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On the Accuracy of Areal Rainfall Estimation: A Case Study

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The issue of the accuracy of areal rainfall estimation is discussed through a case study on a catchment area in the Cévennes region of France. The basic tool for the analysis is a "scaled estimation error variance" which is computed from a scaled climatological variogram model of the rainfall field. We show how this variance provides a theoretical criterion to compare the accuracy that can be expected with three linear estimators (Thiessen, spline, and kriging) for various networks' densities. To support the methodology an experimental validation of the scaled estimation error variance computation is performed, using "reference" areal rainfall values computed with a very high density network.

1. Introduction

Good estimates of mean areal rainfall are needed as inputs to hydrologic models. When based on ground measurements, their accuracy depends on the spatial variability of the rainfall process and on the rain gage network density. Accuracy is of particular importance when areal rainfall estimates must be computed in real time as inputs to runoff forecasting models. The high cost of equipping and maintaining the required telemetering facilities, especially in mountainous regions, often results in low-density observation networks.

Making the most of a given network (whether telemetered or not) is a prime concern shared by all operators and hydrologic designers. Consequently, the interpolation of data collected from scattered rain gages has long been an important research topic in hydrology. *Creutin and Obled* [1982] have compared the performances of commonly used linear estimators. Their study shows that for low to medium density networks sophisticated methods (e.g., spline surface fitting or kriging) give better results than simpler conventional methods (e.g., Thiessen polygons or arithmetic means). Furthermore, various advantages and disadvantages are outlined for each method, providing a basis for evaluating their applicability to specific hydrologic problems. More recently, a similar comparison was applied to annual rainfalls by *Tabios and Salas* [1985], yielding results consistent with those of Creutin and Obled.

However, it is often more useful (but also more difficult) for a hydrologist to evaluate the error involved in areal rather than in point rainfall estimation. The theoretical error variance of any linear areal rainfall estimator can obviously be computed under the assumption that the observations are a realization of a random field with a given covariance model. However, a major difficulty arises in validating the calculated theoretical variance from experimental data, since no direct measurement of the areal rainfall is available.

The main objective of this paper is precisely to perform such an experimental validation through a case study on a catchment area equipped with a very dense rain gage network. The validated error variance is subsequently used as a tool to assess the performance of three different linear estimators (Thiessen polygons, spline surface fitting, and kriging) with varying network densities. These linear estimators are briefly described in section 2. Section 3 shows how the "climatological variogram" concept leads to the computation of a scaled variance of the estimation error for any linear estimator. Sections 4–6 deal with a case study on the Gardon d'Anduze watershed in the Cevennes region (France). Section 4 includes a description of the data and gives a brief account of an extended identification study previously carried out to determine the structure of the climatological variogram in this region. In section 5 we propose that the areal rainfall computed from a very dense network using the Thiessen method [*Thiessen*, 1911] can be considered as a reference value to which estimation from less dense networks can be compared. Finally, the main contribution of this paper (section 6) is the description of a procedure for the experimental validation of the normalized error variance calculation. This normalized variance can then be used as a tool to assess the accuracy of the different estimators and the influence of the network density.

2. Linear Estimators of Areal Rainfall

The areal rainfall over an area S is commonly defined as

$$Z_k^s = \frac{1}{s}\int_S Z_k(x, y)\,dx\,dy \qquad (1)$$

where $Z_k(x, y)$ denotes the point rainfall depth at the point (x, y), for the kth time interval of duration θ, and $s = |S|$. This quantity Z_k^s is obviously unknown, since the rainfall depth is accessible only at a finite number (say, n) of scattered pointwise observations. It is therefore common practice in hydrology to estimate Z_k^s using linear estimators of the form

$$\hat{Z}_k^s = \sum_{i=1}^{n} \lambda_i Z_k^i \qquad (2)$$

i.e., as a weighted mean of the random variables $Z_k^1, Z_k^2, \cdots, Z_k^n$ observed at the rain gages. The three linear estimators we compare (Thiessen polygons, spline surface fitting, and climatological kriging) differ from one another in the values of the

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individual weighting coefficients λ_i. However, in all three cases,

$$\sum_{i=1}^{n} \lambda_i = 1 \qquad (3)$$

In this section the three estimators are briefly described without reference to their accuracy. This issue will be addressed in section 3.

2.1 *Thiessen Polygons*

In this method [*Thiessen*, 1911] the watershed S is divided into n zones of influence S_i, one for each rain gage. The zone of influence of a rain gage is defined by those points which are closer to that gage than to any other station. The weighting coefficients λ_i are then computed as

$$\lambda_i = s_i/s \qquad i = 1, \cdots, n$$

where $s_i = |S_i|$. This method is very familiar to hydrologists and can provide good estimations with dense networks. It does not, however, allow contour mapping.

2.2. *Spline Surface Fitting*

The spline surface fitting method was primarily developed for contour mapping purposes. It is basically a method for interpolation between scattered pointwise observations which avoids the drawback of uncontrolled oscillations arising when polynomial interpolation is used. For this purpose, it obeys an optimal smoothness criterion (see, for example, *Ahlberg et al.*, [1967]). In a two-dimensional space, this criterion is the minimization of the bending energy of a thin elastic sheet supported by the pointwise observations [*Duchon*, 1976].

If we denote the spline surface fitted to the observations of the kth rainfall event by $\hat{Z}_k(x, y)$ then it is natural to approximate the integral (1), i.e., the areal rainfall, by

$$Z_k^s = \frac{1}{s}\int_S \hat{Z}_k(x, y)\, dx\, dy \qquad (4)$$

It is shown in the appendix that $\hat{Z}_k^s$ given by (4) can be written as a linear estimator (2) if appropriate values of the weighting coefficients λ_i are computed by solving a $(n + 3) \times (n + 3)$ algebraic linear system.

2.3. *Climatological Kriging*

Kriging is a method of computing the weighting coefficients λ_i in such a way that $\hat{Z}_k^s$ is a minimum variance estimator of the random variable $Z_{k,}$ under the assumption that the observed rainfall events are realizations of a two-dimensional random field. The pioneers of kriging applications in hydrometeorology were *Delfiner and Delhomme* [1973], *Delhomme and Delfiner* [1973], and *Delhomme* [1978]. A number of years elapsed before hydrologists became widely acquainted with this method. Recently, the theory has been further developed and applied to rainfall related problems [*Chua and Bras*, 1982; *Creutin and Obled*, 1982; *Kitanidis*, 1983; *Bastin et al.*, 1984; *Lebel and Bastin*, 1985; *Tabios and Salas*, 1985]. Since kriging is a linear minimum variance estimation method, it requires knowledge of the mean and covariance function of the random field.

In applications to real-time estimation of areal rainfall, previous studies [*Bastin et al.*, 1984; *Lebel*, 1984; *Lebel and Bastin*, 1985] have led to the development of "climatological kriging," i.e., kriging with a "climatological variogram" as the covariance function. The climatological variogram concept is presented in some detail (definition, advantages, identification, illustrations) in the next section. With this approach, the weighting coefficients λ_i are computed by solving an algebraic linear system (see the appendix).

