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Monteny

Apparent soil thermal diffusivity, a case study: HAPEX–Sahel experiment

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Abstract

In this paper, we determine the apparent soil thermal diffusivity of a dense sandy soil of the HAPEX–Sahel experiment. Several current methods are compared. Two temperature data sets are chosen. In the first, the assumption of steady periodicity is fulfilled and in the second, it is not. In both cases, we compare methods which assume a vertical homogeneity of the soil thermal properties with the NHS method which is based on the vertical inhomogeneity of the thermal diffusivity. Results obtained with both sets of data show that thermal properties are not homogeneous vertically. It is shown that the NHS method is not applicable when the steady periodicity assumption is not valid. In this case, when abrupt change in the temperature wave pattern occurs, as frequently happens in the Sahel just before or just after a rain, a method based on the Lapalce Transform with a corrective factor for the non uniformity of the initial temperature profile (CLTM) must be used. When the steady periodicity assumption is fulfilled, both the Harmonic (HM) and the CLTM methods lead to somewhat greater values of the thermal diffusivity than the NHS method.

1. Introduction

Frequently, energy and moisture balance studies at the earth surface require estimates of soil heat flux and temperature at the soil surface. Coupled models of heat and moisture transfer in bare soils (Milly, 1986; Camillo et al., 1983; Novak and Black, 1985; Passerat de Silans et al., 1989) or in vegetated soils (Dantas-Antonino, 1992;

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Braud et al., 1994) require information about soil thermal properties, such as thermal conductivity or thermal diffusivity, and volumetric heat capacity. This latter can be deduced easily from soil components (Van Wijk, 1963). Thermal conductivity and thermal diffusivity are related by volumetric heat capacity, so only one needs to be determined. Generally the thermal diffusivity is estimated because it describes transient process of heat conduction with temperature boundary conditions. In fact, soil heat transfer is originated by a complex combination of conductive process and intra-porous convective process. We then prefer to consider soil thermal diffusion as a bulk process which will be assimilated to a conductive one. So this paper will be concerned with the *apparent* soil thermal diffusivity. Several methods for determining apparent soil thermal diffusivity or apparent soil thermal conductivity are published. Some involve theoretical models (de Vries, 1963), or semi-empirical models (Johansen, 1975). Although these models are based on the volume fraction of soil constituents, they apply to soils with simple structure. Farouki (1982) presented an exhaustive review of these models with their respective domains of application. Other methods are based on soil temperature measurements in the field. Most of them are deduced from analytical solutions of the one-dimensional heat conduction equation with constant diffusivity in a semi-infinite medium (Horton et al., 1983). Therefore they apply to homogeneous soils (HS methods). Horton et al. (1983) examine several of them which are based on analytical solution of the heat conduction equation, considering that temperature at the upper boundary is well described by a sinusoidal function or by a Fourier series. They show that the Harmonic method (HM) is more reliable than the others examined. The analytical solution used in these methods does not require knowledge of the initial temperature profile. This is due to the hypothesis of steady periodicity which is implicit in these solutions. However this hypothesis is not always fulfilled, particularly in regions where abrupt climatic changes may occur in short periods, for example as during the crossing over of a cold front. Other authors have used methods based on the Laplace Transform (LTM), which require a constant initial temperature profile (Kavianipoor and Beck, 1977; Asrar and Kanemasu, 1982). For these methods the requirement of a steady periodicity assumption is not necessary, and they can be applied during shorter time period than the HM method. For this reason they may better fulfill the condition of constancy of diffusivity. Asrar and Kanemasu (1982) argue that every day it is possible to get a nearly uniform temperature profile when inversion of gradients in soil temperature occurs. However, they did not study the sensitivity of the method to this hypothesis. Passerat de Silans (1986, 1988) and Balabanis (1987) show that the LTM method is very sensitive to the initial temperature profile and propose the introduction of a corrective term taking account of the non-uniformity of the initial temperature profile. They called it the CLTM (Corrected Laplace Transform Method).

HM, LTM and CLTM, all assume vertical homogeneity of the thermal properties. However, in bare soils or in soils of semi-arid regions covered with sparse vegetation, where the upper layer of the soil dries quickly, this assumption may not be valid near the soil surface. Lettau (1954) developed a soil heat transfer theory, accounting for spatially non homogeneous thermal diffusivity in soil (NHS), based on the assumption of steady periodicity. Nassar and Horton (1989) have applied this method to field data for a silty clay loam soil and also to temperature values generated by a numerical model. They

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have investigated the procedure of fitting Lettau's theory parameters, and have concluded that best results are obtained with a cubic-spline fitting procedure.

In this paper, we compare and discuss the HM, LTM, CLTM and NHS methods for determining the soil apparent thermal diffusivity. We apply these methods to two selected temperature data sets collected in the HAPEX–Sahel experiment. In the first data set, the assumption of steady periodicity is fulfilled, while in the other it is not.

