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# Estimating hydraulic conductivity of crusted soils using disc infiltrometers and minitensiometers

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#### Abstract

Although soil crusting has long been recognized as a crucial runoff factor in the Sahel, very few field methods have been developed for the measurement of the crust hydraulic conductivity, which is difficult to achieve because of the small thickness of most surface crusts. A field method, based on the simultaneous use of disc infiltrometers and minitensiometers is proposed for determining the crust hydraulic conductivity and sorptivity near saturation. On crusted soils, the classical analysis of the steady state water flow was found to be inadequate. The proposed method is based on sorptivity measurements performed at different water supply potentials and uses recent developments of transient flow analysis. A minitensiometer, placed horizontally at the crust-subsoil interface, facilitated the analysis of the infiltration regime for the crust solely.

Results are shown for representative soil units of the East Central Super Site of the HAPEX-Sahel experiment: fallow grasslands, millet fields and tiger bush. Non-crusted soils were also considered and validated the transient method as demonstrated by comparison with Wooding's steady state solution. This validation was obtained in the case of fallow grasslands soil but not for the millet fields. In this latter case, the persistent effects of localized working of the soil to remove weeds caused large variations in infiltration fluxes between the sampling points, which tended to dominate over effects of differences in applied potential. For the tiger bush crusted soils, the ratio of the saturated hydraulic conductivity of the crust to that of the underlying soil ranges from 1/3 to 1/6, depending on whether the crust is of a structural (ST) or sedimentation (SED) type. The method also allows the estimation of a functional mean pore size, consistent with laboratory measurements, and 40% less for the crusts in comparison with the underlying soil.

The results obtained here will be used in hydrological models to predict the partition of rainfall between infiltration and runoff.

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#### 1. Introduction

To model the interactions between the continental biosphere and the atmosphere, estimation of the water balance components requires knowledge of the hydraulic properties of the soil including the relationship between unsaturated hydraulic conductivity, K, and the soil water pressure, h, or the soil volumetric water content,  $\theta$ .

Tension disc infiltrometers have become an increasingly popular device for in-situ measurement of K close to natural saturation (Clothier and White, 1981; Perroux and White, 1988; Thony et al., 1991; Mohanty et al., 1994), and papers which compare results obtained from different data analysis methods recently appeared (Hussen and Warrick, 1993; Logsdon and Jaynes, 1993; Cook and Broeren, 1994). More convenient to perform than internal drainage experiments, the methodology is an ideal tool for spatial variability studies and, additionally, it provides estimates of physical variables such as capillary sorptivity, and different characteristic time and length scales.

Based on properties of three-directional axisymmetric infiltration, most existing analytical analysis methods for disc infiltrometer data require the attainment of steady state flow, for which a simple two-term expression was found (Wooding, 1968). Assuming an exponential relationship between *K* and *h* (Gardner, 1958):

$$K = K_{\rm S} \exp(\alpha H) \tag{1}$$

where  $K_s$  is the saturated hydraulic conductivity and  $\alpha$  is a shape factor related to a functional pore size (Philip, 1987), Wooding showed that the unconfined steady state flux density averaged over a source area of radius r can be approximated by:

$$q(h_{\rm f}) = K(h_{\rm f}) + \frac{4\Phi(h_{\rm f})}{\pi r} \tag{2}$$

where  $h_1$  is the water pressure head at the surface ( $h_1 \le 0$ ) and  $\Phi$  is the matric flux potential defined by:

$$\Phi(h_{\Gamma}) = \int_{h_{\Gamma}}^{h_{\Gamma}} K(h) \mathrm{d}h \tag{3}$$

where the subscripts "i" and "f" refer to initial and boundary supply pressure conditions, respectively.

The hydraulic conductivity can thus be calculated, either by using different source radii (Scotter et al., 1982; Thony et al., 1991), or by using multiple supply potentials with the same disc (Reynolds and Elrick, 1991; Ankeny et al., 1991). However, the restrictive assumptions underlying Wooding's solution, i.e. homogeneous and isotropic soil with a uniform initial moisture content, may lead to unrealistic results including negative values of K (Hussen and Warrick, 1993; Logsdon and Jaynes, 1993).

During the HAPEX-Sahel experiment (Goutorbe et al., 1994), the main difficulty in using steady state infiltrometer methods was the presence of surface crusts which play a major role in the hydrology of the Sahelian zone as shown by many authors (Hoogmoed and Stroosnijder, 1984; Casenave and Valentin, 1989; Casenave and Valentin, 1992). Indeed, the partition between infiltration and runoff at the surface of a crusted soil depends on the hydrodynamic properties of both the crust and the underlying soil. While many

attempts at quantifying the effect of a surface crust on one-dimensional infiltration have been reported (Hillel and Gardner, 1969, 1970; Ahuja, 1974; Smiles et al., 1982; Parlange et al., 1984), to our knowledge field experiments under axisymmetric flow conditions have not been performed on crusted soils. We found that classical methods of analysis applied to infiltration tests fail for crusted soils, leading to unrealistic values of K and  $\Phi$  in almost all cases. Indeed, steady state infiltration into a crusted soil involves a complex combination of the hydrodynamic properties of both layers. While it is only the crust properties which play a role at early stages, the hydraulic conductivity of the crust—soil system tends to that of the subsoil at long times. Therefore, to estimate the conductivity of the crust, we developed a specific methodology using a minitensiometer placed at the crust—subsoil interface with transient flow analysis of infiltration into the crust only. The main motivation for this study is in the fact that knowledge of the hydraulic properties of both the crust and the subsoil allows the infiltration of rainfall under any conditions of intensity and duration to be modelled.