3. Computation of the Estimation Error Variance of Linear Estimators

In the previous section, the Thiessen and spline linear estimators were presented within the deterministic framework in which they are commonly used by hydrologists today. In that framework, these methods obviously provide no measure of their accuracy. To provide such a measure, the estimation process must be considered in a probabilistic context. Accordingly, we shall henceforth assume that rainfall observations are realizations of a two-dimensional random field. This point of view allows computation of the estimation error variance of any linear estimator, whatever the method used to compute the weighting coefficients λ_i:

$$(\sigma_k^s)^2 = \mathrm{Var}\,(\hat{Z}_k^s - Z_k^s) = \mathrm{Var}\,(\textstyle\sum \lambda_i Z_k^i - Z_k^s) \qquad (5)$$

which is also written

$$(\sigma_k^s)^2 = \sum_i \sum_j \lambda_i \lambda_j \,\mathrm{Cov}\,(Z_k^i, Z_k^j) + \mathrm{Var}\,(Z_k^s) - 2\sum_i \lambda_i \,\mathrm{Cov}\,(Z_k^i, Z_k^s)$$

It is clear, from this expression, that practical computation of $(\sigma_k^s)^2$ requires knowledge of the random field covariance functions which are not given a priori in most applications. Therefore the preliminary inference of a model of this covariance function from the rainfall observations is the topic of the next section.

3.1. *Climatological Variograms*

One possible method involves separate identification of the covariance function for each realization of the field, i.e., at each time index k. This approach has been followed by *Delfiner and Delhomme* [1973], *Chua and Bras* [1982], and *Kitanidis* [1983]. In our opinion, however, it has two main drawbacks as follows.

1. Most often, a large number of field realizations have been observed and are available for the inference of the covariance function. By treating each realization separately, one makes only very partial use of the global statistical information contained in the whole data set.

2. A careful determination of the random field structure function at each time step may be too time consuming for real-time operation with short time steps. Furthermore, reliable values of the model parameters cannot be obtained from a small number of data points (less than around 15–20).

On the other hand, it would, of course, be unrealistic to adopt a unique covariance function model for all rainfall events irrespective of the season, meteorological conditions, and rainfall intensity. On the basis of several previous investigations [*Creutin and Obled*, 1982; *Bastin et al.*, 1984; *Lebel and Bastin*, 1985], it appears that a reasonable trade off is to adopt an analytical variogram model of the form

$$Y(h; k) = \alpha(k) Y^*(h, \beta) \qquad (6)$$

where h is the Euclidian distance, $\alpha(k)$ is a scaling parameter, and β is a shape parameter. With this structure, all time nonstationarity (i.e., dependence on the time index k) is concentrated in the scale factor $\alpha(k)$, while the component $Y^*(h, \beta)$ (which we call the "scaled climatological variogram") is time invariant.

In a region of relatively regular weather patterns, *Bastin et al.* [1984] successfully used a single climatological variogram Y^* for the estimation of areal rainfall throughout the year. In such a case the scaling factor $\alpha(k)$ mainly reflects the seasonal variation of the spatial structure of the rainfall field. In regions where the climatic variability is stronger (as for the case study of this paper), a unique climatological variogram $Y^*(h, \beta)$ is used only for storms issuing from the same kind of weather conditions. The parameter $\alpha(k)$ then mainly accounts for the scale effect due to the variation in time of the mean rainfall intensity. When the variogram is bounded we can impose, without loss of generality, that

$$\lim_{h\to\infty} Y^*(h, \beta) = 1$$

$\alpha(k)$ is then the variance of the kth field and $Y^*(h, \beta)$, the unique variogram of all the scaled random fields defined by

$$Z_k^* = Z_k/(\alpha(k))^{1/2} \qquad k = 1, \cdots, K \tag{7}$$

We can thus proceed to the identification of a model of the climatological variogram Y^* of Z^* by mixing the realizations of all the fields Z_k^* together. This inference is performed using a "mean-squared interpolation error" (msie) criterion [*Lebel and Bastin*, 1985] and is based on a much larger data set than the one which would have been used in a single-realization context. An example relative to our case study will be briefly described in section 4. An advantage of this approach is that the coefficients λ_i of the kriging estimation are independent of $\alpha(k)$. Hence they depend only on the scaled climatological variogram $Y^*(h, \beta)$. Therefore they can be computed once and for all, as for the Thiessen and spline estimators (note that if the climatological variogram is not used, and if the variogram is identified in real time, then the coefficients λ_i should also be recomputed at each time index k).

3.2. *Using a Scaled Estimation Error Variance to Measure the Estimation Accuracy*

A byproduct of the climatological variogram approach is the possibility of computing a scaled variance of estimation error $(\sigma_u^s)^2$ that can be used as a global (i.e., not relevant to a single event) comparative index of the accuracy of the areal rainfall estimation for various network densities. As a matter of fact, it is easy to show [*Lebel and Bastin*, 1985] that the estimation error variance (5) of any linear estimator can be written

$$(\sigma_k^s)^2 = \alpha(k)(\sigma_u^s)^2 \tag{8}$$

with

$$(\sigma_u^s)^2 = \frac{1}{S}\sum_{i=1}^{n} \lambda_i \int_S Y^*(\overline{u_i u}, \beta)\, du - \frac{1}{S^2}\int_S \int_S Y^*(\overline{uu'}, \beta)\, du\, du' + \mu \tag{9a}$$

where u_i is the location of rain gage i, u and u' are current points in S, $\overline{u_i u}$ is the Euclidian distance between u_i and u, and μ is the Lagrange multiplier (in the case of zero-order drift). In practice, the integrals in (9a) are computed using the following discrete approximation:

$$(\sigma_u^s)^2 = \frac{1}{S}\sum_{i=1}^{n} \lambda_i \sum_{l=1}^{L} Y^*(\overline{u_i u_l}, \beta) - \frac{1}{S^2}\sum_{l=1}^{L}\sum_{m=1}^{M} Y^*(\overline{u_l u_m}, \beta) + \mu \tag{9b}$$

Given expression (8), $(\sigma_u^s)^2$ is the ratio between the areal rainfall estimation error variance and the field variance.

Once the climatological variogram model has been chosen (i.e., once the value of β has been chosen), the scaled estimation error variance $(\sigma_u^s)^2$ can be viewed as depending exclusively on the number and the locations of the rain gages. Therefore $(\sigma_u^s)^2$ is an efficient tool for solving rain gage network optimization problems such as the optimal choice of rain gage locations [*Bastin et al.*, 1984]. Similarly, it will be used in this paper as the basic criterion for comparing the three linear estimators and for analyzing the influence of the network density on the estimation. Of course, the validity of this criterion must be checked, since by definition the kriging estimation error variance is lower than that of the other two methods. It must be further emphasized that $(\sigma_u^s)^2$ is not the variance of the actual areal rainfall estimation error (as can be seen from expression (8)), but provides a theoretical measure of the relative accuracies of the various estimates. This is why in the case study below these theoretical results, i.e., the computation of $(\sigma_u^s)^2$, will be accompanied by an experimental verification. In this case study we intend to (1) assess the magnitude of the increase in accuracy that can be expected in practice when using kriging, (2) evaluate how $(\sigma_u^s)^2$ increases as the network density decreases, and (3) experimentally verify the theoretical values computed in these two previous steps; this will test the reliability of climatological variogram inference and to a larger extent the climatological approach as a whole.

4. CASE STUDY: CLIMATOLOGICAL VARIOGRAMS IN THE CEVENNES REGION

Thirty-four recording rain gages were used to study the areal rainfall over the Gardon d'Anduze watershed (Figure 1).

