &= \sum_\alpha \sum_\beta \lambda^\alpha \lambda^\beta E z_R(x_\alpha{}^R)\cdot z_R(x_\beta{}^R) + \sum_i \sum_j \lambda^i \lambda^j E z_G(x_j{}^G) \\ &\quad \times z_G(x_i{}^G) + 2\sum_i \sum_\alpha \lambda^i \lambda^\alpha E z_G(x_i{}^G)\cdot z_R(x_\alpha{}^R) \\ &\quad + Ez_G(x_0)^2 - 2\sum_\alpha \lambda^\alpha E z_R(x_\alpha{}^R)\cdot z_G(x_0) \\ &\quad - 2\sum_i \lambda^i E z_G(x_i{}^G)\cdot z_G(x_0). \end{aligned} \qquad (5)$$

The minimization of (5) (i.e., bringing its partial derivatives to zero) under the conditions expressed in (4) gives, through the application of Lagrangian techniques, the cokriging system where μ_R and μ_G are Lagrangian multipliers.

$$\begin{bmatrix} Ez_R(x_\alpha^R)z_R(x_\beta^R) & Ez_R(x_\beta^R)z_G(x_i^G) & Ez_R(x_\beta^R) & 0 \\ Ez_R(x_\alpha^R)z_G(x_j^G) & Ez_G(x_i^G)z_G(x_j^G) & 0 & Ez_G(x_j^G) \\ Ez_R(x_\alpha^R) & 0 & 0 \\ 0 & Ez_G(x_i^G) \end{bmatrix} \times \begin{bmatrix} \lambda^\alpha \\ \lambda^i \\ \mu_R \\ \mu_G \end{bmatrix} = \begin{bmatrix} Ez_R(x_\beta^R)z_G(x_0) \\ Ez_G(x_j^G)z_G(x_0) \\ 0 \\ Ez_G(x_0) \end{bmatrix}. \quad (6)$$

Solution of the above system yields the desired weighting vector $[\lambda^\alpha, \lambda^i]$ provided that appropriate hypotheses are used in determining the left-hand matrix and the right-hand vector. The classical hypothesis is the second order stationarity of the random function z_R and z_G, i.e. the invariance of their two first moments for any geographical translation. These moments are therefore expressed as

$$Ez_R(x) = m_R \quad \text{and} \quad Ez_G(x) = m_G$$

$$C_R(h) = Ez_R(x) \cdot z_R(x+h) - m_R^2$$

$$C_G(h) = Ez_G(x) \cdot z_G(x+h) - m_G^2$$

$$C_{RG}(h) = Ez_R(x) \cdot z_G(x+h) - m_R m_G \quad (7)$$

where C_R and C_G are covariance functions and C_{RG} a cross-covariance function.

In the next section, the statistical inference problems posed by such a model, are examined in more detail.

Note that when system (6) is satisfied, the mean square error (5) is simplified to

$$E[z_G^*(x_0) - z_G(x_0)]^2$$

$$= Ez_G^2(x_0) - \sum_{\alpha=1}^{N} \lambda^\alpha Ez_R(x_\alpha^R) \cdot z_G(x_0)$$

$$- \sum_{i=1}^{n} \lambda^i Ez_G(x_i^G) \cdot z_G(x_0) - \mu_G. \quad (8)$$

Thus, cokriging, like MOA, yields a good indication of the accuracy of the estimation in addition to the estimation itself.

b. Statistical inference

The above presentation of cokriging assumes that z_G and z_R are second-order stationary random functions. The statistical inference of this model depends on the study context.

In a **multirealization context,** the mean values of the functions z_G and z_R can be estimated, at least at measurement points. To satisfy mean stationarity conditions, centered functions may be used $[y_G(x) = z_G(x) - Ez_G(x)$ and $y_R(x) = z_R(x) - Ez_R(x)]$. The estimator (1) can be written as

$$y_G^*(x_0) = \sum_{\alpha=1}^{N} \lambda^\alpha y_R(x_\alpha^R) + \sum_{i=1}^{n} \lambda^i y_G(x_i^G).$$

As y_R and y_G are zero mean functions, the unbiasedness condition is unnecessary. The cokriging system is reduced to an MOA system [$n+N$ first rows and columns of system (6) where z is replaced by y]. If the second order moments of y_R and y_G are stationary, the following direct and cross-covariance functions can be defined:

$$C_{R \text{ or } G \text{ or } RG}(h) = Ey_{R \text{ or } G}(x) \cdot y_{R \text{ or } G}(x+h).$$

The MOA system can be determined and solved by fitting these functions with permissible analytic functions (see Christakos, 1984). As the obtained estimator is now a centered value $y_G^*(x_0)$, an estimate of the mean value $Ez_G(x_0)$ is required.

In a **single-realization context,** specific to the cokriging system, three points need to be considered: (i) the classical second-order stationarity model is not valid, but (ii) a local restriction of this model is acceptable; (iii) more sophisticated models exist but are not used here.

i) For a single realization, the mean values of z_R and z_G are unknown, even at measurement points, and these functions therefore cannot be centered to satisfy the constant mean value hypothesis. It would also be unrealistic to assume a constant mean value for functions z_R and z_G over a large area. Mean stationarity therefore cannot be assumed.

ii) The restriction of second order stationarity to a limited area X is theoretically possible, provided that the $(n+N)$ measurement points used in the estimation (1) belong to X. Accordingly the radar and gage means can be expressed by locally constant values:

$$Ez_R(x) = m_R(X) \quad \text{and} \quad Ez_G(x) = m_G(X)$$
$$\text{when} \quad x \in X.$$

The unbiasedness conditions (4) are reduced to

$$\sum_{i=1}^{n} \lambda^i = 1 \quad \text{and} \quad \sum_{\alpha=1}^{N} \lambda^\alpha = 0. \quad (9)$$