#### 2. Theory

The proposed method is based on sorptivity determinations obtained by analyzing transient flow from disc infiltrometer experiments performed at different water supply potentials,  $h_i$ . For each test, matric flux potential is calculated from the corresponding sorptivity value and hydraulic conductivity is obtained by differentiating the matric flux potential with respect to  $h_i$ .

#### 2.1. Transient flow and sorptivity

While one-dimensional soil infiltration is well described analytically, there were, until recently, few theoretical works on three-dimensional unconfined infiltration for a disc source. Turner and Parlange (1974) calculated an approximate analytical solution for the lateral movement at the periphery of a one-dimensional water flow. Warrick and Lomen (1976) proposed an expression for the matric flux potential as a function of time valid for a disc source and a '\(\pi\)-soil', that is, described by Eq. (1). Cumulative infiltration as a function of time in axisymmetric conditions can also be predicted by numerical models (e.g. Warrick, 1992; Quadri et al., 1994) which require the complete soil hydrodynamic description. Their use with the objective to determine the soil's hydraulic conductivity through inverse procedures is thus complicated by the number of parameters to be estimated and subsequent problems dealing with possible non-uniqueness of the solution.

However, restrictions in the use of Wooding's equation, uncertainties about the time at which steady infiltration flux is attained, together with the fact that much useful information is lost by ignoring the transient stage have strengthened the need for a transient three-directional infiltration equation for disc infiltrometers. Two expressions were recently proposed for this purpose (Warrick, 1992; Haverkamp et al., 1994), having in common that the supplementary term introduced by considering unconfined edge flow is linear with time. Then, the expression of Philip (1957) for

one-dimensional infiltrated depth, I<sub>1d</sub>:

$$I_{\rm ld} = S\sqrt{t} + At \tag{4}$$

where t is time, S is the capillary sorptivity, and A is a constant ( $LT^{-1}$ ), is modified into:

$$I_{3d} = S\sqrt{t} + (A+B)t \tag{5}$$

where  $I_{3d}$  is the cumulative three-directional infiltrated depth and B is a constant expressed by (Haverkamp et al., 1994):

$$B = \frac{\gamma S^2}{r(\theta_f - \theta_i)} \tag{6}$$

where  $\gamma$  is a dimensionless constant and  $\theta_i$  and  $\theta_f$  are initial and final volumetric water content, respectively. Sorptivity can be determined from either non-linear fitting of Eq. (4) or Eq. (5) to field data (Bristow and Savage, 1987) or, as suggested by Smiles and Knight (1976), as the intercept of the regression of  $I/\sqrt{t}$  against  $\sqrt{t}$ , using one of the following expressions:

$$\frac{I_{\rm id}}{\sqrt{t}} = S + A\sqrt{t} \tag{7a}$$

$$\frac{I_{3d}}{\sqrt{t}} = S + (A + B)\sqrt{t} \tag{7b}$$

for one- and three-dimensional cases, respectively.

To ensure the hydraulic contact between the disc infiltrometer and the soil, it is often necessary to place a fine layer of sand at the soil surface. Because of this layer, methods using cumulative data, including that of Smiles and Knight, are compromised. This is particularly the case for low permeability soils, due to the relatively large amount of water stored at early time in the sand. Indeed, taking this effect into account modifies Eq. (5) into:

$$I_{3d} = I_0 + S\sqrt{(t - t_0)} + (A + B)(t - t_0)$$
(8)

where  $I_0$  and  $t_0$  are, respectively, the depth of water and the time necessary to wet the sand layer in equilibrium with  $h_0$ . Then, Eq. (7) becomes:

$$\frac{I_{3d}}{\sqrt{t}} = \frac{I_0}{\sqrt{t}} + S\sqrt{\frac{t - t_0}{t}} + (A + B)\frac{t - t_0}{\sqrt{t}}$$
(9)

When  $I_0$  is large compared with S, A, and B, that is when the soil has low conductivity and sorptivity, the relationship between  $I_{3d}/\sqrt{t}$  and  $\sqrt{t}$  is far from linear due to the effect of the first term in the right-hand side of Eq. (9).

The influence of the sand layer is usually neglected in steady state situations and it is generally assumed that it has no effect on the final flux value. Eq. (8) shows that this influence should be taken into account for transient flow analysis, especially when a large amount of sand is applied to overcome surface roughness. Rather than analyzing cumulative infiltration data, a way to circumvent the need for  $I_0$  is to differentiate the cumulative infiltration with respect to the square root of time. Performing this differentiation on

Eq. (5) yields:

$$\frac{\Delta I_{3d}}{\Delta \sqrt{t}} \approx \frac{\partial I_{3d}}{\partial \sqrt{t}} = S + 2(A + B)\sqrt{t}$$
 (10)

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and for Eq. (8), the result is:

$$\frac{\Delta I_{3d}}{\Delta \sqrt{t}} \approx \frac{\partial I_{3d}}{\partial \sqrt{t}} = S \sqrt{\frac{t}{t - t_0}} + 2(A + B) \sqrt{t}$$
(11)

The difference between Eqs. (9) and (11) is that the latter one is not influenced by  $I_0$  and the correction due to  $t_0$  quickly becomes small as time increases.