2. Theoretical considerations

The following equation describes vertical one-dimensional conductive heat transfer in an isotropic medium:

$$C\frac{\partial T}{\partial t}=\frac{\partial}{\partial z}\left(\lambda\frac{\partial T}{\partial z}\right) \tag{1}$$

where T is temperature (K), t the time (s), z the depth (m), C the volumetric heat capacity ($\mathrm{J\,m^{-3}\,K^{-1}}$) and λ the thermal conductivity ($\mathrm{W\,m^{-1}\,K^{-1}}$).

Assuming that both C and λ are independent of depth, i.e., the soil is vertically homogeneous with respect to its thermal properties, Eq. (1) becomes:

$$\frac{\partial T}{\partial t}=K\frac{\partial^2 T}{\partial z^2} \tag{2}$$

where K is the apparent soil thermal diffusivity: $k=\lambda/C$ ($\mathrm{m^2\,s^{-1}}$).

2.1. Harmonic method (HM)

Soil temperatures measured at the upper depth, $T(0,t)$ can be described by a Fourier's series:

$$T(0,t)=T_{\mathrm{m}}+\sum_{i=1}^{n}A_i\sin(i\,\omega t+\phi_i) \tag{3}$$

Here T_{m} is the mean, A_{i} and ϕ_{i} the amplitude and phase shift of harmonic i, respectively, and ω the fundamental frequency:

$$\omega=2\pi/P \tag{4}$$

with P the period of the main harmonic (24 h, generally).

Assuming a steady periodic signal, the solution of Eq. (2) for a semi-infinite soil with boundary conditions given by Eq. (3) and $T(\infty,t)=T_{\mathrm{m}}$ is (Carslaw and Jaeger, 1959):

$$T(z,t)=T_{\mathrm{m}}+\sum_{i=1}^{n}A_i\exp\left(-\frac{z}{d_i}\right)\sin\left(i\,\omega t+\phi_i-\frac{z}{d_i}\right) \tag{5}$$

where $d_i=\sqrt{2K/i\,\omega}$. d_i corresponds to the depth at which the signal is propagated during a period P/i (Van Wijk, 1963). This analytical solution (Eq. (5)) does not require knowledge of the initial temperature profile since the assumption of steady periodicity implies that the initial profile has no more influence on the temperature evolution. Notice also that the mean temperature during the period must be identical at all depths.

Eq. (5) is used to estimate the apparent soil thermal diffusivity K, by a least squares best fit of the calculated $T(z,t)$ to the observed temperatures at depth z.

2.2. *Laplace transformation based method (LTM)*

Solution of Eq. (2), with initial and boundary conditions given by:

$$T(z,0) = T_0 \tag{6a}$$

$$T(0,t) = \Phi(t) \quad \text{with } t > 0 \tag{6b}$$

can be derived using the Laplace transformation (Carslaw and Jaeger, 1959):

$$T(z,t) = T_0 + \frac{z}{2\sqrt{\pi K}} \int_0^t \Phi(\tau) \frac{\exp\left(\frac{-z^2}{4K(t-\tau)}\right)}{(t-\tau)^{3/2}} \mathrm{d}\tau \tag{7}$$

Eq. (7) holds for a semi-infinite medium with an upper boundary condition given by a continuous function of time $\Phi(t)$. It is an impulsional response equation which can be used when abrupt changes in the temperature input signal are observed, e.g., after a rainy period or the passage of a cold front. Restriction of the use of this equation is that the initial temperature profile must be uniform. In the same way as for the harmonic method, the apparent soil thermal diffusivity K is obtained by fitting calculated $T(z,t)$ to measured temperature at depth z, using a least squares procedure. The time for which the fitting process is applied may be much smaller than the period, depending on the depth z to which Eq. (7) applies.

2.3. *Corrected Laplace transformation method (CLTM)*

Considering the linearity of Eq. (2), Passerat de Silans (1986, 1988) and Balabanis (1987) obtained an analytical solution by the superposition of the analytical solution of both problems:

1. with a zero condition at the upper boundary and with an initial temperature profile given by a function of z:

$$\begin{aligned} T(0,t) &= 0 \\ T(z,0) &= F(z) \end{aligned} \tag{8}$$

and

2. with a defined upper boundary condition as a function of time t, and a zero homogeneous initial temperature profile:

$$\begin{aligned} T(0,t) &= \Phi(t) \\ T(z,0) &= 0 \end{aligned} \tag{8'}$$

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Solution of case 1 is given in Carslaw and Jaeger (1959) by the use of image theory:

$$T_a(z,t) = \frac{1}{2\sqrt{\pi K t}} \int_0^{\infty} F(z') \left\{ \exp\left(\frac{-(z-z')^2}{4Kt} \right) - \exp\left(\frac{-(z+z')^2}{4Kt} \right) \right\} \mathrm{d}z' \tag{9}$$

and solution of problem 2 is given by Eq. (7), where $T_0 = 0$.