As the various weights λ simply interpolate or filter these means, $m_R(X)$ and $m_G(X)$ may remain unknown in practice. If the second-order moments of z_R and z_G are locally stationary, the various direct and cross-covariance functions of (7) exist and can be written as

$$C_{R \text{ or } G \text{ or } RG}(h) = Ez_{R \text{ or } G}(x) \cdot z_{R \text{ or } G}(x+h) - m_{R \text{ or } G}(X) \cdot m_{R \text{ or } G}(X) \quad (10)$$

when x and $x+h$ belong to X. The practical computation of these functions in a single-realization context is based on the use of the ergodic principle. The expectations E are replaced by a spatial average in the following manner:

$$C_{R \text{ or } G \text{ or } RG}(h) = \frac{1}{N(h)} \sum_{i=1}^{N(h)} [z_{R \text{ or } G}(x_i) - m_{R \text{ or } G}(X_i)] \times [z_{R \text{ or } G}(x_j) - m_{R \text{ or } G}(X_j)]$$

where $N(h)$ is the number of pairs of measurement points (x_i, x_j) separated by a class of vectors $h \pm \Delta h$. Because estimation of the various local means is very risky, several authors have proposed a substitute for covariance functions in the form of variograms (Matheron, 1965; also called structure functions by Gandin, 1965):

$$\gamma_{R \text{ or } G}(h) = \frac{1}{2} E[z_{R \text{ or } G}(x) - z_{R \text{ or } G}(x+h)]^2$$

$$\gamma_{RG}(h) = \frac{1}{2} E[z_R(x) - z_R(x+h)] \cdot [z_G(x) - z_G(x+h)]$$

where the constant local mean values are filtered. When the cross-covariance function is symmetrical $[C_{RG}(h) = C_{RG}(-h)]$ the following general relation exists between variograms and covariance functions:

$$\gamma_{R \text{ or } G \text{ or } RG}(h) = C_{R \text{ or } G \text{ or } RG}(0) - C_{R \text{ or } G \text{ or } RG}(h). \qquad (11)$$

Using relationships (9), (10), and (11), it can readily be shown that the cokriging system can be solved by replacing the product expectations in (6) either with the corresponding $C_{R \text{ or } G \text{ or } RG}(h)$ or with $-\gamma_{R \text{ or } G \text{ or } RG}(h)$.

iii) More sophisticated hypotheses can be made concerning the general or local behavior of $Ez_R(x)$ and $Ez_G(x)$. Polynomial expressions are often proposed to model these trend (or drift) functions (see Myers, 1982), and more complex unbiasedness constraints are obtained through (3) or (4). Equation (3) can also account for any physically based analytic relationships between these trends (radar range dependence of the relation between radar and gage measurements, for instance).

In this study the simplest assumption of constant and distinct local means for radar and gage measurements is retained since (i) it appears reasonable on the basis of previous studies on the mean local behavior of rainfall fields using a daily time step (Creutin and Obled, 1982), and (ii) our dataset shows no clear evidence of local dependence between mean radar and gage measurements.

c. Possible simplification of the cokriging system for gage–radar combination

Although the above basic principles of cokriging theory are well known, very few full applications have been described in the literature. Only investigators working on poorly sampled phenomena (e.g. in soil sciences, Vauclin et al., 1983) or on clearly multivariate contexts (e.g. in forest sciences, Marbeau, 1978) have applied the complete system. Most frequently "potential cokrigers" use simplified versions of cokriging (e.g. Matheron, 1979, in mining sciences; Chauvet et al., 1976, in meteorology) to avoid difficulties related to computational burden and structure function modelling.

Cokriging appears to be particularly relevant to the problem of radar–raingage rainfall measurement for three main reasons: (i) raingages provide good direct measurements of precipitation but the networks are sparse; (ii) spatial resolution of radar pictures is very high but the strong variability of the relationship between rainfall intensity and reflectivity limits the reliability of this measurement; (iii) radar and raingage measurements are thought to be fairly well correlated in space and time. However the high density of radar data leads to too large a system even if the estimator given by (1) is considered over a geographically reduced neighborhood (e.g. for a 5 × 5 km radar grid combined with a one gage per 1000 km^2 network, a neighborhood containing ten gages covers 400 radar pixels and the dimension of the resulting cokriging system is 410).

A very major simplification, in terms of reducing system size, can be made when each gage may be connected to a radar pixel. Let us consider that the ground rainfall depth can be split into two orthogonal terms:

$$z_G(x) = \hat{z}_R(x) + \epsilon(x) \qquad (12)$$

where $\hat{z}_R(x)$ represents the conditional expectation of z_G knowing the radar measurement z_R and where $\epsilon(x)$ is a residual, assumed to be statistically independent of $\hat{z}_R(x)$. In practice, $\hat{z}_R$ can be deduced from z_R through a linear regression relationship between z_R and z_G over the gage network. The residuals are therefore available experimentally at gage locations x_i^G:

$$\epsilon(x_i^G) = z_G(x_i^G) - \hat{z}_R(x_i^G).$$

The linear estimator (1) is modified in two ways: (i) the radar measurements z_R are replaced by $\hat{z}_R$ and (ii) the gage measurements are expressed as in (12). Grouping the term $\hat{z}_R$ gives

$$z_G^*(x_0) = \sum_{\alpha=1}^{N} \lambda^\alpha \hat{z}_R(x_\alpha{}^R) + \sum_{i=1}^{n} \lambda^i \epsilon(x_i{}^G) \qquad (13)$$

since each gage point x_i^G can be connected to a radar grid point $x_\alpha{}^R$. The replacement of z_R by $\hat{z}_R$ is fully justified when the regression relationship is linear. As the mean $\hat{z}_R$ value matches the mean ground rainfall depth $[Ez_G(x) = E\hat{z}_R(x) = m$, so $E\epsilon(x) = 0]$ the unbiasedness constraint is given by

$$\sum_{\alpha=1}^{N} \lambda^\alpha = 1. \qquad (14)$$

Finally the minimization of the mean square error

$$E[z_G^*(x_0) - z_G(x_0)]^2$$

$$= E[\sum_{\alpha=1}^{N} \lambda^\alpha \hat{z}_R(x_\alpha{}^R) + \sum_{i=1}^{n} \lambda^i \epsilon(x_i{}^G) - \hat{z}_R(x_0) - \epsilon(x_0)]^2$$

yields, after simplifications due to orthogonality between $\hat{z}_R$ and ϵ:

$$\sum_{\beta=1}^{N} \lambda^\beta E\hat{z}_R(x_\alpha{}^R)\cdot\hat{z}_R(x_\beta{}^R) - E\hat{z}_R(x_\alpha{}^R)\cdot z_G(x_0) = 0 \quad \text{for} \quad \alpha = 1 \text{ to } N$$

$$\sum_{j=1}^{n} \lambda^j E\epsilon(x_i{}^G)\cdot\epsilon(x_j{}^G) - E\epsilon(x_i{}^G)\cdot\epsilon(x_0) = 0 \quad \text{for} \quad i = 1 \text{ to } n.$$

The above system can in fact be split, under condition (14), into two independent subsystems. The first defines a simple kriging estimate of what can be considered as a "guessfield" $\hat{z}_R$:

$$\begin{bmatrix} E\hat{z}_R(x_\alpha{}^R)\cdot\hat{z}_R(x_\beta{}^R) & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} \lambda^\alpha \\ \mu_R \end{bmatrix} = \begin{bmatrix} E\hat{z}_R(x_\alpha{}^R)\cdot z_G(x_0) \\ 1 \end{bmatrix}. \quad (15)$$