Sorptivity, initially introduced as the variable driving horizontal absorption, is commonly considered to control the early stages of vertical infiltration as well, when the effect of gravity is minor. S depends on both initial and boundary conditions. Although its exact analytical expression is not known, many approximations have been proposed (Elrick and Robin, 1981 present a review of these). White and Sully (1987) showed that S is related to the matric flux potential through the expression:

$$\Phi(h_{\rm f}) = \frac{bS^2(h_{\rm i}, h_{\rm f})}{(\theta_{\rm f} - \theta_{\rm i})} \tag{12}$$

where b is a parameter depending on the shape of diffusivity and having a value in the range  $1/2 \le b \le \pi/4$ . A reasonable intermediate value of 0.55 can be taken for most field (Smettern and Clothier, 1989) and theoretical (Warrick and Broadbridge, 1992) situations. No dependence of the b parameter on  $h_f$  was considered in our study.

# 2.2. From sorptivity to conductivity

Following Smiles and Harvey (1973), White and Perroux (1989) proposed to estimate conductivity values from sorptivity measurements performed at different supply water potentials. Indeed, Eq. (3) shows that K can be deduced by differentiation of  $\Phi$  against  $h_i$ :

$$\frac{\partial \Phi}{\partial h_i} = K_i - K_i \tag{13}$$

where  $K_i$  is negligible as compared with  $K_f$  in most field situations. Combination of Eqs. (12) and (13) enables deducing K from two or more S values. To use, simultaneously, the entire set of  $(\Phi, h_f)$  data obtained for each test by Eq. (12), an analytical form of the  $\Phi(h_f)$  function is required. It is convenient to keep the exponential form of Eq. (1) for its ease of integration, which gives:

$$\Phi(h) = \frac{K_s}{\alpha} \exp(\alpha h) \tag{14}$$

Parlange (1972) claims that  $\alpha$  should not be considered as a constant over the whole range of h. Thus, it is simply assumed here that variations of  $\alpha$  with  $h_f$  are small within the range of potentials covered by the suction disc infiltrometer (typically between 0 and 150 mm of water). Moreover, it must be assumed that the decrease in  $\alpha$  for  $h \to h_i$ , which is very

likely, has little effect on the total area covered by  $\Phi$  between  $h_i$  and  $h_f$ . This assumption is justified if the K(h) function is concave upwards.

Eq. (14), which can be fitted to the experimental values for an estimation of  $K_s$  and  $\alpha$ , has the advantage to provide, through the  $\alpha$  parameter, an effective pore size  $(\lambda_m)$  from simple capillarity theory (Philip, 1987):

$$\lambda_{\rm m} = \frac{\sigma \alpha}{\rho g} \tag{15}$$

where  $\sigma$  is surface tension,  $\rho$  is water density, and g is acceleration due to gravity.

Knowledge of sorptivity and conductivity enables estimation of  $t_{grav}$ , the time after which gravitational forces dominate capillary effects (Philip, 1969):

$$t_{\text{grav}} = \left(\frac{S}{K}\right)^2 \tag{16}$$

For  $t \ll t_{\text{grav}}$ , Eq. (4) can be reduced to its first term:

$$I_{\rm Id} = S\sqrt{t} \tag{17}$$

This provides yet another sorptivity estimate,  $S_1$ , by knowing the installation depth of the minitensiometer,  $Z_1$ , and the time,  $t_1$ , at which it is reached by infiltrating water. Assuming a 'Green and Ampt' behavior, the integral defining S:

$$S = \int_{\theta}^{\theta_1} z(\theta) t^{-1/2} d\theta \tag{18}$$

where z is depth and  $\theta_1$  is the volumetric water content of the crust at  $t_1$ , becomes:

$$S_1 = \frac{Z_1(\theta_1 - \theta_i)}{\sqrt{t_1}} \tag{19}$$

As the infiltration is pursued long after  $t_1$ ,  $\theta_1$  is not necessarily equal to  $\theta_1$  measured at the end of the experiment. Indeed, the water content of the crust may still increase after  $t_1$  and the gradient of  $\theta$  behind the wetting front might be significant. Consequently,  $\theta_1$  has to be calculated from the cumulative infiltration  $I_1$  at the instant  $t_1$ , subtracting the amount of water stored into the contact layer (estimated by knowing the volume and the porosity of the sand) and the amount of water corresponding to the edge effect (estimated by lateral wetting front measurements at  $t_1$  and assuming a parabolic profile into the soil around the disc).

Nevertheless, the uncertainty in  $t_1$  is the main source of poor accuracy on  $S_1$  estimates and values calculated by Eq. (19) do not provide good estimates of  $\Phi$  by Eq. (12). However, their comparison with values obtained from Eq. (11) allows any systematic bias to be checked, as well as revealing the potential for water blockage at the crust–subsoil interface.

Due to the usual difficulties in field measurements of sorptivity, little use was made of the White and Perroux (WP) method. Here, an entire set of S estimates is used to fit a single relationship between  $\Phi$  and  $h_f$ , but because of the need for a same initial water content for all measurements, tests must be performed at different locations. Consequently, spatial variability of S (Talsma, 1969; Sharma et al., 1980) may require a large number of

replications to obtain reliable estimates. Indeed, the slope of the  $\Phi(h_i)$  relationship must be estimated with enough accuracy for the differentiation operation to be performed (see Eq. (13)). The scatter in the  $\Phi(h_i)$  dataset may be important as a result of the use of squared sorptivity values.

