# ON ETHNOMATHEMATICS AND THE TRANSMISSION OF MATHEMATICAL KNOWLEDGE IN AND OUTSIDE SCHOOLS IN AFRICA SOUTH OF THE SAHARA

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## Culture, education and development

Three important publications on the challenges to the South in general and to education in Africa in particular appeared in 1990.

The first is *"The challenge to the South"*, the Report of the South Commission, led by the former President of Tanzania, Julius Nyerere (1). It criticizes development strategies that minimize cultural factors. Such strategies only provoke indifference, alienation and social discord. The development strategies followed until today "have often failed to utilize the enormous reserves of traditional wisdom and of creativity and enterprise in the countries of the Third World" (*ibid.*, p. 46). Instead, the cultural wellsprings of the South should feed the process of development.

An important feature of *"African Thoughts on the Prospects of Education for All"* (2) is the fact that two themes keep recurring in all contributions: the focus on the crisis in African contemporary culture and the theme of African languages (as vehicles of culture and media of education). The crux of the crisis of African cultures is the issue of African cultural identity. A people's cultural identity (including their awareness of such an identity), is seen as the springboard of their development effort (*ibid.*, p. 10). Africa needs *culture-oriented education*, that would ensure the survival of African cultures if it emphasized originality of thought and encouraged the virtue of creativity. Scientific appreciation of African cultural elements and experience is considered to be "one sure way of getting Africans to see science as a means of understanding their cultures and as a tool to serve and advance their cultures" (*ibid.*, p. 23).

In *"Educate or Perish: Africa's Impasse and Prospects"* (3) the historian Ki-Zerbo stresses that today's existing African educational system is "unadapted and elitist" and "favors foreign consumption without generating a culture that is both compatible with the original civilization and truly promising". Reminding us of the African proverb *"When lost, it's better to return to a familiar point before rushing on"*, Ki-Zerbo underlines that "Africa is in serious trouble, not because its people have no foundations to stand on, but because ever since the colonial period, they have had their foundations removed from under them" (*ibid.*, p. 82). Probably this is particularly true in the case of mathematics.

African countries face the problem of low "levels of attainment" in mathematics education. Math anxiety is widespread. Many children (and teachers too!) experience mathematics as a rather strange and useless subject, imported from outside Africa; as something that exists only in schools. Does this image of mathematics correspond to reality?

## First examples of mathematical ideas outside school

*First example: A cowrie game in Côte d'Ivoire*

In 1980 a research-seminar on *"Mathematics in the African socio-cultural environment"* was introduced at the Mathematical Research Institute of Abidjan (IRMA, Côte d'Ivoire). The seminar was directed by S.Doumbia. One of the interesting themes analysed by her and her colleagues is the mathematics of traditional West-African games, like Nigbé Alladian.

One plays Nigbé Alladian with four cowrie shells. On their turn, each of the two players casts the cowrie shells. When all four land in the same position, i.e. all "up" or all "down", or when two land in the "up" position and the other two in the "down" position, the player gets points. In the other cases, one "up" and three "down", or, three "up" and one "down".a participant does not get points. The researchers of IRMA found experimentally that the chance of a cowrie shell to fall in the "up" position is \f(2,5). The following table shows the probabilities of four "up", four "down", etc.:

| position | probability | |
|---|---|---|
| all "up" | \f(16,625) | |
| all "down" | \f(81,625) | \f(313,625) |
| two "up", two "down" | \f(216,625) | |
| one "up", three "down" | \f(96,625) | |
| three "up", one "down" | \f(216,625) | \f(312,625) |

One sees that the rules of the game had been chosen in such a way that the chance to win points is (almost) the same as to get no points. S.Doumbia concluded that "the players have managed ...to adopt a clever counting system, in order to balance their chances. The probability of scoring some points is \f(313,625) as against \f(312,625)". (Doumbia, 1989, p.175) (4) Doumbia's research team explores now the possibilities of using this and other traditional games in the mathematics classroom (cf. Pil & Doumbia, 1992).

*Second example: Peasants in Nigeria*

Shirley (5) and his students at the Ahmadu Bello University in Nigeria conducted oral interviews with unschooled, illiterate members of the students' home communities.