The second defines a regression estimate for the residual ϵ:

$$[E\epsilon(x_i{}^G)\cdot\epsilon(x_j{}^G)][\lambda^i] = [E\epsilon(x_i{}^G)\cdot\epsilon(x_0)]. \quad (16)$$

The main simplifications arising from this breakdown are related to (i) the possibility of working on different neighborhoods in the two subsystems and thus of significantly reducing the size of the first system and (ii), the degeneration of the first system (15) when the estimated point x_0 belongs to the radar network ($x_0 = x_\alpha{}^R$), and thus the reduction of the cokriging system to a residual estimation through system (16) of size n. Equation (13) then becomes

$$z_G^*(x_0) = \hat{z}_R(x_0) - \sum_{i=1}^{n} \lambda^i\epsilon(x_i{}^G), \quad (17)$$

the associated estimation variance being

$$E[z_G(x_0) - z_G^*(x_0)]^2 = E\epsilon^2(x_0) - \sum_{i=1}^{n} \lambda^i E\epsilon(x_i{}^G)\cdot\epsilon(x_0). \quad (18)$$

3. Dataset used

In spite of the present development of the French weather radar network, only one radar site was available with digitally processed records at the outset of this study. Located near Paris-Roissy Airport, this radar system covers the central north part of France shown on the icon map in Fig. 1.

a. Radar processed data

The standard "Melodi Onera" radar used has the following characteristics: wavelength 10 cm, 2° beam at half power, 700 kW peak power, 2 μs pulse length and −106 dBm minimum detectable signal. The video radar output is processed by the "Saphyr" system developed by the French Meteorological Service. (Details can be found in Gilet et al., 1980.) Using a numerical integrator, a microcomputer and a magnetic tape recorder, this system controls the pulse repetition frequency (250 Hz), converts the video signal into 256 levels, averages 64 pulses at 256 bins extending 500 m radially and 2° in azimuth, and records a complete PPI every 5 min at elevation 0.7°. In the case studied, the presence of permanent echoes due to the Paris agglomeration results in a minimum recorded range of 52 km, a severe limitation for hydrologic applications.

Subsequent to this on-site processing, a minicomputer program was used to transform this basic data into daily rainfall depths over a rectangular grid. After range correction for decreased power density with range due to gas attenuation, an appropriate radar equation and the classical Marshall–Palmer relation ($Z = aR^b$, with $a = 200$ and $b = 1.6$) yielded a reflectivity equivalent rainfall rate for each bin. Next these elementary rates were cumulated in time and averaged in space to obtain a daily value for each 5 × 5 km mesh of the resulting grid. To avoid problems due to permanent echoes, the grid was reduced to the "cleanest" part of the radar area (see Fig. 1). Special care was taken to avoid anomalous propagation effects by visual examination of each basic picture; doubtful pictures led to the elimination of the concerned day.

From the available set of radar pictures, recorded between October 1980 and October 1982, 11 days were selected with full recording over 24 h starting from 0600 (this UTC starting time constraint is due to the time at which ground measurements are taken).

b. Raingage data

The gage network corresponding to the radar grid counts, after critical selection, 98 stations measuring daily rainfall depths. This network has been split into two subnetworks: (i) a merging network of 29 stations used in the gage–radar combination and (ii) a test network of 69 stations providing independent information to test radar and gage–radar combination results.

4. Practical aspects of the statistical inference

A main step in gage and radar data processing through either complete or simplified cokriging is the statistical inference of the second order stationarity model. As demonstrated in section 2b, local application of this model to raw gage and radar measurements is an acceptable compromise in this context. First, this restriction permits a mathematical solution of the proposed merging systems within a reduced neighborhood—including for instance five gages. Second, over such areas the expected values of radar and gage measurements may reasonably be assumed to be constant. Of course, this must be considered a very crude approximation of the merging problem since the systematic errors inherent in the radar device should first be eliminated. However, the only way to correct such er-