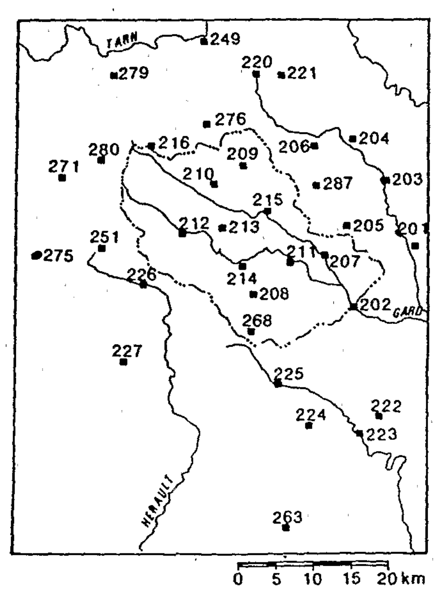

Fig. 1. Recording rain gage network over the Gardon d'Anduze watershed.

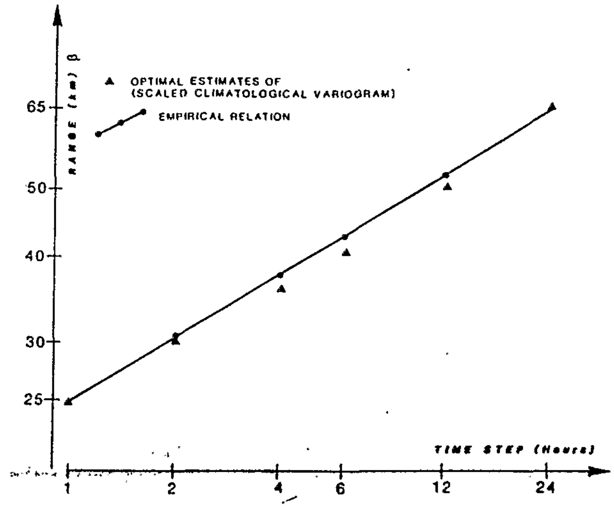

Fig. 2. Fitting an empirical relation between the rainfall duration of accumulation and the range of the spherical variogram model.

The basic data were the strongest hourly autumn rainfalls between 1971 and 1980. These rainfalls were accumulated to obtain 2, 4, 6, 12, and 24 hour rainfalls. Almost every severe flashflood in this region takes place during the fall, and it has been shown [*Tourasse*, 1981] that the high intensities (up to 100 mm of water depth in 1 hour) are the result of similar meteorological patterns. Thus the climatological approach of inferring the variogram seems very well-suited to this case.

For each of the six time steps (1, 2, 4, 6, 12, and 24 hours), a careful identification study was carried out using the MSIE method, including an extensive cross-validation. A detailed description can be found in the work by *Lebel* [1984] and *Lebel and Bastin* [1985]. We give only the main results here as follows.

1. A locally constant drift model and a spherical isotropic scaled climatological variogram model were selected; it should be emphasized that this structural choice (stationary drift, bounded and isotropic variogram...) is not arbitrary, but results from a careful analysis (this point will be further commented in section 7).

2. The spherical variogram model is of the form

$$Y^*(h, \beta) = \frac{1}{2}\left\{\left(3\frac{h}{\beta}\right) - \left(\frac{h}{\beta}\right)^3\right\} \qquad 0 \le h \le \beta$$

$$Y^*(h, \beta) = 1 \qquad h > \beta$$

where β is the range of the variogram. The optimal estimates of β arising from the identification study are illustrated in Figure 2, along with the empirical relation

$$\beta(\theta) = 25\theta^{0.3}$$

that has been derived from these values. This relation allows computation of β (and hence of the areal rainfall) at time steps for which the variogram was not inferred.

3. As is shown in the previous section, $\alpha(k)$ is the field variance (since the variogram is bounded). This field variance was found to be well-estimated by the sample variance of each realization, i.e.,

$$\hat{\alpha}(k) = \frac{1}{n}\sum_{i=1}^{n} |z_k^i - m(k)|^2$$
$$m(k) = \frac{1}{n}\sum_{i=1}^{n} z_k^i \qquad (10)$$

An example of the scaled climatological variogram model corresponding to the time step $\theta = 1$ hour is shown in Figure 3. It can be seen that the model fits the experimental scaled variogram very well. According to (7) this experimental variogram was obtained from the accumulation of 103 scaled field realizations:

$$Y^*(h_{ij}) = \frac{1}{2}\sum_{k=1}^{K} \{(z_k^i)^* - (z_k^j)^*\}^2$$

where $(z_k^i)^* = z_k^i/(\hat{\alpha}(k))^{1/2}$ and z_k^i is the observed value of Z_k^i. For further details, see *Lebel and Bastin* [1985].

5. COMPUTATION OF AREAL RAINFAL REFERENCE VALUES

Using the dense network of Figure 1, hourly areal rainfalls were computed with the three linear estimators. The basic

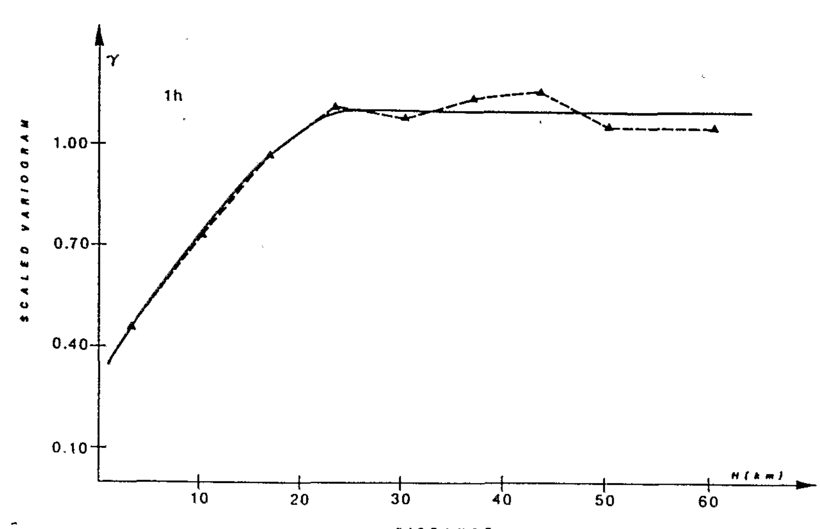

Fig. 3. Fitting of an analytical model to a climatological experimental variogram.