They found that "although some of the (arithmetical) algorithms used by the informants are similar to those taught in schools, some interesting non-standard techniques were also found" (Shirley, 1988, p. 5). "Several respondents displayed a good number sense, especially in using decomposition, associativity, and distributivity to simplify into round numbers" (*ibid.*, p.6). For instance, to determine 18 + 19 = ...?, some used the following procedure: 1 removed from 18, leaves 17 and 1 combined with 19 gives 20, therefore 17 + 20 gives the same result as 18 + 19. Schematically:

$$\underline{18 + 19 = \ldots?}$$
$$18 - 1 = 17 \text{ and } 1 + 19 = 20$$
$$17 + 20 = 37$$

Yet another approach to the same problem rounded both 18 and 19 to 20, noting that 2 and 1 had been respectively added; the two 20s give 40, and then the 2 and 1 are subtracted to yield 37. The researchers noted that the illiterate respondents presented creative solutions to division problems. "For 45 ÷ 3 = ...?, one respondent apparently knew that 21 ÷ 3 is 7, and so decomposed 45 into 21 + 21 + 3. Dividing all the terms of this sum by 3 and adding gave 7 + 7 + 1 = 15" (*ibid.*, p. 7). Shirley gives the advise to assign teacher-student to find algorithms in their communities – literate or illiterate, rural or urban, as "too often, school lessons leave the impression that there is only one way to do a given task" (*ibid.*, p. 9).

*Third example: Market women in Mozambique*

Lecturers and students of the Master's Degree Programme in Mathematics Education for Primary Schools at Mozambique's *Instituto Superior Pedagógico* (Beira Branch) have been analysing arithmetic in and outside school. On interviewing "unschooled" women to know how they determine sums and differences, it was found that the women "solved easily nearly all the problems, using essentially methods of oral/mental computation, i.e., computation based on the spoken numerals. The methods used were very similar to those suggested by the present day mathematics syllabus for primary education, but including some interesting alternatives" (J. Draisma, 1992, p. 110) (6). For instance, 59% of the interviewed women calculated mentally 62 – 5 = ...? by first subtracting 2 and then 3. Schematically:

$$\underline{62 - 5 = \ldots?}$$
$$62 - 2 = 60$$
$$60 - 3 = 57,$$

i.e. they used the same method as is emphasized in the schoolbook. Another 29% of the women subtracted first 5 from 60 and then added 2:

$$\underline{62 - 5 = ..?}$$
$$60 - 5 = 55$$
$$55 + 2 = 57,$$

and 12% subtracted first 10 from 62, and added the difference between 10 and 5, i.e. 5. Schematically:

62 – 5 = ...?
62 - 10 = 52
10 - 5 = 5
52 + 5 = 57.

Did these women *(re)invent* their methods? Did they *learn* them? From whom and how?

When multiplying, most of the interviewed women solved the problems by doubling. An example illustrates the process 6 x 13 = ...?

Schematically the solution is the following:

6 x 13 = ...?
2 x 13 = 26
4 x 13 = 2 x 26
2 x 26 = 52
6 x 13 = 26 + 52
26 + 52 = 78

(J. Draisma, oral communication, 1992.) Does each of these women (re)invent the doubling method spontaneously? Or does there exist a tradition? If so, how is the method taught and learnt?

From these three examples, a first conclusion may be drawn.

Although African countries face the problem of low "levels of attainment" in mathematics education; although math anxiety is widespread; although many children (and teachers too!) experience (school) mathematics as a rather strange and useless subject, imported from outside Africa, there exists, there "lives" mathematics outside school.

One of the causes of their experience is that the goals, contents and methods of mathematics education are not or not sufficiently adapted to the cultures and needs of the African peoples, as *ethnomathematicians* stress.

## Concerns of ethnomathematics and ethnomathematicians

Looking back historically we may see that Ethnomathematics has been defined at different levels. Defined as the cultural anthropology of mathematics and mathematical education, it is a relatively new field of interest. As the view of Mathematics as "culture-free", as a "universal, basically aprioristic form of knowledge" has been dominant, Ethnomathematics emerged later than other ethnosciences. Among mathematicians, ethnographers, psychologists and educationalists, G. Luquet (1929), O.Raum (1938), L. White (1947), E. Fettweis, and R. Wilder (1950) may be registered as isolated forerunners of Ethnomathematics (7). Their reflections did not find a lot of echo. A reductionist tendency tended to dominate mathematics education, relying on culture-free cognition models (8).

*Ethnomathematics as the mathematics which is practised among identifiable cultural groups*