These difficulties may explain why most of the authors have used classical steady state methods when they are applicable. It seems that only Cook and Broeren (1994) compared conductivities obtained in-situ from steady state methods with those obtained from the WP method. Thus, it appeared suitable to perform a comparison between WP and steady state approaches on non-crusted soils before applying the former one to crusted soils.

#### 2.3. Error analysis

Logarithmic differentiation of Eq. (12) yields:

$$\frac{\mathrm{d}\Phi}{\Phi} = \frac{\mathrm{d}b}{b} + 2\frac{\mathrm{d}S}{S} + \frac{\mathrm{d}\theta_{\mathrm{i}}}{\theta_{\mathrm{i}}} + \frac{\mathrm{d}\theta_{\mathrm{i}}}{\theta_{\mathrm{i}}} \tag{20}$$

The standard error due to the use of the approximate value of 0.55 for the parameter b is difficult to estimate. The theoretical range  $1/2 \le b \le \pi/4$  corresponds to a maximum error and not to a standard deviation. A value of 0.1 for the relative error of b was arbitrarily taken in this study. For each test, the error on S determination, as the intercept of the line corresponding to Eq. (11), is given by the regression analysis. The errors made on  $\theta$  estimations (wet and dry weighing of samples, bulk density estimation) were found to be negligible as compared with the previous errors.

The  $\log_e \Phi$  values are plotted against  $h_1$  to determine the parameters  $K_s$  and  $\alpha$  by fitting the linearized form of Eq. (14):

$$\log_{c} \Phi = \log_{c} \left( \frac{K_{s}}{\alpha} \right) + \alpha h_{1} \tag{21}$$

For the sake of simplicity, we put:

$$\beta = \log_{c}\left(\frac{K_{s}}{\alpha}\right) \tag{22}$$

Because each point was obtained at a single spot, the errors on  $K_s$  and  $\alpha$  due to scatter of the data account for spatial variability of soil properties, causing a location error. The linear regression analysis gives standard deviation errors,  $\Delta \alpha$  and  $\Delta \beta$ , on the slope  $\alpha$  and the intercept  $\beta$ , respectively.

Define the following bounds:

$$\alpha^{+} = \alpha + \Delta \alpha \tag{23}$$

$$\alpha^{-} = \alpha - \Delta \alpha \tag{24}$$

$$\beta^{+} = \beta + \Delta \beta \tag{25}$$

$$\beta^{-} = \beta - \Delta \beta \tag{26}$$

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According to Eq. (22),  $K_s$  is simply calculated by:

$$K_{\rm s} = \alpha \exp \beta \tag{27}$$

It should be noted that the errors made on the intercept  $\beta$  and the slope  $\alpha$  cannot be considered as independent. Indeed, if the slope is overestimated for example, the intercept is very likely to be overestimated also (since  $h_f$  values are in the negative domain of the abscissa axis). Consequently, the location error made on  $K_s$  estimates is maximized by the product operation between  $\alpha$  and  $\beta$ . There is no analytical expression for the error of the product of variables affected by non-independent errors. Thus, only the following bounds can be obtained:

$$K_s^+ = \alpha^+ \exp \beta^+ \tag{28}$$

$$K_{s}^{-} = \alpha^{-} \exp \beta^{-} \tag{29}$$

The location error on  $K_s$  is log-normally distributed and characterized by a factor f defined by:

$$f = \sqrt{\frac{K_s^+}{K_s^-}} \tag{30}$$

with:

$$K_s^- = K_s / f \le K_s \le K_s^+ = f K_s$$
 (31)

#### 3. Materials and methods

Millet crops and fallow grasslands covers are found, on sandy soils, all over the East Central Super Site of the HAPEX-Sahel experiment, except on the plateaus. Thus, they represent the main soil feature to be considered in the study of the biosphere—atmosphere interactions and their hydrodynamic characterization is essential for modeling purposes (Braud et al., 1997). Millet and fallow soils are subject to crusting (Casenave and Valentin, 1989). In non-crusted areas, soil can be considered to be vertically homogeneous.

The tiger bush, covering lateritic plateaus is characterized by a sandy-loamy-clay soil with a small hydraulic conductivity, producing high values of runoff, increased by the existence of large crust-covered areas separating vegetative strips. During the rainy season (June-September 1993), two types of crust, structural (ST) and sedimentation (SED), were selected for their spatial representativity and their case of identification. The former can be found downslope of the vegetative strips and are formed by the sieving effect of raindrop impacts which concentrate fine particles at the basis of the structure with sand on top. Gravels are frequently included at the surface (type ST3, following the classification proposed by Valentin and Bresson, 1992). SED crusts, abundant in zones of accumulation of water, are found upslope of the vegetative strips and are formed by sedimentation of particles in small pools after rainfall events. Detailed description and analysis of Sahelian soil crusting is given in Casenave and Valentin (1989) and Valentin and Bresson (1992). Infiltration tests were also performed on the underlying soil to estimate difference in

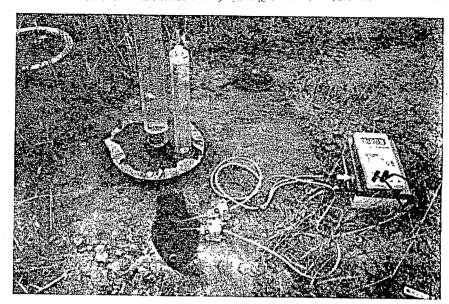


Fig. 1. Fallow grassland; disc infiltrometer D1 (left) and soil water pressure monitoring (here at two depths) with the pressure transducers (middle) connected to the alimentation and the voltmeter (right).

conductivity between crust and subsoil. For the tiger bush tests, initial volumetric water content was always lower than 5%, with an average of 1.9%.