Writing $F(z) = T_{z0} + f(z)$, T_{z0} being the temperature at a depth z_0 below which the temperature remains constant during time t so that $f(z) = 0$ for $z \geq z_0$, Eq. (9) can be written as:

$$T_a(z,t) = T_{z0}\, \mathrm{erf}\left(\frac{z}{2\sqrt{Kt}} \right) + \frac{1}{2\sqrt{\pi K t}} \int_0^{z_0} f(z') \left\{ \exp\left(\frac{-(z-z')^2}{4Kt} \right) - \exp\left(\frac{-(z+z')^2}{4Kt} \right) \right\} \mathrm{d}z' \tag{10}$$

and the analytical solution used in the CLTM is:

$$T(z,t) = T_a(z,t) + \frac{z}{2\sqrt{\pi K t}} \int_0^{t} \Phi(t) \frac{\exp\left(\frac{-Z^2}{4K(t-\tau)} \right)}{(t-\tau)^{3/2}} \mathrm{d}\tau \tag{11}$$

2.4. Lettau's non homogeneous soil method (NHS)

The three methods presented above, hold for homogeneous soils in terms of thermal properties. However, due to the moisture gradients in the vadoze zone and eventually to variation of the bulk dry density with depth, soil near the surface may be far from homogeneous.

Nassar and Horton (1989), using Lettau's theory of the thermal diffusion in non-homogeneous soils (Lettau, 1954), show that more accurate results can be obtained when temperature parameter dependence with depth and their gradients are determined by fitting a cubic spline to the discrete experimental values. According to these authors and Lettau's theory, and assuming a steady periodic temperature evolution, we can write at any depth z:

$$\begin{aligned} T(z,t) &= T_m + A(z)\sin(\omega t + \phi(z)) \\ G(z,t) &= G_m + B(z)\sin(\omega t + \beta(z)) \end{aligned} \tag{12}$$

considering only the first harmonic. G is the heat flux given by:

$$G(z,t) = -\lambda \left. \frac{\partial T}{\partial z} \right|_z \tag{13}$$

and time dependency is given by the energy conservation equation:

$$\frac{\partial G}{\partial z} = -C \frac{\partial T}{\partial t} \tag{14}$$

when no heat sinks are considered.

If one considers several harmonics as in the HM, Lettau's theory will lead to several possible close values of K. However, Lettau considers that best results are obtained with the first harmonic. Amplitudes $A(z)$ and $B(z)$, and phases $\phi(z)$ and $\beta(z)$ are depth-dependent. Then:

$$\frac{\partial T}{\partial z} = \frac{\partial T_m}{\partial z} + A'(z)\sin(\omega t + \phi(z) + \epsilon(z)) \tag{15}$$

where:

$$\tan(\epsilon(z)) = \left[\frac{\partial \phi(z)}{\partial z} \bigg/ \frac{\partial \ln A(z)}{\partial z} \right]$$

$$A'(z) = A(z) \frac{\partial \phi(z)}{\partial z} \bigg/ \sin(\epsilon(z)) \tag{16}$$

In the same way:

$$\frac{\partial G}{\partial z} = \frac{\partial G_m}{\partial z} + B'(z)\sin(\omega t + \beta(z) + \delta(z)) \tag{17}$$

where:

$$\tan(\delta(z)) = \left[\frac{\partial \beta(z)}{\partial z} \bigg/ \frac{\partial \ln B(z)}{\partial z} \right]$$

$$B'(z) = B(z) \frac{\partial \beta(z)}{\partial z} \bigg/ \sin(\delta(z)) \tag{18}$$

Using Eqs. (12)–(18) simultaneously with Eq. (1) and assuming that C does not vary with time, we obtain:

$$\beta(z) = \pi + \phi(z) + \epsilon(z)$$

$$\delta(z) = \phi(z) - \beta(z) - \pi/2$$

$$K(z) = \frac{\lambda(z)}{C} = \frac{\omega \sin\epsilon(z) \sin\delta(z)}{\dfrac{\partial \phi(z)}{\partial z} \dfrac{\partial \beta(z)}{\partial z}} \tag{19}$$

Then it is possible to estimate $K(z)$, by fitting the temperature time dependence at each depth by a Fourier series with one harmonic and calculating by a cubic spline procedure the gradients $\partial\phi(z)/\partial z$, $\partial \ln A(z)/\partial z$ and $\partial\beta(z)/\partial z$.