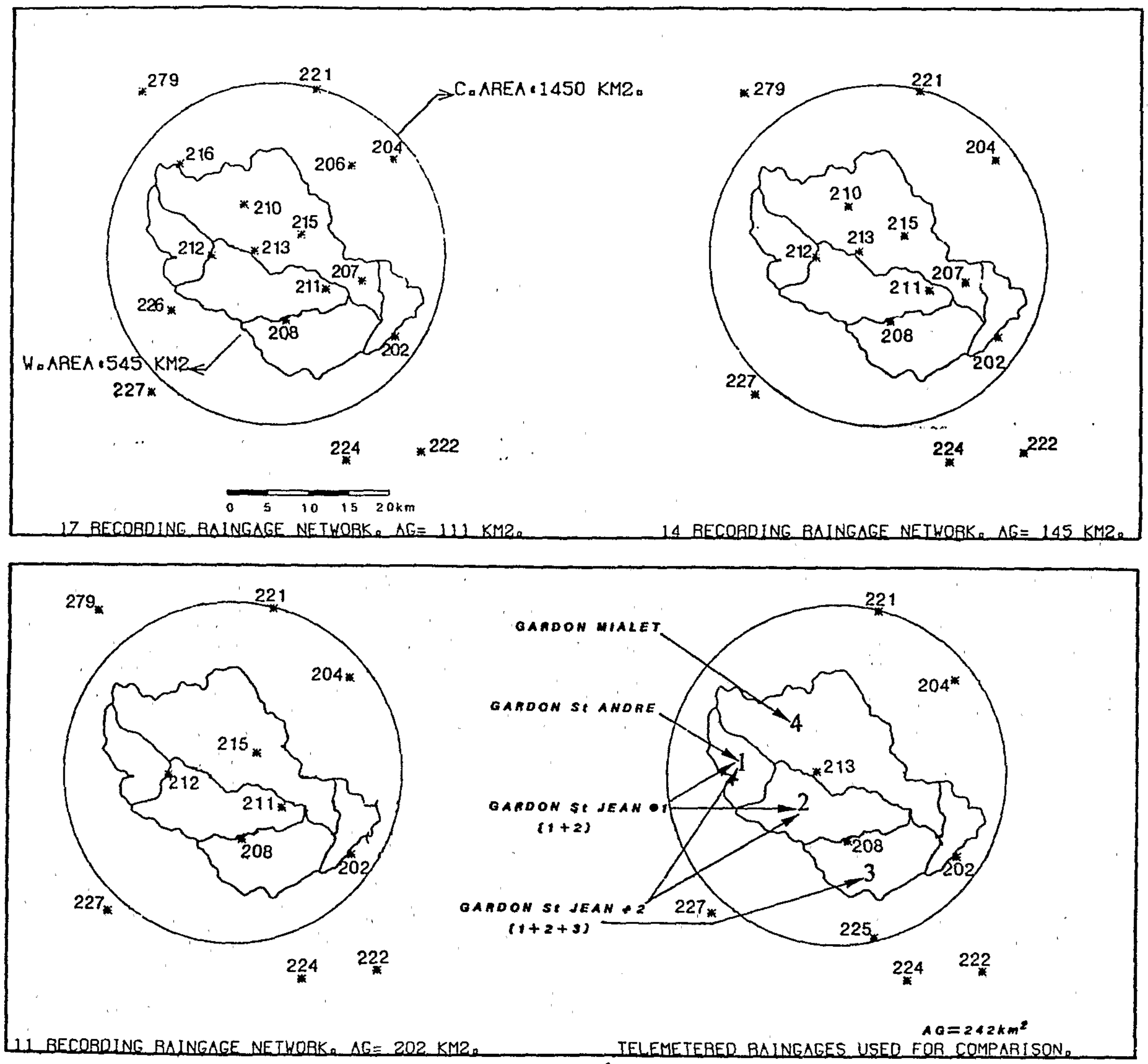

Fig. 4. Subwatersheds of the Gardon d'Anduze watershed and the network of varying density used to evaluate the decrease in accuracy of areal rainfall estimation resulting from a lowering of the network density.

sample used to compare the three estimators is made up of the 103 hourly events that allowed identification of the climatological variogram (section 4). In addition, in order to account for the influence of the surface area, the main watershed of 545 km² was divided into subwatersheds as shown in Figure 4.

It was found that the three estimators are very close to each other and highly correlated (Table 1), especially for large catchments. The explanation is given in Table 2, where it can be seen that the weighting coefficients are very similar whatever the estimator. Furthermore, the value of the scaled variance $(\sigma_u^s)^2$ was found to be 0.01 for the Gardon d'Anduze watershed. According to (8), this means that the scaled variance of the estimation error is only 1% of the field variance, whatever the estimator used. Since all three methods yield equivalent estimates and the theoretical error involved is fairly low, we will consider the areal rainfall $(z_k^s)^T$ computed by the

TABLE 1. Correlation Coefficients r Between the Three Estimates of Hourly Areal Rainfall Over the Main Watershed (545 km) and Subwatersheds (Dense Network)

Watersheds, km²	Gardon St. André, 53	Gardon St. Jean No. 1, 165	Gardon St. Jean No. 2, 265	Gardon Mialet, 237	Gardon Anduze, 545
Kriging spline	0.96	0.99	0.99	0.99	0.99
Kriging Thiessen	0.97	0.99	0.99	0.99	0.99
spline Thiessen	0.96	0.99	0.99	0.99	0.99

TABLE 2. Weighting Coefficients of the Three Linear Estimators of Hourly Areal Rainfall Over Two Watersheds (18 Largest Values of 34)

Gardon Anduze, 545 km²				Gardon Mialet, 237 km²			
Rain Gage	Thiessen	Spline	Kriging	Rain Gage	Thiessen	Spline	Kriging
212	128	134	129	210	189	190	190
211	84	83	83	215	189	185	186
207	73	83	69	209	189	156	144
210	82	82	77	216	113	106	115
202	75	77	72	207	109	94	99
215	82	73	76	213	80	60	58
208	67	73	68	212	29	56	62
209	77	64	58	287	29	46	35
268	58	61	54	211	29	45	12
214	75	59	69	276	21	38	19
216	56	57	53	206	0	20	7
213	60	49	67	205	8	17	21
226	18	24	27	280	0	12	8
251	3	20	25	279	0	−11	−2
280	7	18	17	251	0	11	4
205	22	18	27	204	0	−10	3
287	14	14	13	226	0	−10	7
276	9	13	24	208	0	−9	−2

Weighting coefficients are times 1000.

TABLE 4. Scaled Variance of Hourly Areal Rainfall Estimation Error

	No. of Stations			
	17 (112)*	14 (145)*	11 (207)*	Telemetered (242)*
Gardon a St. Andre (53 km²)				
Kriging	0.152	0.221	0.230	0.563
Spline	0.234	0.274	0.298	0.899
Thiessen	0.194	0.327	0.327	0.814
Gardon a St. Jean (No. 1, 165 km²)				
Kriging	0.040	0.054	0.063	0.148
Spline	0.060	0.097	0.080	0.203
Thiessen	0.047	0.067	0.078	0.210
Gardon de Mialet (237 km²)				
Kriging	0.028	0.040	0.113	0.185
Spline	0.053	0.059	0.132	0.235
Thiessen	0.035	0.063	0.153	0.289
Gardon de St. Jean (No. 2, 265 km²)				
Kriging	0.033	0.039	0.042	0.080
Spline	0.060	0.068	0.057	0.100
Thiessen	0.039	0.049	0.054	0.110
Gardon D'Anduze (545 km²)				
Kriging	0.015	0.020	0.039	0.074
Spline	0.020	0.030	0.049	0.091
Thiessen	0.020	0.031	0.055	0.108

*Area per gage, km².

Thiessen [1911] method with the dense network of Figure 1 as the "true" reference value to which estimations from less dense networks will be compared.

6. Estimation Accuracy Versus Network Density

6.1. *The Telemetered Network*

In the early 1970s, the decision was made to set up a flood warning and forecasting system over the Cevennes Region, to which the Gardon d'Anduze watershed belongs. Presently, 13 telemetered stations are available in the upstream watersheds, nine of which are located within or not far from the Gardon d'Anduze watershed (Figure 4). It is of major importance to assess the loss of information involved when using the telemetered network for real-time computation of areal reainfalls as compared to the high-density network. Because the density of the telemetered network is fairly low compared to that of the entire network, three other intermediate density networks were considered (Figure 4). The network density is defined as the area (1450 km²) of a circle containing the main watershed, divided by the number of gages located inside this circle. An important point here is that the information is collected not only from inside the watersheds, but from close outside areas as well. The stations used for the areal estimation cover the main watershed and the surrounding area, preventing border effects from influencing the estimation. As can be seen in Table 3, for few 1-hour events, substantial differences may exist between the results of the three estimation methods when using the telemetered networks, thus justifying concerns about the accuracy of the estimation and the need to examine it for each method.