Infiltration tests were carried out during two consecutive summers (1992 and 1993) with disc infiltrometers of 125 (D1) and 40 mm (D2) radius (Thony et al., 1991; Vauclin and Chopart, 1992). To monitor the time evolution of the soil water pressure, a minitensiometer (20 mm length, 2.2 mm diameter) connected to a pressure transducer (Data Instruments, Lexington, MA, USA) through a capillary tubing (1.45 mm internal diameter), was inserted horizontally under the disc at a depth ranging from 1 cm for crusted soils to 5 cm for sandy non-crusted soils. An overall view of the equipment is presented in Fig. 1.

In the millet fields and fallow sites (85% sand) infiltration tests were performed until a steady state flow condition was attained. While tests on millet plots were done between the plants, tests on fallow plots were carried out after removing about 1 cm of soil and cutting the above-ground vegetative cover while leaving the roots in place. The multi-radii method was used with  $h_1 = -10$ , 40, 70, and 100 mm of water, with three replications for both discs, representing together a total of 48 tests. For each test, measurement of initial and final water content, performed by taking disturbed gravimetric samples, allowed determination of corresponding sorptivity and matric flux potential values (Eq. (12)) and comparison of steady state and transient analyses results. The single-radius multiple-potential method (Reynolds and Elrick, 1991; Ankeny et al., 1991), was also used for comparison.

In the tiger bush, the minitensiometer was installed only under the large disc infiltrometer (D1) because of a lower relative disturbance of soil. Moreover, the sorptivity

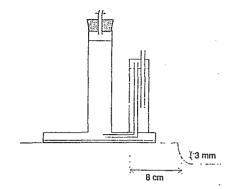


Fig. 2. Schematic implementation outline of the minitensiometer.

estimation with the small disc infiltrometer (D2), is quite inaccurate as the first term at right-hand side in Eq. (5) becomes relatively small as the radius decreases (see Eq. (6)). Thus, the sorptivity estimations used for this study were all obtained with D1. Because of the fragility of the crusts, the minitensiometer cannot be installed in dry conditions without the risk of significant soil disturbance (detachment, fracture, or cracking). Consequently, it was necessary to drill a hole into the soil, while injecting by small increments a total of about 1 cm<sup>3</sup> of water with a syringe. Installation of the tensiometer into this hole and infiltration test were performed 24 h later when the soil has dried. The porous cup was inserted at about 1 cm depth (its exact depth was measured after the test), at about middistance between the center and the edge of the disc (Fig. 2). Fig. 3 shows the minitensiometer under a SED crust after an infiltration test.

A ring of the same radius as the disc infiltrometer was placed on the soil surface and sand was placed in it and leveled. The quantity of sand was measured to estimate  $I_0$  in Eq. (8) and  $t_0$  in Eq. (11). The disc infiltrometer was then placed on this bed of sand and the cumulative infiltration was monitored. During infiltration, the tensiometer response to the arrival of the vertical wetting front, defined by the maximum of  $\partial h/\partial t$ , facilitates restricting the analysis to the stage corresponding to the crust only. At that moment  $(t_1)$  the progression of the lateral wetting front at the surface was also measured to estimate the ratio,  $I_1 \omega I_{3d}$ .

In order to quantify the deficit of moisture in the subsoil due to the crust impeding effect, infiltration tests were continued after  $t_1$ , until the tensiometer showed an approximately constant soil water pressure. At the end of every test, disturbed soil samples were taken at two depths and also around the disc for determination of final gravimetric water content. Two sampling operations were systematically performed, with about 30 s interval, so that the effect of the delay of sampling after the removal of the disc could be estimated (a decrease of about 0.01 g g<sup>-1</sup> was found).

Gravimetric water contents were converted into volumetric values through dry bulk density ( $\rho_d$ ) measured from undisturbed samples collected at the end of the season, following the method proposed by Fies and Zimmer (1982). Due to the impossibility to take undisturbed samples of ST crusts, a value of 1.7 g cm<sup>-3</sup> was calculated by assuming a

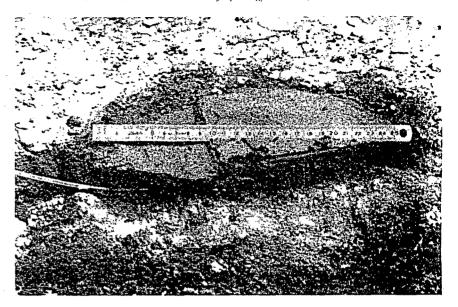


Fig. 3. Tiger bush, sedimentation crust: the inserted micro-tensiometer after removing half of the infiltrated zone at the end of a test. Note the sand contact layer partially unremoved (right).

specific density of 2.65 for the solids and assuming that measured saturated gravimetric water content represents 85% of porosity (this percentage was obtained from other crust types). However, this is a minor uncertainty compared with other sources of error.

In the tiger bush, 59 infiltration tests were performed with the disc D1, including 20 on ST crusts, 23 on SED crusts, and 16 on the underlying soil.