3. Materials and methods

The HAPEX–Sahel project took place in Niger during the 1992 wet season, from mid-august to mid-October (Goutorbe et al., 1994). The region is generally covered by aeolian sand where a semi-arid vegetation growths. The field experiments were con-

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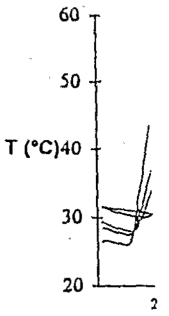

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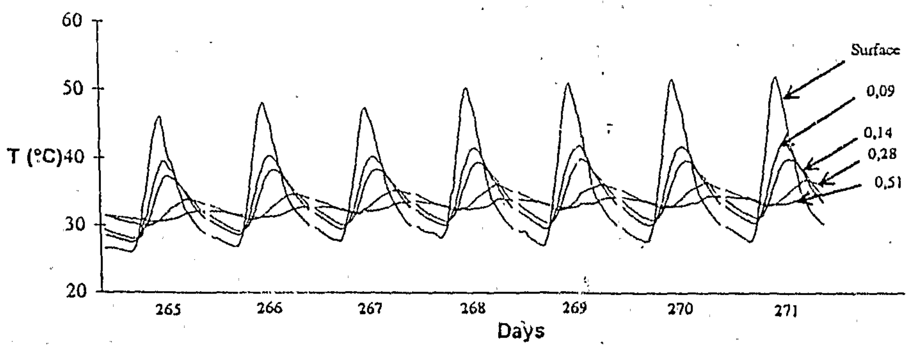

Fig. 1. Measured temperatures during week 39. The depth of the measurements is indicated.

ducted at the Eastern Super Site of the HAPEX square (13°29′21″–2°36′77″; 13°29′21″–2°47′88″; 13°40′–2°36′77″; 13°40′–2°47′88″) at different sub-sites (Monteny, 1993). One of them concerns a fallow savannah area which consists of *Guiera S.* bushes with an undergrowth of sparse grasses and herbs. This sub-site was equipped for measuring energy balance and moisture balance in the soil-vegetation-atmosphere continuum. In the soil, temperatures were measured by thermocouple probes installed at different depths (0.002, 0.02, 0.09, 0.14, 0.28, 0.51 and 1.01 m). The exact depth of each sensor was measured after the experiment when they were removed. Temperature at the surface was also routinely measured by an IR radiometer. When the soil was bare, i.e., before grasses cover the area where temperature probes were installed, radiometric temperatures agree well with the temperatures measured at 0.002 m depth. Hence, we have taken the soil temperature at 0.002 m depth as the soil surface temperature. Signals from temperature probes were sampled each 10 s with a Campbell Scientific datalogger and multiplexer and 20-min averages were computed for final storage.

Because of some problems with the measurements of the temperatures at 0.02 m, they were discarded in this study.

4. Results and discussion

The first temperature data set used in this paper corresponds to week 39 (the last week of September 1992, from DOY 266 to DOY 271). During this week, the daily pattern of temperature waves at all instrumented depths is steady periodic and sinusoidal, as can be seen in Fig. 1, because of clear sky conditions. During this week the soil was drying. The basic assumption, of steady periodicity, in the HM and NHS method is fulfilled during this week. This data set can be used to compare the performances of both these methods. Comparisons can also be made with the CLTM, when an identical duration is used in the fitting procedure (24 h). The LTM was also applied to this data set from 9:00 am, when temperature profile appears to be uniform, to 24:00, every day. These results are compared to those from the CLTM when applied with the same 15 h of data. With these methods, the apparent soil thermal diffusivity is calculated for the layers 0–0.09; 0.09–0.14 and 0.14–0.28 m.

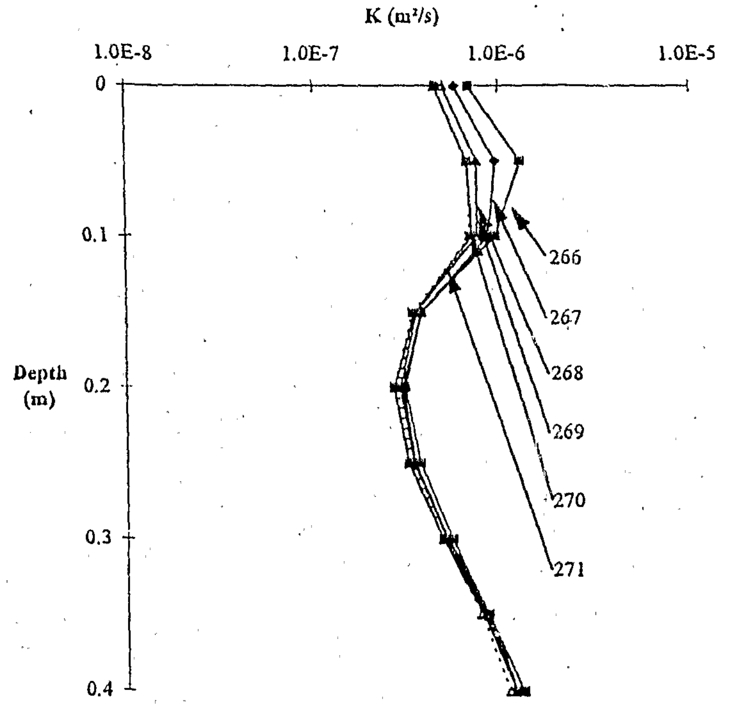

Fig. 2. Apparent soil thermal diffusivity profiles given by the NHS method, estimated from temperature profiles in successive days. The day number is indicated for each curves.