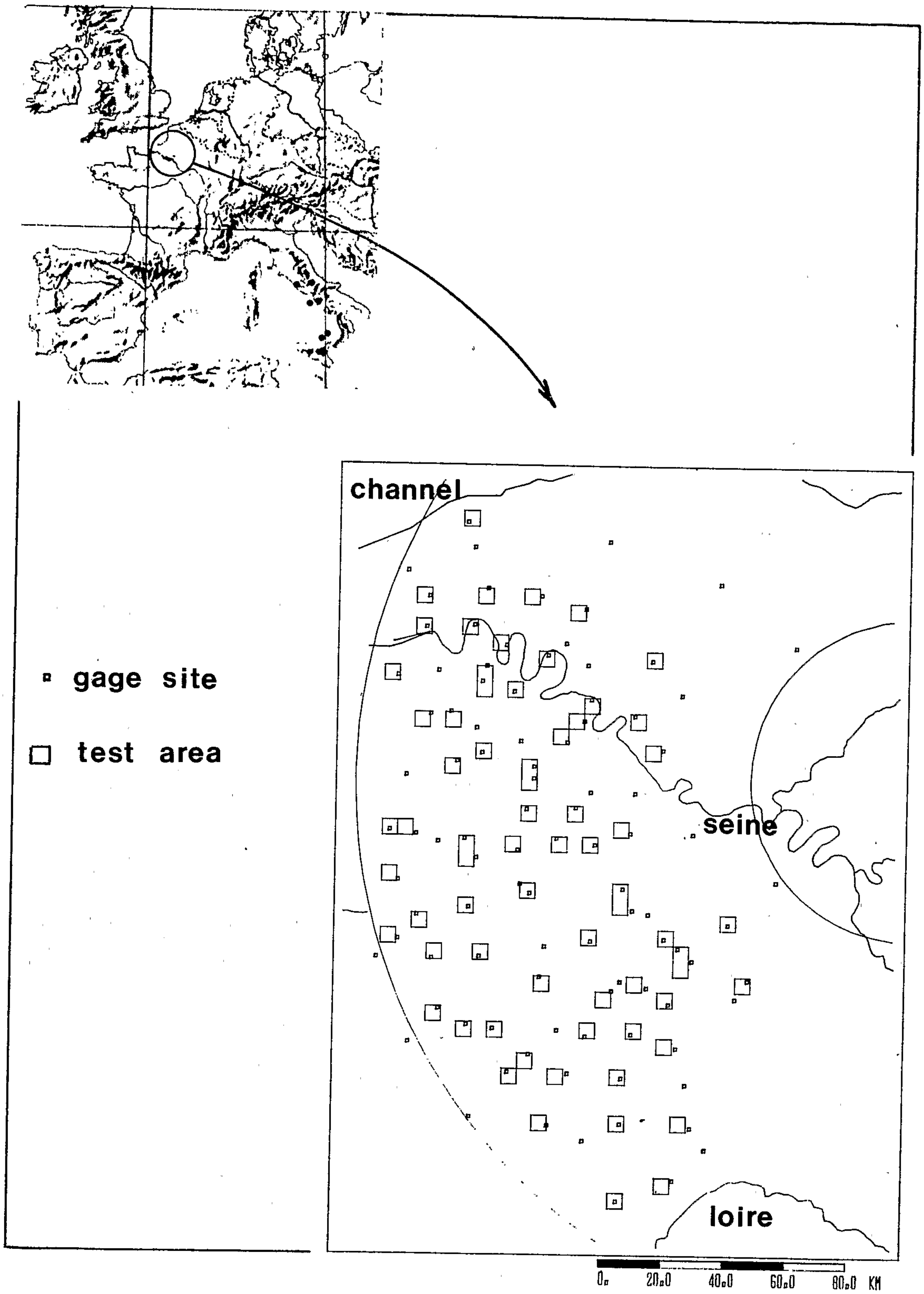

FIG. 1. Radar and gage network locations.

rors statistically is to observe their long-term spatial organization, and this possibility is excluded in our experimental context.

When second order stationarity is reduced to limited areas, the most appropriate tool for analysis of the statistical structure of the studied functions is the variogram (see section 2b). The main statistical characteristics of each event are recalled in Table 1, and three aspects of the structure analysis for the 11 available rainfall fields are presented: (a) gage and radar measurements are first considered separately, showing good agreement between the identified structures and leading to a distinction between convective and frontal meteorological situations; (b) cross variograms are then used to characterize the cofluctuation of the two measurements in order to predict the effectiveness of their combination; (c) finally, the structure of the residuals $\epsilon(x)$ between gage measurements and corresponding radar guess field values is modeled in order to perform the simplified cokriging procedure.

Since the number of studied rainfall fields is too large to present each analysis separately, only a few selected variograms will be displayed (Figs. 2 and 3) as an illustration of the description of the various models fitted, given in Table 2. As shown in this table, two kinds of models were selected to represent direct and cross variograms of the radar and gage measurements:

- Range models were used for variograms presenting variation stabilization around a given value (referred to as the sill, theoretically equal to the field variance) for distances greater than the range (or decorrelation distance). Spherical, exponential or Gaussian expressions can be used to represent such variograms (e.g., for 8 October 1980 and 13 July 1982 events in Fig. 3).
- Linear models were used when a range could not be identified (e.g., for 2 August 1981 and 22 July 1982 convective events in Fig. 3).

TABLE 1. Mean and standard deviations of ground measurements (computed over 98 stations) and multiplicative correction factors (ratios of mean ground value to mean radar value).

Date of event	Mean value of the 98 gage measurements (mm)	Standard deviation of the 98 gage measurements (mm)	Multiplicative correction factor
7 Oct 1980	5.5	5.0	5.7
8 Oct 1980	8.6	10.6	3.1
10 Oct 1980	13.9	5.4	2.2
14 Dec 1980	7.9	3.0	1.8
15 Dec 1980	4.3	3.7	2.6
2 Aug 1981	1.2	3.4	2.4
23 Jun 1982	4.1	2.8	2.5
13 Jul 1982	9.4	10.4	2.0
15 Jul 1982	3.2	2.8	2.6
22 Jul 1982	0.9	2.9	2.4
22 Oct 1982	22.5	6.9	2.8

Some fitted variograms present a discontinuity for short distances (a white noise effect) pointing out the existence of microregionalization for the studied phenomenon (e.g., in convective situations 2 August 1981 or 22 July 1982 events in Fig. 3, where the size of individual showers relative to the gage density results in a white noise effect).

An intuitive manual fitting of these theoretical models to the experimental variograms was performed, paying special attention to the following points:

- *Short distances.* A white noise effect is assumed when a set of three or four significant [i.e., computed with $N(h) > 20$] experimental variogram values clearly do not converge to the origin. Special attention must be paid to this part of the variograms since it is the only part used when the estimation neighborhoods are reduced. In addition, the results obtained are very sensitive to the amplitude of the white noise effect which has a smoothing influence on the estimator.
- *Large distances.* The variogram becomes sensitive to the overall trend of the studied functions [i.e., the difference between local mean values $M(X_i) - M(X_j)$ becomes predominant in the computation of $(z(x_i) - z(x_j))^2$]. It is generally possible to detect a geographical direction along which this trend is lower (the chosen direction is indicated in Table 2 by (1) for E.W. and (3) for N.S.). Notice that this large distance modeling does not have any influence on the results.

a. *Gage and radar measurement structure*

The ground measurement variograms have been computed over the 98 stations of the complete network for reasons of robustness. The dataset reflects results commonly observed for daily rainfall field structures: frontal meteorological situations (mainly in autumn, e.g., 8 October 1980) produce continuous rainfall structures (no white noise effect) with generally an identifiable range; convective situations (in summer) produce more choppy fields whose structure may be overlooked by the ground network even if its density is relatively high (one gage per 200 km^2 for the complete network).

The radar measurement variograms may be established using the whole 5×5 km^2 grid. As illustrated in Fig. 4, where the variogram of the complete raw radar picture for the 8 October 1980 field is compared with the preprocessed one (ground clutters removed), the variogram is very responsive to anomalous values. This observation was used here to recognize ground clutters. Practically speaking, it is simply necessary to identify, in the short distance classes, the pair of points giving anomalous squared differences, and then to eliminate any points found in several of these outlying pairs.

For consistency with ground variograms, the radar variograms presented in Table 2 are computed using the same set of 98 points. Considering the properties