#### 4. Results and discussion

#### 4.1. Homogeneous soils: fallow and millet

For the fallow grassland, sorptivity and matric flux potential estimates obtained by Eqs. (11) and (12), respectively, are shown in Fig. 4, together with the  $\Phi(h)$  curve obtained by fitting Eq. (14) to the data. Its derivative (K(h)) is plotted against results from steady state methods in Fig. 5, which shows the good agreement between all the methods for the fallow. Both multi-radii and multi-potential experiments exhibit a shift in conductivity near saturation. This change of the general slope, at about -20 mm, is characteristic of a change in functional pore size. The  $\alpha$  values, about  $10 \text{ m}^{-1}$  for h < -20 mm and  $40 \text{ m}^{-1}$  for h > -20 mm, corresponds to pore radii of 80 and 330  $\mu$ m, respectively. We attribute the shift to larger functional pore size to the influence of grass roots. This shift does not appear with the transient method result, which uses a single exponential function for the

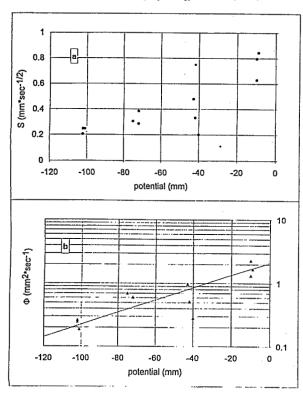


Fig. 4. Fallow grasslands: (a) sorptivity and (b) matric flux potential calculated (Eq. (12), triangles) and fitted (Eq. (14), continuous line), as a function of supply water potential.

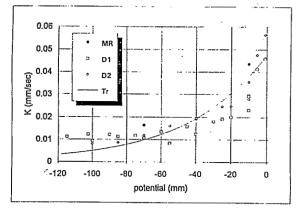


Fig. 5. Fallow grasslands: hydraulic conductivity as a function of water potential, by multi-radii analysis (MR), multi-potential analyses with the large (D1) and the small (D2) discs and transient analysis (Tr).

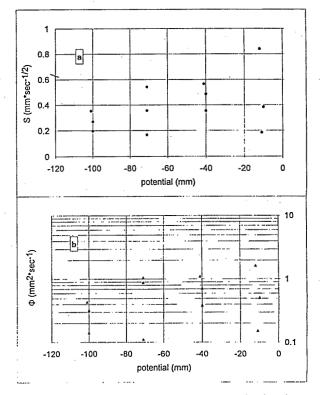


Fig. 6. Millet fields: (a) sorptivity and (b) matric flux potential, as a function of water potential,

whole domain. However, it is remarkable that the 'transient'  $\alpha$ , 22 m<sup>-1</sup> corresponds to a good mean value and that  $K_s$  is almost unchanged (Fig. 5).

For the millet cover, the transient method was found to be inadequate due to the high spatial variability of S. Indeed, the effects of localized working of the soil act to increase the infiltration flux only for part of the tests. Peugeot et al. (1997) found that these effects may persist up to 80 mm of rainfall. The slope of  $\Phi$  versus h (Fig. 6(b)) cannot be estimated with reasonable accuracy, since the effects of spatial variability dominate those of supply potentials. In such a case, the differentiation operation becomes hazardous and thus, it is unrealistic to expect a correct estimation of conductivity. Consequently, the transient method should not be applied in the case of a high field heterogeneity, unless a very large number of tests is performed.

#### 4.2. Crusted soils: tiger bush

# 4.2.1. Infiltration

Fig. 7 shows, for a typical infiltration test carried out at  $h_f = -10$  mm of water on a ST crust, measured cumulative infiltration and soil water pressure as a function of time

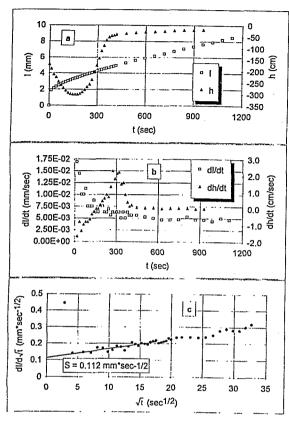


Fig. 7. Tiger bush, infiltration test at  $h_1 = -10$  mm; (a) measured cumulative infiltration depth (I) and soil water pressure (h); (b) infiltration flux (dI/dt) and soil water pressure variation with time (dh/dt); (c) sorptivity estimation by Eq. (11).

(Fig. 7(a)), the corresponding derivatives with time (Fig. 7(b)) and the sorptivity value obtained by using Eq. (11) (Fig. 7(c)). The early decrease in h is explained by the initial non-equilibrium of the tensiometer with its soil environment. Indeed, as soon as the porous cup is in contact with the dry soil (that is, a few minutes before the infiltration test begins), the water pressure decreases sharply to reach equilibrium with the low soil water potential. To avoid loosing the hydraulic contact between the porous surface and the soil, the infiltration test should be performed as quickly as possible after installing the tensiometer.