The apparent soil thermal diffusivity profile is calculated by the NHS method, assuming vertical inhomogeneity of the thermal soil properties. Following Nassar and Horton (1989), the gradients $\partial\phi/\partial z$ and $\partial \ln A/\partial z$ are calculated with a cubic spline-fit on the data from the six depths instrumented. Then $\beta(z)$, $\epsilon(z)$ and $\delta(z)$ are calculated using Eqs. (16) and (19), and gradients $\partial\beta/\partial z$ are generated. $K(z)$ values are calculated by Eq. (19) at depths varying from surface to 40 cm, with a step of 5 cm. Results are shown in Fig. 2. It can be observed in this figure that variation of the apparent soil thermal diffusivity with depth occurs. In week 38, a rainy period occurs during the first four days with an amount of 23.6 mm. In DOY 265, the first day of week 39, a very small rain of 0.5 mm occurs at 6:00 am. Hence the upper soil layer was drying from DOY 266 to DOY 271. In Fig. 2, it can be observed that the thermal diffusivity decreases as the upper soil layer (0–0.15 m) dries. The soil dry bulk density profile was measured up to 0.175 m at the experimental site (Monteny, 1993) and shows variation with depth (Fig. 3). Soil moisture was routinely measured at twelve access tubes by neutron probe. The tubes were distributed along a transect in the sub-site. Measurements were made every 10 cm from the soil surface. In Fig. 4, the average profiles for DOY 265, 268 and 272 are drawn. They show the pattern of the drying process of the soil, but they do not correspond to the soil moisture values where the temperature probes were installed because no soil moisture measurements were provided there. The inhomogene-

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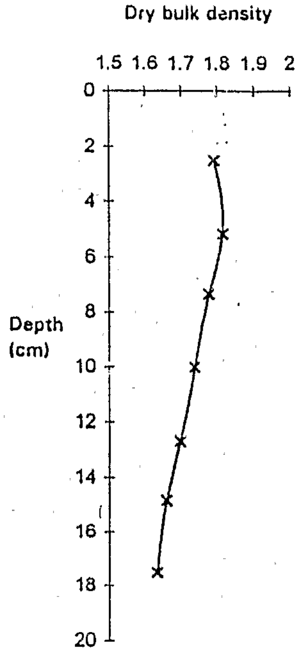


Fig. 3. Soil dry bulk density profile.

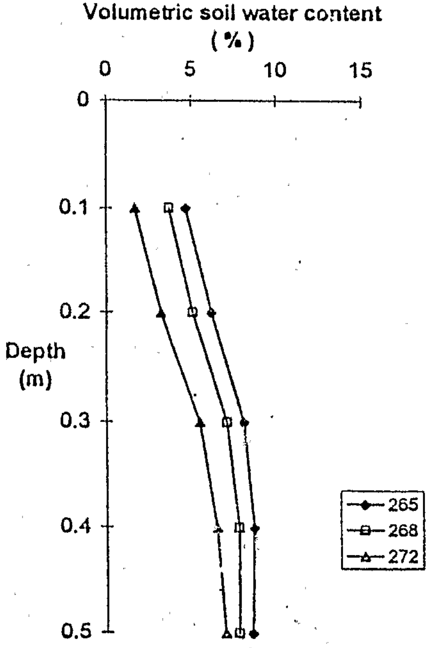


Fig. 4. Volumetric soil moisture profiles (average of twelve neutron probe access tubes) in DOY 265, 268 and 272.

ity of the apparent soil thermal diffusivity observed in Fig. 2 must be explained in great part by both the soil moisture and the dry bulk density variations with depth.

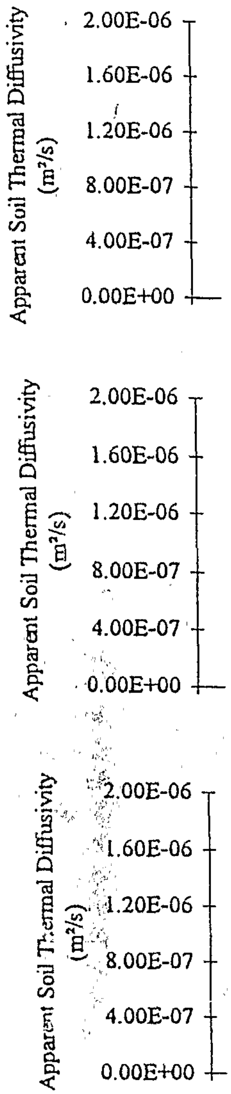
From the profiles shown in Fig. 2, we calculate the maximum and the mean values of the thermal diffusivity for each soil layer: 0–0.09; 0.09–0.14; and 0.14–0.28 m. These values are compared with estimates of the thermal diffusivity from the homogeneous methods in Table 2 (see later).