TABLE 3. Areal Rainfall Estimation (1/10 mm) Over the Gardon St. André Watersheds (53 km²), Storm of September 12, 1976

	Hour					
	4–5	5–6	6–7	7–8	8–9	9–10
Dense Network						
Thiessen	0	2	179.4	151	13	11.5
Telemetered Network						
Kriging	27	134	272	240	124	80
Spline	10	118	446	466	52	28
Thiessen	20	250	470	505	175	30

6.2. *Theoretical Variances of Estimation Error*

The scaled variances of estimation error computed using (9*b*) provide overall comparison criteria, regardless of the magnitude of a given event. The results of these computations are summarized in Table 4 and illustrated in Figure 5. We can draw the following conclusions.

1. Whatever the method considered, the estimation error variance increases in a fairly regular way as the network density decreases; it tops at 90% of the scaled field variance when the spline estimate is used with the telemetered network on the smallest watershed.

2. The spline estimate is, in general, not much better than the Thiessen estimate [*Thiessen*, 1911] even though it is often considered as a more sophisticated (hence more accurate) method. For several small watersheds, the spline estimates are even less accurate than the Thiessen estimates.

3. By contrast, one can observe the large differences that always exist between kriging and the other estimates, especially when it comes to low-density networks. This latter observation, however, does not necessarily lead to the conclusion that climatological kriging is better. It can simply reflect the fact that the estimation error variances of Table 4 were computed with the same variogram used to estimate the kriging weighting coefficients (hence biasing the results in favor of kriging).

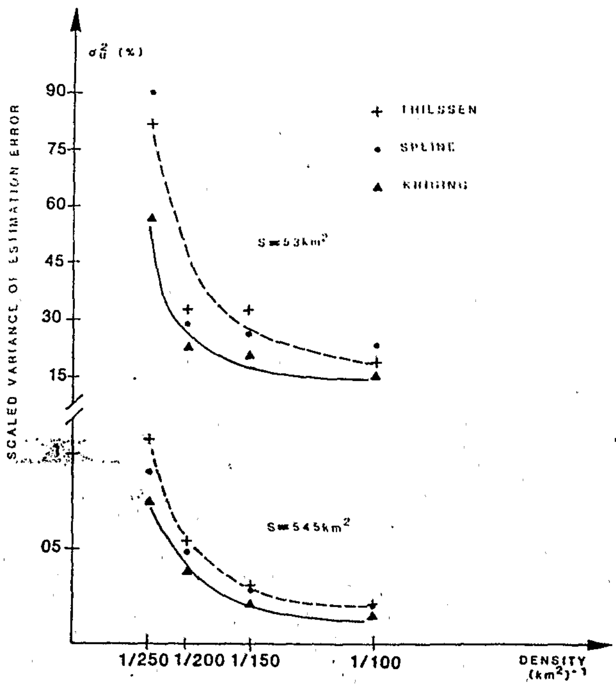

Fig. 5. Scaled estimation variance of hourly areal rainfall as a function of watershed area for various network densities.

Actually, the conclusion that kriging is better (in a way that can be evaluated with Table 4) holds only if the theoretical variance $(\sigma_u^s)^2$ computed from the variogram model is a realistic measure of the actual estimation variance (whatever the estimator considered). This is the reason why we believe that an experimental validation analysis based on the "true" reference values defined in section 5 is needed to check the validity of the results of Table 4. This validation analysis will be the main point of this section.

6.3. *Experimental Validation*

Experimental validation was carried out in two ways: (1) by computing correlations between the true reference values $(z_k^s)^T$ (section 5) and the low-density network estimates z_k^s on a sample of hourly rain events and (2) by counting the number of times that the reference value belongs to the theoretical confidence interval (CI) of the estimate. Computation of correlation coefficients allows an a posteriori assessment of the cofluctuation of tested estimations and reference values. It is worth noting that these true values are independent of the variogram model used to compute the estimation error variance, since Thiessen estimates [*Thiessen*, 1911] were taken as the reference.

The data set used for the computation of the correlation coefficients between the reference values and the various areal estimates was enlarged to include 200 hourly events; i.e., 97 events not used in the variogram model inference were taken into account. The Thiessen estimations [*Thiessen*, 1911] using the dense network were computed for the 200 events, making up five reference data sets (one for each subwatershed). Thiessen, spline, and kriging values were computed using four other networks of decreasing densities thus making up twelve data sets for each subwatershed to be compared to the corresponding reference data set.

The histogram in Figure 6 shows the distribution of the reference Thiessen values $(z_k^s)^T$ over the Gardon d'Anduze watershed for the 200 events. Simultaneously, with the correlation coefficient, the following accuracy criterion E_r was computed:

$$E_r = \left\{\left\{\frac{1}{K}\sum_{k=1}^{K}|z_k^s-(z_k^s)^T|^2\right\}\right\}^{1/2} \Big/ \frac{1}{K}\sum_{k=1}^{K} z_k^s$$

The variation of correlation coefficients (Table 5) with the network density and the watershed area is very similar to the variation of the scaled estimation variances (Table 4) and so is the variation of the accuracy criterion E_r (Figure 7). It is especially noteworthy that the smaller the watershed and the lower the density, the greater the difference between estimates. In addition, kriging data sets are always the most correlated with reference data sets and display the lowest values of E_r as well.

These results are a strong indication that kriging is more accurate than the two other estimates, when low-density networks are used. The interest of this test is that it applies to the actual rainfall process Z_k and not to the scaled random field Z_k^*. It must be noted that both the correlation coefficients and the values of the accuracy criterion E_r are in good agreement with the theoretical scaled variances of estimation error given in Table 4. An experimental procedure was then derived to test both the accuracy of the estimators and the reliability of the theoretical variances of estimation error. This procedure is as follows.

Computation of $(\sigma_u^s)^2$ and the sampling variance of the kth field $\alpha(k)$ allows computation of the theoretical unscaled variance of the estimation error $(\sigma_k^s)^2$ using (8). For each subwatershed, the value of $(\sigma_u^s)^2$ is given in Table 4, while the value $\alpha(k)$ is the same for every watershed since it is a characteristic statistical parameter of the field; $\alpha(k)$ is computed with the dense network in order to provide an accurate as possible estimation of the true variability of the field.

Once this has been done, the theoretical confidence interval

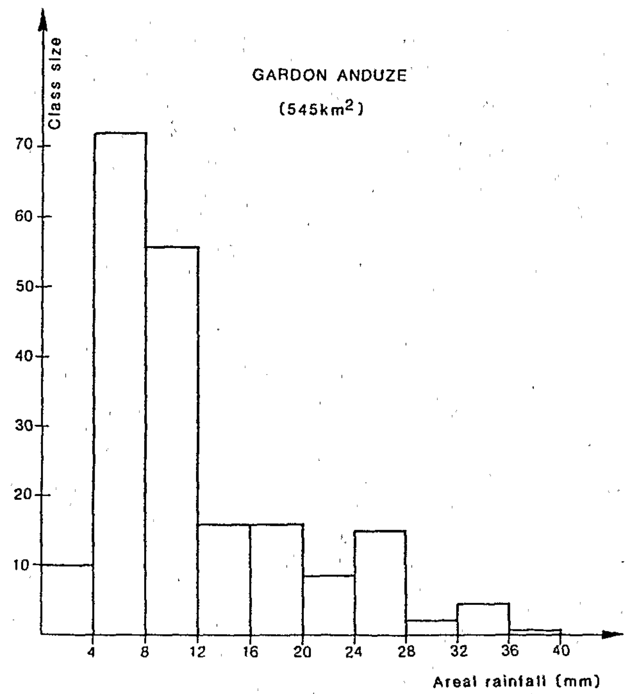

Fig. 6. Class repartition of the 200 reference values used for the correlation test (*Thiessen* [1911] estimates, dense network).