It can be noted that the final value of h does not correspond to the pressure applied, because of the well-known moisture deficit of the subsoil due to the impeding effect of the crust (Aboujaoudé et al., 1991; Touma, 1992). The measured value of h is an intermediate value between the water pressures in the crust and the subsoil which are not necessarily equal. The wetting front arrival can be identified at  $t_1 = 300$  s (maximum of  $\partial h/\partial t$ , as seen in Fig. 7(b)). Thus, the cumulative infiltration data after 300 s are not considered in the

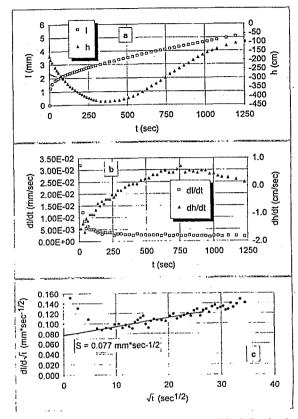


Fig. 8. Tiger bush, infiltration test at  $h_1 = -100$  mm; (a) measured cumulative infiltration depth (*I*) and soil water pressure (*h*); (b) infiltration flux (d*l/dt*) and soil water pressure variation with time (d*h/dt*); (c) sorptivity estimation by Eq. (11).

regression analysis of Eq. (11) to estimate sorptivity. The obvious early influence of the sand layer is revealed by the first point of Fig. 7(c) which falls far away from the linear behavior of the plotted data.

Similar graphs corresponding to another infiltration test carried out at  $h_1 = -100$  mm of water (Fig. 8) show a smoother response of the tensiometer, decreasing the accuracy of the  $t_1$  estimate (about 750 s), both smaller flux and sorptivity, and a longer sand influence (about 25 s as seen in Fig. 8(c)). The inaccuracy on  $t_1$  has obviously a large influence on the precision of  $S_1$  estimates (see Eq. (19)), but its effect on S is small.

Measurements of lateral wetting front advance, both at  $t_1$  and at the end of infiltration, showed an accentuated anisotropy of the wetting zone since the ratio of the lateral to the vertical front was found to be about 3.5 for ST and 2.6 for SED. This is probably one of the reasons why classical steady state methods lead to negative values of K for layered soils. It should be noted that our method avoided the effect of soil anisotropy

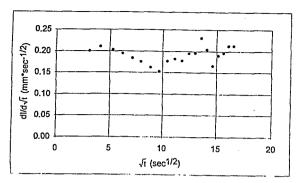


Fig. 9. Tiger bush: infiltration test rejected for sorptivity determination (Eq. (11)).

since only the intercept and not the slope of Eq. (11) is being analyzed. However, the method is flexible enough to account for anisotropy through the parameter B in Eqs. (5) and (11).

When infiltration data are plotted according to Eq. (11), the linearity is essential for sorptivity determination. Correlation coefficients higher than to 0.95 were commonly obtained, but in 35% of the cases, a curved behavior led to rejection of the results for  $\Phi$  calculations. We suspect that the reason for which some of the experiments do not exhibit good linearity is related to the excessively large amount of sand required for situations with high surface roughness. Indeed, when a thick layer of sand is present between the disc and the soil, it is difficult to determine the number of early readings which should be rejected for sorptivity determination. As a matter of fact, the transition moment from infiltration into the sand layer to infiltration into the soil, is then difficult to see because of overlap between the two phenomena. This is illustrated in Fig. 9 which corresponds to a rejected test.

No significant difference was found between results for soil underlying ST crusts and soil underlying SED crusts. Fig. 10 presents the  $\log_c \Phi$  versus  $h_1$  data for the three categories, ST, SED, and subsoil (SUB). The correct linearity which appears, despite the important scatter due to the use of squared S values, validates the choice of the exponential form (Eqs. (1) and (14)). Values of the different hydraulic parameters are summarized in Table 1. As shown in Peugeot et al. (1997), the  $K_s$  values obtained here for the crusts are quite consistent with the results of Casenave and Valentin (1992) who measured, in initially wet conditions and on the same crust types, critical values of rainfall intensity below which runoff does not occur.

Table 1

Tiger bush: hydraulic parameters for structural (ST) and sedimentation (SED) crusts and for the subsoil (SUB)

· <u> </u>	ρ <sub>d</sub> (g cm <sup>-3</sup> )	θ (cm³ cm²³)	K, (mm s <sup>-1</sup> )	S (mm s <sup>-1/2</sup> )	α (mm ·1)	λ <sub>m</sub> (μm)	t <sub>grav</sub> (h)
ST	1.7	0.31	8.5e - 4	0.18	0.014	105	13
SED	1.47	0.35	5.2e - 4	0.15	0.015	115	22
SUB	1.56	0.34	2.8e - 3	0.27	0.023	175	2.5

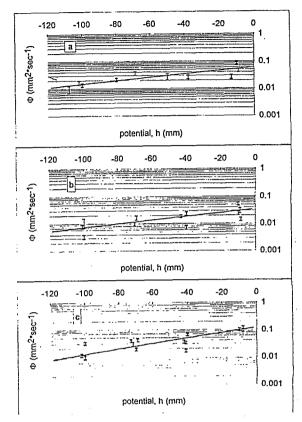


Fig. 10. Tiger bush, matric flux potential calculated (Eq. (12), bars) and fitted (Eq. (14), continuous line) as a function of water potential: (a) structural crusts, (b) sedimentation crusts, (c) subsoil. Error bars correspond to Eq. (20).

#### 4.2.2. Error analysis

The ratio dS/S, given by the regression analysis (Eq. (11)) performed for each infiltration test, varies between 0.013 and 0.28 with a mean value of 0.056. This error is of the same order of magnitude as the error due to the use of the approximate value of 0.55 for the parameter b. Finally, Eq. (20) gives a mean relative error of about 20% for the matric flux potential estimates. The corresponding error bars are shown in Fig. 10.