In Fig. 5, the values of the apparent soil thermal diffusivity calculated with the HM, CLTM-24 (CLTM when applied with a 24-h time fitting), CLTM-15 (CLTM when applied with a 15-h time fitting from 9:00 to 24:00) and LTM methods are compared for each soil layer. In the latter case, the fitting procedure initiates at 9:00 am, when the temperature profile is closest to an uniform one. The LTM method is used with a constant initial profile given by the arithmetic mean of the temperatures measured from 0 to 0.51 m at 9:00 am (Fig. 6). However, in all three layers, LTM gives smaller values than the other methods. This is due to the assumption of uniform initial profile not being valid because the correction introduced in CLTM improves the results significantly. In the first two layers and more especially in the former, the HM, CLTM-24 and CLTM-15 give similar values of thermal diffusivity.

In the third layer, CLTM-15 gives erratic values. In Table 1, the depth of penetration after 24 hours of the main harmonic temperature wave is calculated for several values of the thermal diffusivity by the expression for d_i in Eq. (5), doing $i = 1$. During the 15 h where CLTM-15 is applied, the depth of penetration of the temperature wave is yet smaller than that in Table 1. Hence the temperature wave reaches 0.14 m depth, but does not penetrate to 0.28 m depth during the 15 h when the fitting procedure is used. Therefore the CLTM cannot be applied during this short time to such a thick layer. Indeed, with an impulsional response formulation of the analytical solution of the heat conduction equation, the minimum time required for the fitting procedure is a function of the thickness of the layer considered.

In Table 2, the values of the apparent soil thermal diffusivity calculated by the HM and CLTM-24 methods are compared with the mean and maximum values in the layer estimated from the thermal diffusivity profiles given by the NHS method. Results show that when the vertical inhomogeneity of the thermal properties is ignored, the values of the apparent soil thermal diffusivity, calculated by both HM and CLTM are greater than the mean and the maximum values given by the NHS method.

DOY 235, 239 and 252 constitute another data set, where time dependence of temperature is well fitted by a Fourier series, but the assumption of steady periodicity is not valid. In the night of DOY 234 to 235, an intensive rain which reached 44.9 mm occured just before midnight. DOY 239 was preceded by three rainy days with a much smaller intensity and DOY 252 was preceded by an intense precipitation 24 h before. Temperatures measured on these three days were used to calculate soil apparent thermal diffusivity using the HM, CLTM, and NHS methods. Results are shown in Table 3 for the first two of these methods. The NHS method leads to inconsistent results (values of thermal diffusivity varying from negative to 10^{-4} $m^2 s^{-1}$) and is inapplicable. The reason for this might be that the assumption of steady periodicity is invalid for these three days. In Table 3, generally, the values calculated with both methods are quite different. It is expected that the CLTM will give better results as the method is more



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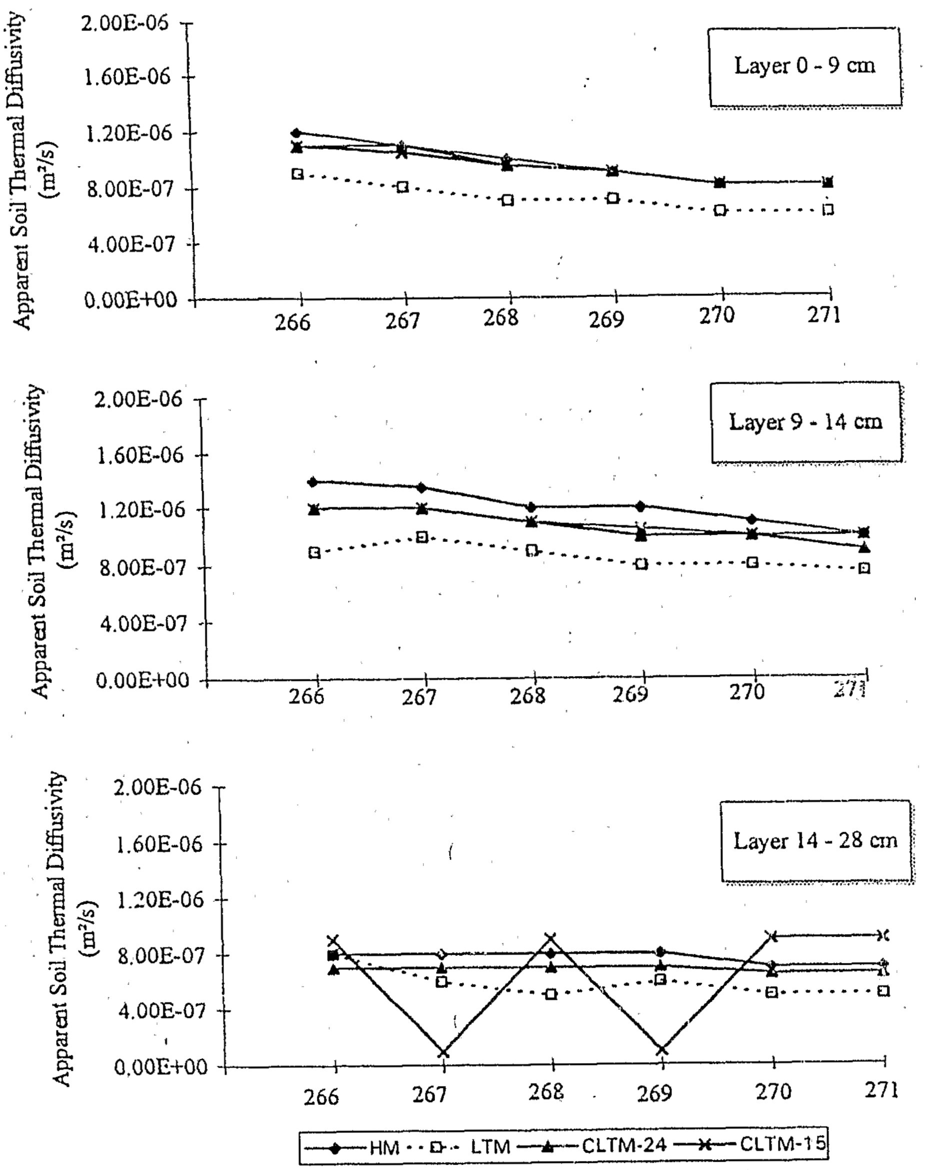