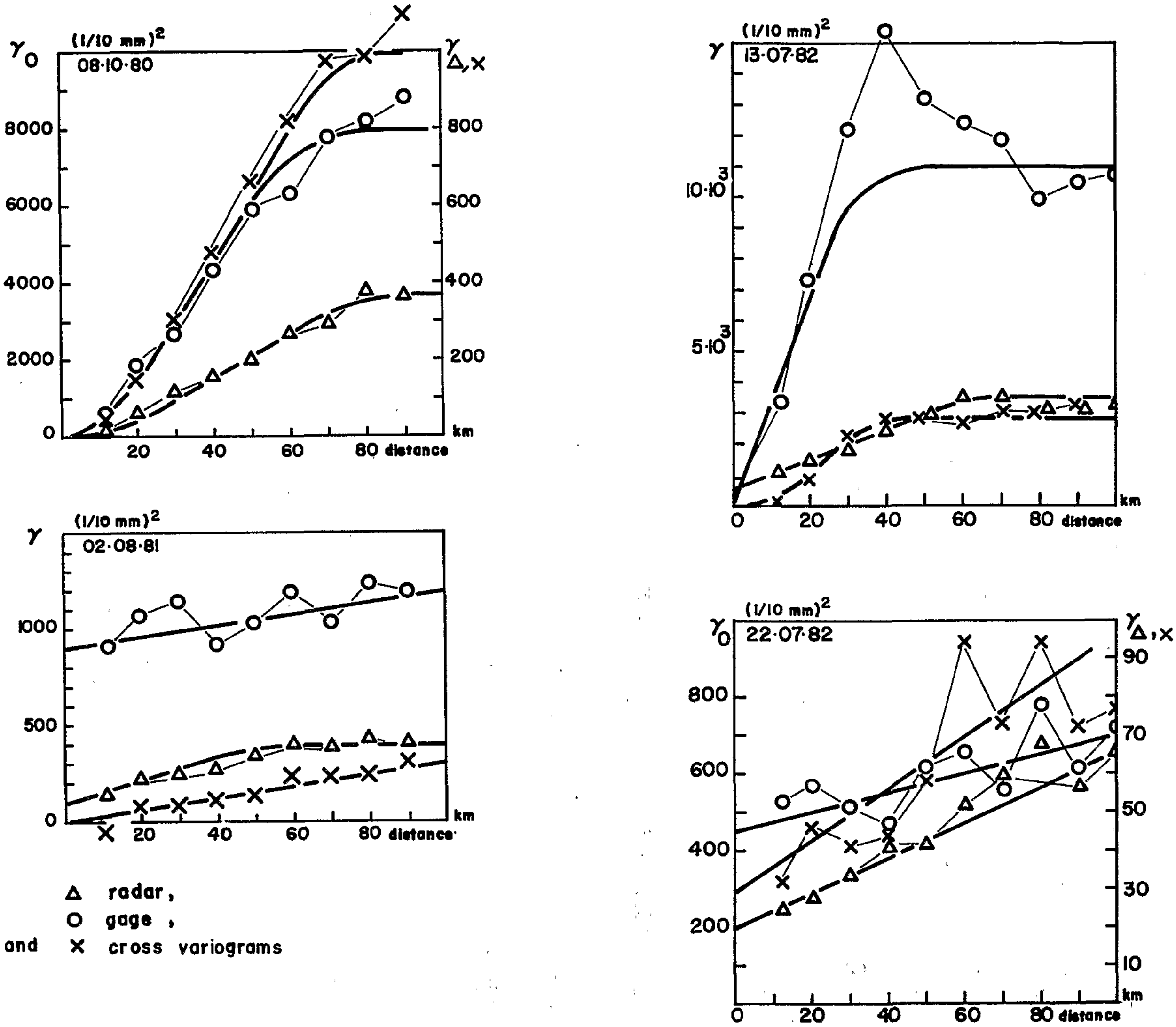

FIG. 2. Radar and gage direct and cross variograms for four typical days.

of the two measurement devices, a comparison of their corresponding variograms would be expected to show (i) good overall agreement since they basically measure to same phenomenon, and (ii) a longer range and a lower sill (i.e., variance) for radar measurements because of the integrating effect of remote sensing.

In practice, when radar and gage variograms are compared side by side (see selected examples in Fig. 3), the expected likeness between fitted models can be observed for most winter days (8 October 1980) and some summer days for which the convective situations have a sufficient spatial extent (13 July 1982). When the rainfall field results from very localized showers (other summer days such as 2 August 1981 or 22 July 1982 mapped below) the spatial resolution of radar gives, as expected, a more accurate structure identification, especially for short distances (e.g., in Fig. 3, direct and cross variograms of the 2 August 1981 fields show a range-type radar variogram while the gage variogram is almost a pure white noise effect). Comparing the range and sill values of the radar and gage variograms when they are existing, the spatial integrating effect of radar measurements can be seen.

b. Cross structure analysis

The cross variograms of gage and radar measurements have been calculated over the 98 measurement points where both gage and radar data are available. This is required for the mathematical expression of the cross variogram given in section 2b.

The results lead to the following comments:

- For four days (7 and 8 October 1980, 15 December 1980, and 13 July 1982) the direct and cross-variogram