TABLE 5. Correlation Coefficients Between Reference Areal Rainfalls Computed With the Dense Network of 34 Stations and Various Estimates (200 Events)

	No. of Stations			
	17 (112)*	14 (145)*	11 (207)*	Telemetered (242)*
	Gardon a St. Andre (53 km²)			
Kriging	0.98	0.96	0.91	0.49
Spline	0.96	0.92	0.86	0.32
Thiessen	0.96	0.84	0.84	0.14
	Gardon a St. Jean (No. 1, 165 km²)			
Kriging	0.96	0.94	0.93	0.78
Spline	0.95	0.92	0.89	0.68
Thiessen	0.92	0.91	0.86	0.64
	Gardon de Mialet (237 km²)			
Kriging	0.99	0.96	0.87	0.85
Spline	0.99	0.96	0.78	0.74
Thiessen	0.98	0.93	0.76	0.76
	Gardon a St. Jean (No. 2, 265 km²)			
Kriging	0.99	0.96	0.87	0.85
Spline	0.99	0.96	0.78	0.74
Thiessen	0.98	0.93	0.76	0.76
	Gardon D'Anduze (545 km²)			
Kriging	0.98	0.96	0.93	0.87
Spline	0.98	0.96	0.91	0.86
Thiessen	0.97	0.94	0.91	0.83

*Area per gage, km².

of the estimation is expressed as

$$(z_k^s)^T \pm c\sigma_k^s \quad (11)$$

where c is a constant whose value defines the amplitude of the confidence interval, and σ_k^s is the theoretical unscaled standard deviation of estimation error.

Next it is determined whether or not the reference value z_k^s belongs to the theoretical confidence interval. If the distribution of errors is assumed to be Gaussian, z_k^s should belong to the confidence interval 68 times out of a hundred for $c = 1$, and 95 times out of a hundred for $c = 2$.

The test was performed using the reference data sets of 200 hourly events set up to compute the correlation coefficients above. The scores of Table 6 are the average over the 200 events for each watershed and each network. It can be seen that except for the smallest watershed, the proportion of true values belonging to the theoretical one standard deviation confidence interval remains around the expected value of 0.68. For the Gardon St. André watershed, the ampliitude of the confidence interval appears to be overestimated: kriging and Thiessen scores are greater than 0.80, and spline scores are greater than 0.70, which means that respectively less than 20 and 30%, respectively, of the true values are outside the interval, while the expected proportion is 32%. Concerning the two standard deviation intervals, the experimental proportions of "hits" are very close to the expected theoretical value of 0.95, except for the second smallest watershed (Gardon St Jean number 1; 165 km²). For this watershed, the CI amplitudes are slightly underestimated, since every score is under 0.95.

Another general pattern of Table 6 is that spline CI amplitudes are often underestimated (scores smaller than the expected value). While it is difficult to explain why the theoretical computations of σ_k^s are more erroneous for the spline estimate than for the two others, the overall performance of the test is good. Although it may be argued that the distribution of errors is not really Gaussian, the theoretical values of the estimation error variances obtained with the climatological variogram are thus experimentally valid indicating that kriging is by far the most accurate estimate among the three studied here. As a consequence, the curves of Figure 5 may be deemed relevant in assessing the performance of one of the network considered herein, with respect to the area of the watershed and the network density.

7. COMMENTS AND CONCLUSIONS

The two main concerns of this paper were first to derive a methodology for assessing the accuracy of areal rainfall estimation, and second, to study in a region of intense precipitation the variations of this accuracy when networks of varying densities are employed, using three different linear estimators. This methodology is based on a so-called scaled climatological variogram which accounts for the general pattern of the rainfall spatial organization over a given region. Whenever the inference of this variogram is possible, the scaled variance of estimation error of any linear estimator is computable, thus providing theoretical criterion for overall comparison of these estimators. The climatological variogram is also the basic tool of climatological kriging, a kind of kriging that is especially well-suited to rainfall analysis.

In the case study, an experimental confirmation of the theoretical values of the estimation error variance was obtained. This was carried out not only with the events used for the inference of the climatological variogram, but with additional events as well, thus proving the robustness of the climatological variogram inference process. Concerning the transformation of the scaled variance of estimation error into the unscaled variance of the kth field, it proved better to compute the scale parameter $\alpha(k)$ as the spatial sample variance over

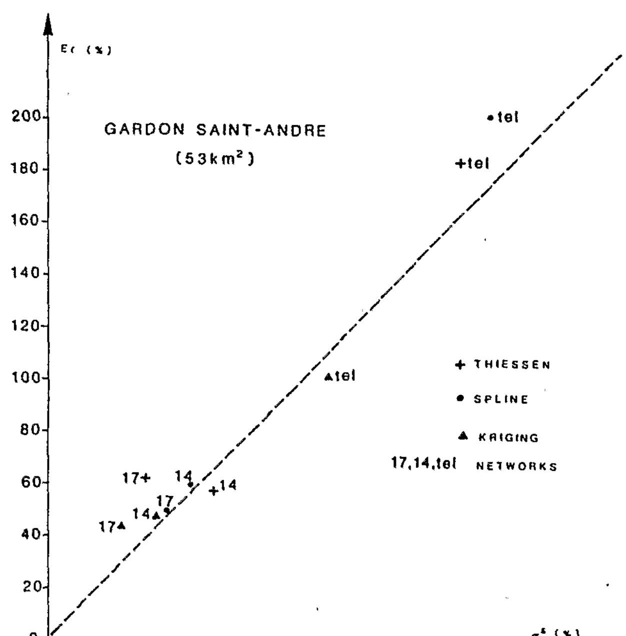

Fig. 7. Accuracy criterion E_r versus scaled variance of estimation error $(\sigma_u^s)^2$.