Parameters of the location error analysis on  $K_s$  estimates (Eqs. (22)–(31)) are summarized in Table 2 for ST, SED, and SUB. The location error factor, ranging from 1.6 to 1.8, shows that spatial variability has the larger effect on the uncertainty of  $K_s$  estimates. This means that, to improve the global accuracy of the method, it is more important to increase the number of replications, than to perform a more accurate S determination for each test. Taking into account location error and error of  $\Phi$  estimates, the global error made in  $K_s$  determination is log-normally distributed with a standard deviation of a factor  $\pm 2$ .

Table 2

Tiger bush: parameters of the location error analysis (Eqs. (22)–(31)) for structural (ST) and sedimentation (SED) crusts and for the subsoil (SUB)

	ST	SED	SUB
β+	- 2.66	- 3.25	- 2.05
β-	- 3.12	- 3.76	- 2.56
α <sup>+</sup> (mm <sup>-l</sup> )	0.017	010.0	0.023
$\alpha^{-}$ (mm <sup>-1</sup> )	0.010	0.010	0.015
$K_{\rm s}^{+}$ (mm s <sup>-1</sup> )	1.2e - 3	7.3e - 4	2.9e - 3
K- (mm s-1)	4.6e - 4	2.4e - 4	1.1e - 3
f	1.61	1.75	1.61

Mohanty et al. (1994) carried out a spatial analysis of hydraulic conductivity measured using disc infiltrometers and obtained coefficients of variation for  $K_s$  between 75% and 125%. Thus, the error that we obtain here for  $K_s$  can be considered reasonable.

# 4.2.3. Characteristic length and time scales

The characteristic mean pore size  $\lambda_m$  (see Table 1), calculated by Eq. (15), with a precision of 30% (see Table 2), shows a decrease of 40% between subsoil and crust. These values were compared with results obtained by the mercury porosimetry technique (Fies, 1992a) on samples collected at the same place where the infiltration tests were performed. Fies showed that the porosity is bi-modal: one domain, characteristic of the clay fraction, has a pore size ranging from  $10^{-2}$  to  $10^{-1}$   $\mu$ m. The second domain, with values ranging from 1  $\mu$ m to a maximum value of 40  $\mu$ m for ST and SED crusts and  $100 \,\mu$ m for the subsoil, characterizes the coarse fraction (Fies, 1992b). Our field values of  $\lambda_m$  (110  $\mu$ m for ST and SED, 175  $\mu$ m for SUB, Table 1) appear markedly superior to laboratory measured maximum pore radius, which is in agreement with general findings reported by White and Sully (1987). Not surprisingly, our results illustrate the predominant hydraulic role of the coarse porosity on infiltration.

Estimation of  $t_{\rm grav}$  (Table 1) by Eq. (16) shows that gravity effects can be neglected during at least 1 h of infiltration into a crust, which makes possible the use of Eq. (19) to obtain a second sorptivity estimate,  $S_1$ . S values, obtained by Eq. (11), are plotted against  $S_1$  in Fig. 11, for tests providing the two estimates without any ambiguity. As no consistent underestimation appears in  $S_1$  compared with S values, no water blockage effect is shown at the crust-soil interface. Indeed, such a phenomena would have entailed a delay in the tensiometer response to the wetting front arrival and thus decrease  $S_1$  values. Therefore, it seems that ST and SED crusts can be effectively seen, from a hydrodynamic point of view, simply as impeding layers, by having a lower hydraulic conductivity than the subsoil.

Crust conductivity values obtained by this method were used in a soil-vegetation-atmosphere-transfer model (Braud et al., 1997) and led to improve the prediction of the soil water content as compared with measurements performed during the HAPEX-Sahel experiment. Results presented here for the tiger bush will be used to compute cumulative infiltration to predict pointwise runoff amounts for natural rainfall conditions and compare them with measured values obtained by Peugeot et al. (1997).

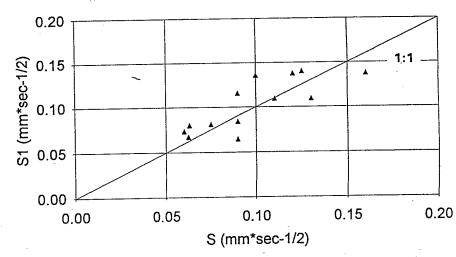


Fig. 11. Comparison of S (Eq. (11)) and  $S_1$  (Eq. (19)) sorptivity estimates for structural and sedimentation crusts.

#### 5. Conclusions

Faced with the inability of classical tension disc infiltrometry methods, based on the analysis of steady state regime of infiltration, to provide conductivity estimates of crusted soils, a new method has been developed, using transient flow analysis and minitensiometer implemented below the crust. Saturated hydraulic conductivity, obtained with a precision of a factor 2, decreased in the crust from three-fold to six-fold compared with the subsoil. The effective pore size was found to be significantly reduced by the crust formation and consistent with porosimetry measurements. The method would be particularly suitable for sandy crusts, for which the decrease in conductivity must be more accentuated. It is not recommended to use the method where crusts either have a high surface roughness (because of the need for a thick layer of sand) or are thinner than 1 cm (because of the difficulty of placing the minitensiometer at the crust—subsoil interface). More generally, the method could be used for any case of layered soils, the extra effort required for minitensiometers installation being largely offset by avoiding the steady state flow requirement.

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