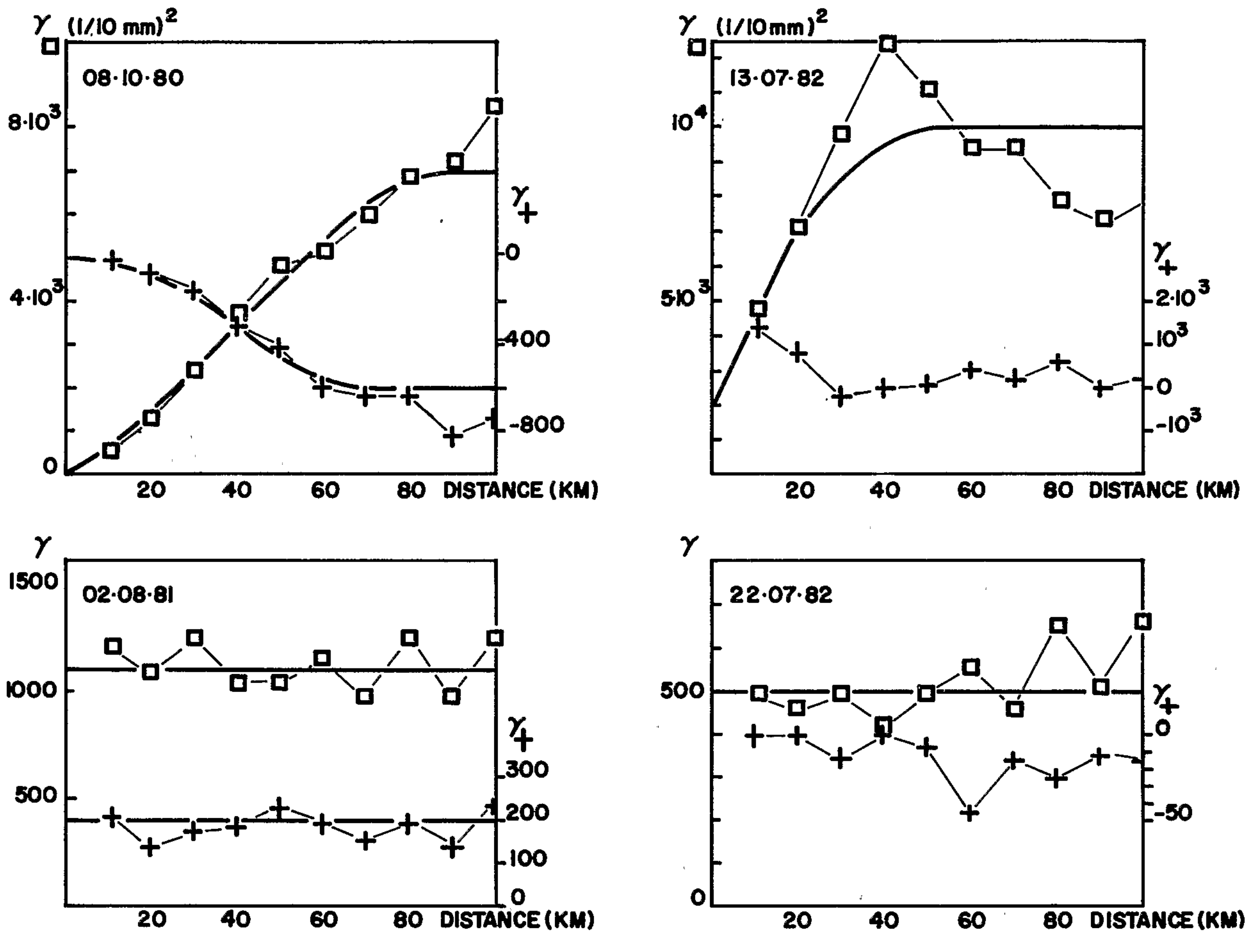

FIG. 3. Variograms of the residuals (open squares) and cross variograms between residuals and radar measurements (plusses) for four typical days.

models are almost homothetic. Consider the orthogonal decomposition of the function $z_G(x)$ [already proposed in (12)]:

$$z_G(x) = az_R(x) + b + \epsilon(x), \tag{20}$$

where $\epsilon(x)$ is a random function statistically independent of $z_R(x)$. In this case, it can easily be proven that $\gamma_{RG}(h)$ and $\gamma_R(h)$ are homothetic:

$$\gamma_{RG}(h) = a\gamma_R(h).$$

Furthermore

$$\gamma_G(h) = a^2\gamma_R(h) + \gamma_\epsilon(h) \tag{21}$$

where $\gamma_\epsilon(h)$ is the variogram of the function ϵ. If (as in our case), γ_G, γ_R and γ_{RG} are homothetic, (21) may be written as

$$\gamma_G(h) = (a^2 + e)\gamma_R(h),$$

denoting γ_ϵ as

$$\gamma_\epsilon(h) = e\gamma_R(h) \tag{22}$$

where e is a positive constant or zero.

As $a \leqslant \sqrt{a^2 + e}$, the variograms satisfy the following condition:

$$\gamma_{RG}(h) \leqslant \sqrt{\gamma_R(h) \cdot \gamma_G(h)}.$$

Equality is obtained when the function ϵ is a constant value, i.e., when z_G and z_R are linearly related.

The inequality is significant, for these four days, indicating strong spatial discrepancies between radar and raingage measurements.

- for three other winter days (10 October 1980, 14 December 1980, and 22 October 1982) the cross-variograms are equal to zero, indicating the statistical independence of z_R and z_G fluctuations.
- the four remaining convective days (2 August 1981, 23 June 1982, 15 and 22 July 1982) are very difficult to model. The strong white noise effects on the gage variograms indicate that the gage density is too low for this meteorological situation.

TABLE 2. Description of the variogram models fitted for the 11 available days.

Date of the event	Direct variogram of		Cross variogram of gages vs radar	Direct variogram of residuals
	gages	radar		
7 Oct 1980	S* 1800 50 0	S 80 60 0	S 150 60 0	S 1600 80 0
8 Oct 1980	S 8000 80 0	S 380 100 0	S 1000 80 0	S 7000 90 0
10 Oct 1980	S(1) 1200 60 0	S(3) 1200 100 0	W† 0	S 3200 100 0
14 Dec 1980	S(1) 500 80 0	S(3) 300 150 0	W 0	S 650 100 0
15 Dec 1980	L‡ 12 0	L 1.8 0	L 2.5 0	L 10 0
2 Aug 1981	L 3 900	S 400 70 100	L 3 0	W 1100
23 June 1982	L 6 300	S(1) 50 100 0	L 1 30	W 500
13 July 1982	S 11 000 50 0	S 3500 70 500	S 2800 50 0	S 10 000 50 0
15 July 1982	L 2.4 600	S 70 60 10	S 100 50 20	W 600
22 July 1982	L 2.5 450	L 0.45 20	L 0.65 30	W 500
22 Oct 1982	L 50 0	S 1250 100 0	W(3) 0	L 60 0

* S, with range: sill, range, white noise effect.
† W, pure white noise effect.
‡ L, linear: slope, white noise effect.

c. Residuals structure

Table 2 offers variogram models for the residuals $\epsilon(x)$. Figure 3 displays these variograms for four typical days. As for cross variograms, the three sets of days may be examined separately:

- When the direct and cross variograms are homothetic the variogram of $\epsilon(x)$ should be written as in (22). The fitted models satisfy this relation. In addition, the sill of these variograms is systematically lower than the sill of the gage variograms. This means that the error variance of the residuals will be lower than for the gages. For these days simplified cokriging should give better results than the interpolation using gages alone.
- When the cross variograms are equal to zero, the variograms of $\epsilon(x)$ show a higher variability than for the gages. No improvement should be expected from simplified cokriging over gage interpolation.
- When the gage density is too low, the variograms of $\epsilon(x)$ exhibit a pure white noise effect. In other words, the gage information should give no improvement over the radar measurements.

Finally, to test the validity of the hypothesis of orthogonality between the radar signal and the residuals, their experimental cross variograms were plotted. As shown in Fig. 4, orthogonality can be assumed since the cross structure functions present a clear white noise effect (e.g., 2 August 1981) or a very low variability.

5. Test results

The simplified cokriging method was applied to the described dataset. To judge the quality of the results

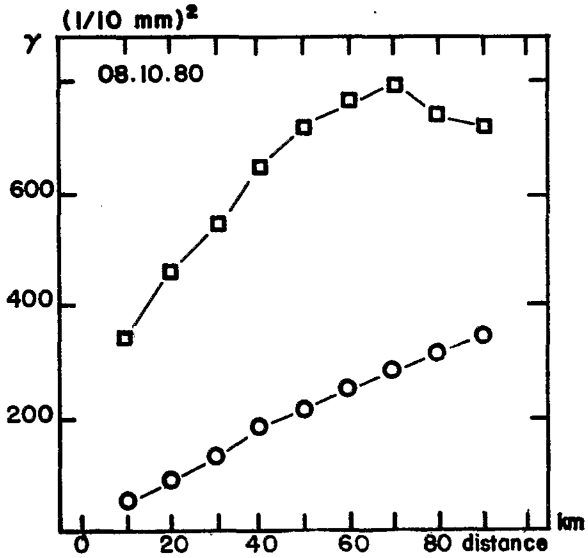

FIG. 4. Response of the radar data variogram to the presence of ground clutters. The computation of the variogram with squares includes ground clutters, while they are excluded from the computation of the variogram with circles.

obtained, a validation process was defined and a comparison was made with the results of either the radar alone or the interpolation of the merging network alone.

a. Validation process

When defining a validation process, two choices must be made. The first concerns the reference value, which should be as close as possible to the true value of the phenomenon. The second choice concerns the likeness indicator, measuring the distance between the reference value and the estimate provided by the validated device or method.