TABLE 6. Percentage of Reference Values Belonging to the Theoretical Confidence Interval Computed With (11)

	Gardon a St. Andre, 53 km²		Gardon a St. Jean, No. 1, 165 km²		Gardon de St. Jean, No. 2, 265 km²		Gardon de Mialet, 237 km²		Gardon D'Anduze, 545 km	
	1	2	1	2	1	2	1	2	1	2
					Kriging					
17	0.88	1.00	0.60	0.79	0.72	0.88	0.81	0.96	0.69	0.97
14	0.82	0.96	0.58	0.85	0.69	0.90	0.67	0.92	0.68	0.93
11	0.85	0.96	0.65	0.87	0.69	0.91	0.70	0.96	0.72	0.94
Telemetered	0.82	0.98	0.68	0.90	0.70	0.95	0.69	0.95	0.73	0.96
					Spline					
17	0.70	0.93	0.53	0.85	0.57	0.87	0.73	0.98	0.60	0.90
14	0.71	0.95	0.56	0.89	0.61	0.90	0.63	0.91	0.61	0.87
11	0.77	0.96	0.61	0.86	0.65	0.88	0.65	0.96	0.71	0.95
Telemetered	0.85	0.98	0.69	0.95	0.64	0.95	0.66	0.96	0.72	0.97
					Thiessen					
17	0.87	0.98	0.67	0.87	0.80	0.94	0.78	0.96	0.75	0.96
14	0.79	0.94	0.66	0.91	0.77	0.92	0.70	0.92	0.71	0.92
11	0.79	0.96	0.68	0.86	0.72	0.92	0.56	0.95	0.63	0.95
Telemetered	0.81	0.97	0.69	0.94	0.79	0.96	0.65	0.96	0.70	0.97

Here 1, 1 standard deviation interval $(Z_k^*)^T + \sigma_k^s$ (theoretical percentage assuming a Gaussian distribution of errors is 0.682); 2, 2 standard deviation interval $(Z_k^*)^T \pm 2\sigma_k^s$ (theoretical percentage assuming a Gaussian distribution of errors is 0.954).

the whole network area rather than the variance over only the watershed concerned. As can be seen in Table 7, this point is especially important for watersheds of small size compared to the range of correlation. In this table, the confidence intervals obtained when computing $\alpha(k)$ in two different ways are compared. The watershed is 53 km² in area, which means a characteristic length of 7–8 km, while the correlation range is 25–30 km. It is clear that computing $\alpha(k)$ as the variance of the kth event over the watershed results in an overestimation of the error variance for heavy rainfalls and in an underestimation of the error variance for light rainfalls.

On the other hand, the network area must not be too large, because the larger the area, the more zero rainfalls might be included in the sample, thus leading to an artificial decrease of the field variance (this is the "hole effect" well-known to kriging users). Finally, the area covered by the network used to perform any areal rainfall estimation should be related to the correlation range of the rainfall, rather than to the area of the watershed over which the estimation is performed. It is, of course, also necessary to select the stations in such a way that the watershed(s) is located in the center of the network area.

While the above comments may be deemed generally valid, providing the climatological variogram is a realistic model of the spatial covariance function of the rainfall, further conclusions may be drawn that apply to regions of similar geomorphoclimatic characteristics to the one studied here. The first such conclusion is related to the structure of the spatial covariance model used throughout this paper.

Despite the rough topography of the region and intense precipitation, a spherical variogram associated with a zero-order drift allowed computation of accurate values of the variance of estimation error. Of course, this is probably not the only model that would have yielded good results, but it was also shown that a not too realistic model, such as spline gener-

TABLE 7. Variations of the Theoretical Confidence Intervals With $\alpha(k)$, Gardon St. Andre

	Dense Network			Telemetered Network				
Event k	Reference Value, mm	$(\alpha(k))^{1/2}$*	$(\alpha(k))^{1/2}$†	Kriging Estimate	(σ_k^s)*	CI*	(σ_k^s)†	CI†
Aug. 28, 1976, 3–4 P.M.	23.9	8.8	15.8	19.1	6.6	25.7 12.5	11.8	30.9 7.3
Oct. 23, 1977, 11–12 A.M.	10.9	10.7	3.5	4.6	8.1	12.7 −3.5	2.7	7.3 1.9

Telemetered network: $\sigma_u^s = 0.75$.
*Spatial sample variance of the kth event computed over the whole network area.
†Spatial sample variance of the kth event computed over the watershed area (53 km²).

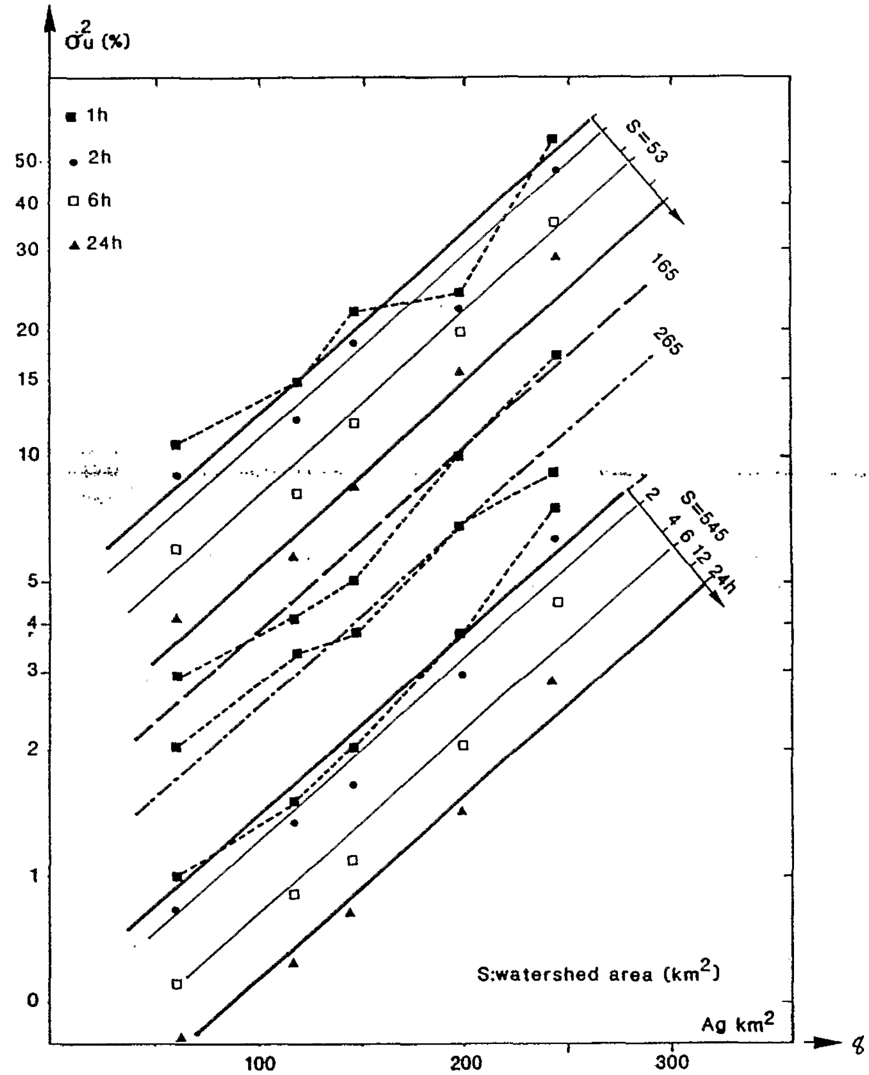

Fig. 8. Theoretical scaled estimation variance of areal rainfall as a function of watershed area, gage area, and time step.

alized covariance, performs far poorer than the spherical model. As was shown by *Lebel* [1984], the question of the order of the drift is not of prime importance. Two reasons may contribute to that: (1) the drift actually used in the estimation process is a local drift since kriging was performed over a sliding neighborhood and (2) for time steps shorter than 24 hours the relation of rainfall depth of a particular event to the topography is not as strong as it can be for greater time steps. *Chua and Bras* [1980] also found that a zero-order drift model provide accurate values of the variances of estimation error for storm data collected in the San Juan Mountains. This supports the idea that using complicated covariance models in rainfall analysis is often not worthwhile, at least in the best linear unbiased estimator context.