Fig. 5. Comparison of apparent soil thermal diffusivity calculated by HM, LTM, CLTM-24 and CLTM-15 when the assumption of steady periodicity is valid.

appropriate to an abrupt change in temperature patterns in relation to the previous days. This seems true for DOY 239 and 252 where the thermal diffusivity values are what would be expected from the values calculated on week 39 given that the soil is wetter, especially in the first layer (0–9 cm). For both days, the CLTM method is also used with a fitting time of 15 h instead of 24 h, leading to the same results except for the deeper

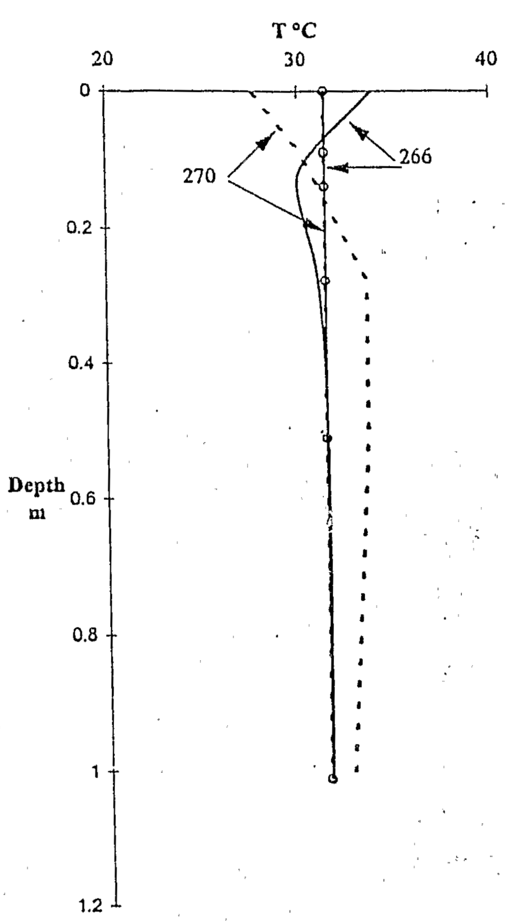

Fig. 6. Two examples of initial temperature profiles measured at 9:00 (DOY 266 and 270). These profiles were aproximated by uniform profiles (vertical lines) for the LTM application.

layer. However, on DOY 235, the apparent soil thermal diffusivity value estimated for the (0–9 cm) layer is too high. When running the CLTM method with the 15-h time fitting, beginning at 9:00 am, the apparent thermal diffusivity for this layer drops to $1.5.10^{-6}$ $m^2 s^{-1}$. The high value obtained with the larger time fitting can be explained by simultaneous liquid water convection since infiltration occured early in the morning (between 0 and 9:00) of DOY 235. The surface soil layer was saturated on DOY 235 in the morning and heat transport by intra-porous vapour convection could not occur.