Different reference values have been chosen in the other studies already mentioned. Most authors consider the ground measurement as the standard value (Wilson and Brandes, 1979), possibly taking into account the effect of gage inaccuracy through practical comparison of mean areal rainfall amounts computed with various gage densities (e.g., Woodley et al., 1975; Hildebrand et al., 1979). Other authors make the strong assumption of good qualitative identification of rainfall patterns by radar and use a "radar-derived" value for the actual rainfall (e.g. Harrold et al., 1974).

To avoid the drawback of this last approach, which includes radar data in the standard, and to take into account the effect of gage inaccuracy, the reference values considered in this study are computed by integration over radar pixels (5 × 5 km) containing one test raingage (i.e., the gage is not used for merging purposes) by means of an optimal interpolation technique (here kriging—the simple reduction of cokriging to a single random function) relying on the test network. This procedure satisfies two objectives:

(i) Integration of the point measurements of ground rainfall over the sampling area of radar measurements leads to more comparable values. The chosen targets remain sufficiently small and scattered in space to validate the radar perception of the spatial variations of the rainfall fields.

(ii) The reference values are independent of the ground measurements used for merging since the merging and test networks are distinct.

Concerning the second choice, two different sets can be distinguished among the range of available likeness indicators:

1) Indicators reflecting both the mean adequacy and the cofluctuation of the compared values. Among these indicators, we have selected in the following comparison the percent mean absolute error:

$$e_4 = \frac{100}{n_d} \sum_{d=1}^{n_d} \Big[\sum_{t=1}^{n_t} |z_T(t,d) - z_G^*(t,d)| / \sum_{t=1}^{n_t} z_G^*(t,d) \Big]$$

and the distribution of the so-called factors of difference:

$$\max[z_T(t,d)/z_G^*(t,d), z_G(t,d)/z_T^*(t,d)]$$

as proposed by Woodley et al., 1975, where $z_T(t, d)$ represents the reference and $z_G^*(t, d)$ the estimated value for target t and day d. The first indicator provides a general quality index, and the second visually displays the evolution of the reconstitution quality in more detail.

2) Indicators sensitive only to the cofluctuation of the compared values, such as the correlation coefficient between reference and estimated values. This coefficient remains unchanged when a linear transformation is performed over a given set of values (for instance when calibrating radar measurements using a constant multiplicative factor). It has therefore been used day by day in the following comparison to evaluate the cofluctuation of two signals in space alone. Note that this day to day correlation does not take into account the time cofluctuation, which can be very strong if the mean characteristics of the various days considered differ significantly. It is therefore obvious that a day by day correlation is more selective in determining what should be the main quality of radar measurements, i.e., a good appreciation of the spatial variability.

b. Comparison of the various results obtained

Two sets of results were obtained for two merging gage network densities. Twenty-nine gages were used in the first (i.e., a density of one gage per 700 km²) and ten gages in the second (i.e. one gage for 2000 km²). Each set is composed of (i) raw radar measurements, only preprocessed to eliminate ground clutters, (ii)

TABLE 3. Global percent mean-square error and correlation coefficients between reference and estimated rainfall values for the three compared methods.

Measurement method	Percent mean absolute error		Correlation coefficient*	
	10 gages	29 gages	10 gages	29 gages
Raw radar	64.6	64.6	.61	.61
Gage interpolation	27.3	19.8	.50	.76
Simplified cokriging	25.2	22.2	.63	.76

* To avoid time-cofluctuation effects, reference and estimated values have been centered day by day.

rainfall depth estimates of the merging gages using kriging, and (iii) results of the gage–radar combination through simplified cokriging.

A general comparison of the results obtained for each method is given in Table 3, based on the percent mean absolute errors and the correlation coefficient calculated over all the available data, and in Fig. 5 which displays the corresponding distributions of the factors of difference.

In examining each basic device separately, gages appear to offer a better overall performance than radar for both network densities. This observation must be considered in the light of two points: (i) the daily time step is long enough to favor gage performance and (ii) the distance from the radar site is a major handicap for radar performance here. In addition, when the effect of systematic errors is removed (e.g., by correlation coefficient computation after day by day centering) the performance of radar results falls between those of the two network densities.

The combination of these devices through simplified cokriging improves the overall results of each separate device. This improvement, however, is very slight, especially for a dense ground network.

A day by day display of the results of each method is offered in Fig. 6. The squares of the correlation coefficients (i.e., the explained variance) between reference and estimated values over the 69 test targets throw a different light on the results, pointing out the spatial agreement of the compared values.

For convective meteorological situations (summer days), radar shows more steady performance than the ground network. The merging process rarely improves the radar measurements, especially for the low density network.

During frontal situations (fall and winter days), radar results are systematically worse than gage interpolation results. The merging process works effectively when the ten-gage network is considered, proving its capability to combine the complementary possibilities of the two devices.

Based on this correlation criterion, the calibration method using a spatially constant multiplicative factor does not improve raw radar performance. Simplified cokriging, therefore, appears to upgrade the results of this simplest calibration method systematically.

6. Conclusion

The methodology presented herein is obviously only a preliminary approach to the problem of raingage–