The spherical model is a convenient tool, for it provides a value of the decorrelation distance. In the Cevennes region, the relation between this distance and the time step of rainfall accumulation seems to be well-approximated by a power type function. This allows computation of the range (decorrelation distance) for any time step between 1 hour (25 km) and 24 hours (65 km). Since the procedure of theoretical estimation error variance computation was validated experimentally on hourly data, extension of the method to other time steps appeared founded. The kriging scaled variances were computed for six time steps (1, 2, 4, 6, 12, and 24 hours) with the variogram model inferred from experimental variograms (see values of the range in Figure 2). An accuracy measure of the kriging interpolation process was thus available for various network densities, watershed areas, and time steps of rainfall accumulation. This information is summarized in Figure 8. For clarity, only a few values are marked in the chart, but other values are easy to infer because the distance between the parallel straight lines is relatively small.

Using the chart of Figure 8, it is possible to evaluate whether or not the density of any network meets a desired accuracy with respect to a given time step and a given watershed area. This is of interest in assessing existing networks as well as in designing a future network. Furthermore, values of the estimation error variance may be derived for time steps at which no data were collected, providing that a good estimation of the correlation range is possible. This is of particular interest, since the time desired is often dependent on the watershed response to the rainfall input. In many cases, no data are available at that time step, but the chart allows an a priori assessment of the expected variance of estimation error, before any further study or investment are considered. Although the general patterns of this chart are similar to those given by *Huff* [1970] in Illinois and *Woodley et al.* [1975] in Florida, it would be, of course, unwise to extrapolate the results to other areas.

Concerning the accuracy of areal rainfall estimation with the telemetered network, note that the scaled kriging variance of estimation error remains lower than 10% for watersheds of areas greater than 100 km^2. This indicates that ground-based networks provide a sufficiently accurate estimation for hourly areal rainfall in this region, since for smaller watersheds one would probably have to work at a time step smaller than 1 hour. However, it must be kept in mind that this region is very well-instrumented, and conversely this approach has proved that in many other French region an accurate real time estimation of areal rainfall is still impossible with currently available data. Given the results of this paper, the increase in accuracy that can be expected from the use of meteorological radar, compared with relatively dense telemetered networks, should be studied carefully to help hydrologists decide which solution is best suited to a given problem.

APPENDIX: UNIVERSAL KRIGING AND SPLINE FUNCTIONS

The objective of this appendix is to recall briefly the technique of universal kriging for spatial averaging from a finite set of pointwise observations and to recall that the spline surface fitting method is in fact a special case of the universal kriging technique. We use the following notations; cartesian coordinates of a current point in the plane R^2:

$$u = (x, y)$$

vector of available point wise observations:

$$(Z^n)^T = [z(u_1), \cdots, z(u_i), \cdots, z(u_n)]$$

with $u_1 = (x_1, y_1) \cdots u_n(x_n, y_n)$ the coordinates of the data points. The superscript T denotes the vector transpose.

Universal Kriging

The spatial phenomenon of interest (here the rainfall depth accumulated over a given time step) is assumed to be a two-dimensional nonstationary random field that can be written

$$Z(u) = m(u) + Y(u)$$

where $Y(u)$ is a zero-mean stationary field, and $m(u)$ is the mean of the field $Z(u)$, also called the drift. It is assumed that at least locally, it can take the following polynomial form:

$$m(u) = E[Z(u)] = \sum_{l=1}^{L} \alpha_l f_l(u)$$

where the functions $f_l(u)$ are known monomials in u while the coefficients α_l are unknown.

The spatial mean over the surface S of the random field $Z(u)$ is defined as the random variable

$$Z^s = \frac{1}{s} \int_S Z(u)\, du$$

The universal kriging problem is then the problem of finding an optimal (linear, unbiased, minimum variance) estimator of Z^s. It is well-known [e.g., *Matheron*, 1973] that whatever the polynomial drift considered, a so-called "generalized covariance function" allows for the solution of this estimation problem. The generalized covariance function is denoted $C(u_i, u_j)$ for any (u_i, u_j) current points in R^2 and is implicitely defined by the following expression:

$$\sum_{i=0}^{n} \sum_{j=0}^{n} \lambda_i \lambda_j C(u_i u_j) = \mathrm{Var}\left[\sum_{i=0}^{n} \lambda_i Z(u_i)\right]$$

where

$$\sum_{i=0}^{n} \lambda_i Z(u_i)$$

is a zero-mean generalized increment filtering out the drift. This means that the λ_i are such as satisfying L relationships:

$$\sum_{i=1}^{n} \lambda_i f_l(u_i) = 0 \qquad l = 1, \cdots, L$$

Then, the optimal estimate of Z^s is given by

$$\hat{Z}^s = \sum \lambda_i Z(u_i) = \Lambda^T Z^n \tag{A1}$$

where the vector Λ is computed by

$$\begin{bmatrix} \Lambda \\ \mu \end{bmatrix} = \begin{bmatrix} C & E \\ E^T & 0 \end{bmatrix}^{-1} \begin{bmatrix} C_s \\ E_s \end{bmatrix} \tag{A2}$$

where C is a $(n \times n)$ matrix with entries $C(u_i, u_j)$ and E^T is a $(L \times n)$ matrix of the form

$$E^T = \begin{bmatrix} f_1(u_1) & \cdots & f_1(u_n) \\ f_2(u_1) & \cdots & f_2(u_n) \\ f_L(u_1) & \cdots & f_L(u_n) \end{bmatrix}$$

C_s is a column n vector with entries

$$\frac{1}{s} \int_S u, u_i)\, du \qquad i = 1, \cdots, n$$

E_s is a column l vector with entries

$$\frac{1}{s} \int_S f_l(u)\, du \qquad l = 1, \cdots, L$$

μ is a vector of Lagrange multipliers. Since $\hat{Z}^S$ is unbiased, we have

$$\sum_{i=1}^{n} \lambda_i = 1$$

Therefore it turns out that $\hat{Z}^S$ is a weighted mean of the observations $z(u_i)$.

Spline Surface Fitting

As it has been pointed out by *Matheron* [1980], the spline surface fitting method to compute a linear estimate of Z^s can be shown to be a special case of the universal kriging technique (see also *Dubrule* [1982]), corresponding to the following structural choice. A first-order drift:

$$m(u) = Y_1 + Y_2 x + Y_3 y$$

A generalized covariance function of the form

$$C(u_i, u_j) = |u_i - u_j|^2 \log |u_i - u_j|$$

From (A1) and (A2) we can write

$$\hat{Z}^S = [C_s{}^T, E_s{}^T] \begin{bmatrix} \Psi \\ Y \end{bmatrix} \tag{A3}$$

where the vectors $\Psi^T = (\Psi_1, \cdots, \Psi_n)$ and $Y^T = (Y_1, Y_2, Y_3)$ are defined by

$$\begin{bmatrix} \Psi \\ Y \end{bmatrix} = \begin{bmatrix} C & E \\ E^T & 0 \end{bmatrix}^{-1} \begin{bmatrix} Z^n \\ 0 \end{bmatrix} \quad (A4)$$

Then the link with the usual mechanical interpretation of the spline technique is as follows.

1. It can be easily shown that the optimal estimate $\hat{Z}^S$ given by (A3) is equivalent to the following where $Z(u)$ is the "spline surface":

$$Z(u) = Y_1 + Y_2 x + Y_3 y + \sum_{i=1}^{n} \Psi_i C(u_i, u) \quad (A5)$$

2. The coefficients (Ψ, Y) given by (A4) correspond to the minimization of the bending energy of a thin elastic plate supported by the observations [*Duchon*, 1976]. Therefore the function (A5) is called a "thin plate" spline surface.

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