Table 1
Depth of temperature wave penetration d, as a function of the thermal diffusivity K

$K \times 10^6$ ($m^2 s^{-1}$)	1.2	1.1	1.0	0.8	0.6	0.4
d (m)	0.182	0.174	0.165	0.148	0.128	0.105

Table 2
Comparison of the ap
inhomogeneous (NHS)

DOY	H
Layer 0 – 0.09 m	
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271	
Layer 0.00 – 0.1	
266	
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268	
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Layer 0.14 – 0.2	
266	
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Table 3
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0.6	0.4
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Table 2
Comparison of the apparent soil thermal diffusivity calculated with the homogeneous (HM, CLTM) and inhomogeneous (NHS) methods during week 39

DOY	HM × 10^6	CLTM × 10^6	NHS_{mean} × 10^6	NHS_{max} × 10^6
Layer 0–0.09 m				
266	1.2	1.1	1.2	1.3
267	1.1	1.1	0.89	0.96
268	1.0	0.95	0.71	0.77
269	0.9	0.90	0.71	0.77
270	0.8	0.80	0.64	0.69
271	0.8	0.80	0.64	0.69
Layer 0.00–0.14 m				
266	1.4	1.2	0.82	1.1
267	1.35	1.2	0.71	0.89
268	1.2	1.1	0.66	0.76
269	1.2	1.0	0.66	0.76
270	1.1	1.0	0.62	0.72
271	1.0	0.9	0.62	0.70
Layer 0.14–0.28 m				
266	0.8	0.7	0.31	0.52
267	0.8	0.7	0.30	0.50
268	0.8	0.7	0.28	0.45
269	0.8	0.7	0.28	0.45
270	0.7	0.65	0.27	0.43
271	0.7	0.65	0.27	0.42

Table 3
Apparent soil thermal diffusivity ($m^2 s^{-1}$) with the assumption of vertical homogeneous soil when the hypothesis of steady periodicity is not fulfilled

DOY	Layer	HM × 10^6	CLTM × 10^6
235	0–0.09	1.8	3.2
	0.09–0.14	0.8	1.6
	0.14–0.28	0.3	0.9
239	0–0.09	2.6	2.2
	0.09–0.14	1.3	1.2
	0.14–0.28	0.8	0.9
252	0–0.09	2.6	2.6
	0.09–0.14	1.6	1.2
	0.14–0.28	0.9	0.9

Therefore a value of the thermal diffusivity lower than for a wet unsaturated soil would be expected. Thus the value calculated with the 15-h fitting procedure is realistic. Notice that, with this fitting time, the thermal diffusivity of the layer (9–14 cm) is 1.3×10^{-6} $m^2 s^{-1}$ instead of the value of $1.6.10^{-6}$ $m^2 s^{-1}$ given in Table 3. From these results, it seems that when abrupt changes occur in the temperature pattern, only the CLTM method is able to give satisfactory values of the apparent soil thermal diffusivity. As abrupt changes are associated with rainfall events in this experiment, only the CLTM method is able to provide realistic values of the apparent soil thermal diffusivity when soil is very wet.

5. Conclusions

Two data sets from the HAPEX–Sahel experiment are used to estimate the apparent soil thermal diffusivity. These sets allow for testing the sensitivity of several current methods to their basic assumptions. Methods used in this work are compared according to whether the soil thermal properties are considered vertically homogeneous or not. Results presented in this paper show that:

1. Methods based on the Laplace transform should not be used without a correction for non-uniform initial profile of temperature, even if the initial profile appears quasi-uniform. This correction is made in the CLTM method.
2. If the assumption of steady-periodicity for temperature is valid, both the Harmonic Method (HM), and the Corrected Laplace Transform based Method (CLTM), lead to identical results. As the HM is more simpler to program and less CPU time-consuming than the CLTM, the use of the HM is recommended.
3. The harmonic method, when used on days just after a abrupt climate modification when the assumption of steady-periodicity is not valid, leads to errors in the estimation of thermal diffusivity. In these situations, the CLTM is more appropriate. For the HAPEX–Sahel experiment, only the CLTM is able to give values of the apparent soil thermal diffusivity when the upper soil layer is very wet. Moreover, the CLTM can be used with a shorter fitting time and does not require the surface temperature to be fitted by a Fourier series. For instance, Singh and Sinha (1977) considered temperature evolution through time fitted partially to a linearly rising or falling curve or to an exponentially rising or falling curve. In these cases, the time span of the data needed for fitting may be reduced but the CLTM can then only be applied on a limited depth of soil. This depth of soil depends on the velocity of the temperature wave penetration.
4. The NHS method, based on Nassar and Horton's procedure from the Lettau's soil heat transfer theory for non homogeneous soil is used with both data sets. When used with the first data set (week 39), where the steady-periodic assumption is valid, the profile shape of the dependency of K with depth could be explained partially by the vertical variation of the soil dry bulk density and soil moisture. Comparing the values given by this method with those obtained from the HM or CLTM, we deduce that these last two methods overestimate the soil thermal diffusivity when the assumption of vertical homogeneity is not observed. In the other data set, where the assumption

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of steady periodicity is not valid, the NHS method leads to inconsistent values and is not applicable. In this case only the CLTM gives satisfactory values.

For the CLTM method, the initial temperature profile must be described accurately, and to a greater depth than the deapest layer which soil thermal diffusivity is to be calculated for. For the NHS method, accurate interpolations must be done for the Lettau's theory parameters. This implies that at any instant the temperature profile must be known accurately. Therefore for both these methods, temperature must be measured at several points even beneath the depth of the deeper studied layer. This is not necessary with the HM method.

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