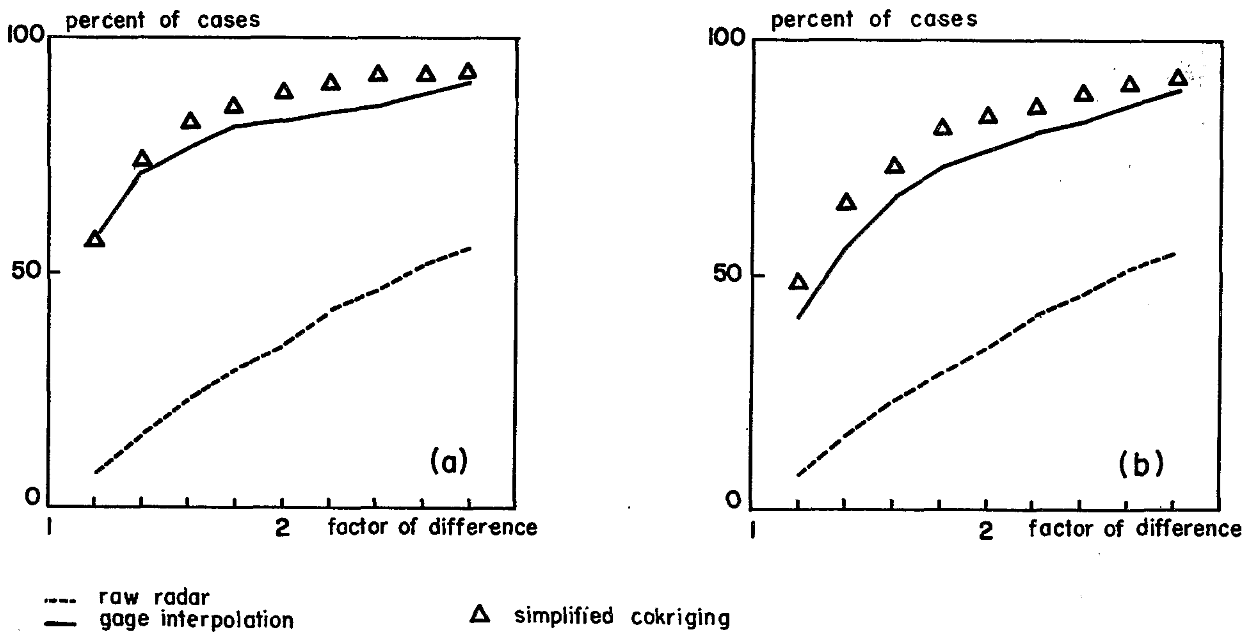

FIG. 5. Distributions of the so-called factors of differences for the two merging network densities. (a) 29 and (b) 10 gages.

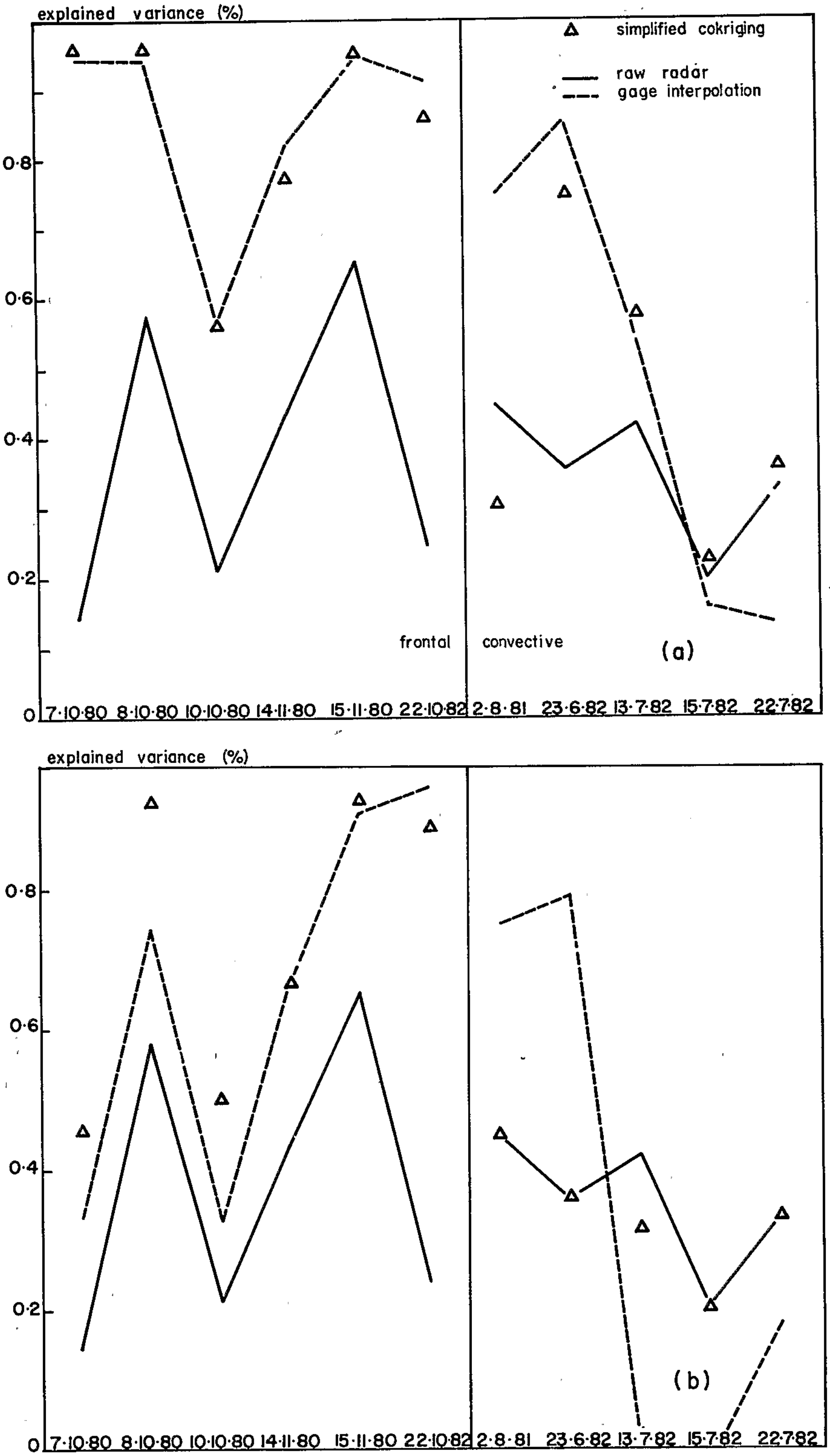

FIG. 6. Day by day correlation coefficients between the values of ground rainfall depth integration over small 5 × 5 km targets and the results of the three compared methods for two merging network densities. (a) 29 and (b) 10 gages.

radar merging. Further efforts will have to be made to take into account, more explicitly, the errors inherent in the two devices combined. Several possibilities are available in the proposed model to deal with such corrections. For instance, (i) the linear relation giving $\hat{z}_R(x)$ by regression can be replaced by a more complex relation, or (ii) more sophisticated assumptions can be made concerning the mean behavior of the residuals $\epsilon(x)$.

In its present form, the simplified cokriging method already offers promising properties:

(i) The proposed simplification drastically reduces the size of the system, making it tractable in an operational context.

(ii) The obtained estimator satisfies the classical mean-square-error minimization criterion provided that certain relatively weak assumptions can be made. In spite of an unfavorable dataset, the results obtained are satisfying in comparison with conventional radar calibration using gages. Further applications of this method (in progress) using a more appropriate dataset (dense network of recording raingages near the radar site) are necessary to confirm these initial results.

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Rain Measurement by Raingage–Radar Combination: A Geostatistical Approach

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