At the end of the 1970s and beginning of the 1980s, there developed a growing awareness among mathematicians of the societal and cultural aspects of mathematics and mathematical education. It is in that period that U. D'Ambrosio proposes his *ethnomathematical programme* as a methodology to track and analyse the processes of generation, transmission, diffusion and institutionalization of (mathematical) knowledge in diverse cultural systems (9). In contrast to "academic mathematics", i.e. the mathematics which is taught and learned in the schools, D'Ambrosio calls *ethnomathematics* "the mathematics which is practised among identifiable cultural groups, such as national-tribal societies, labor groups, children of a certain age bracket, professional classes, and so on" (1985b, p. 45). "The mechanism of schooling replaces these practices by other equivalent practices which have acquired the status of mathematics, which have been expropriated in their original forms and returned in a codified version" (1985b, p. 47). Before and outside school almost all children in the world become "matherate", i.e. they develop the "capacity to use numbers, quantities, the capability of qualifying and quantifying and some patterns of inference" (1985a, p. 43). In school the "learned" matheracy eliminates the so-called "spontaneous" matheracy: the former spontaneous abilities are downgraded, repressed and forgotten while the learned ones are not assimilated either as a consequence of a learning blockage, or of an early drop-out, or even as a consequence of failure or many other reasons. The question which arises then is what to do: "should we...give up school mathematics and remain with ethnomathematics? Clearly not..." (1985a, p.70). In D'Ambrosio's view one should compatibilize cultural forms, i.e. "...the mathematics in schools shall be such that it facilitates knowledge, understanding, incorporation and compatibilization of known and current popular practices into the curriculum. In other words, recognition and incorporation of ethnomathematics into the curriculum" (1985a, p. 71).

Let us now briefly review other concepts that have been proposed and that are related to D'Ambrosio's ethnomathematics.

*Emergence of concepts related to ethnomathematics*

During the 1970s and 1980s, there emerged among teachers and didacticians of mathematics in developing countries and later also in other countries a growing resistance against the racist and (neo)colonial prejudices related to mathematics, against the eurocentrism in mathematics and its history (10). It was stressed that beyond the "imported school mathematics" there existed and continues to exist also *other mathematics.*

In this context, various concepts have been proposed to contrast with the "*academic* mathematics" / "*school* mathematics" (i.e. the school mathematics of the transplanted, imported curriculum):

- "*indigenous* mathematics" [Cf. e.g. Gay & Cole, 1967 (11); Lancy, 1976 (12)]: Criticizing mathematics education of Kpelle children (Liberia) in "western-oriented" schools – they "are taught things that have no point or meaning within their culture" (Gay & Cole, p. 7) – Gay and Cole propose a creative education that uses the indigenous mathematics as starting point;

- *sociomathematics* of Africa [Zaslavsky, 1973]: "the applications of mathematics in the lives of African people, and, conversely, the influence that African institutions had upon the evolution of their mathematics" (Zaslavsky, p.7) (13);
- *informal* mathematics [Posner, 1978, 1982 (14)]: mathematics that is transmitted and that one learns outside the formal system of education;
- *mathematics in the (African) socio-cultural environment* [S. Touré, S. Doumbia (Côte d'Ivoire), 1980]: integration of the mathematics of African games and craftwork that belongs to the social-cultural environment of the child in the mathematics curriculum;
- *spontaneous* mathematics [D'Ambrosio, 1982 (15)]: each human being and each cultural group develops spontaneously certain mathematical methods;
- *oral* mathematics [Carraher *et. al.*, 1982 (16); Kane, 1987 (17)]: in all human societies there exists mathematical knowledge that is transmitted orally from one generation to the next;
- *oppressed* mathematics [Gerdes, 1982 (18)] in class societies (e.g., in the countries of the "Third World" during the colonial occupation) there exist mathematical elements in the daily life of the populations, that are not recognized as mathematics by the dominant ideology;
- *non-standard* mathematics [Carraher *et. al.*, 1982; Gerdes, 1982, 1985 (19); Harris, 1987 (20)]: beyond the dominant standard forms of "academic" and "school" mathematics there develops and developed in the whole world and in each culture mathematical forms that are distinct from the established patterns;
- *hidden* or *frozen* mathematics [Gerdes, 1982, 1985]: although, probably, the majority of mathematical knowledge of the formerly colonized peoples has been lost, one may try to reconstruct or "unfreeze" the mathematical thinking, that is "hidden" or "frozen" in old techniques, like, e.g., that of basketmaking;
- *folk* mathematics [Mellin-Olsen, 1986 (21)] the mathematics (although often not recognized as such) that develops in the working activity of each of the peoples may serve as a starting point in the teaching of mathematics;
- mathematics *codified in know-how* [Ferreira, 1987 (22)].

These concept proposals are provisional. They belong to a tendency that emerged in the context of the "Third World" and that later on found an echo in other countries.

The various aspects illuminated by the aforementioned provisional concepts have been gradually united under the more general "common denominator" of D'Ambrosio's ethnomathematics. This process has been accelerated by the creation of the *International Studygroup on Ethnomathematics* [ISGEm] in 1985.

*Ethnomathematics as the field of research*

In the previous section, ethnomathematics was the mathematics of a certain (sub)culture. In that sense socalled "academic mathematics" is also a concrete example of ethnomathematics. When all ethnomathematics is mathematics, why call it ethnomathematics? And not simply the mathematics of this and that (sub)culture?

Doing so, ethnomathematics may be defined at another level, as a research field. The emergence of this research field reflects the consciousness of the existence of many mathematics, particular in a certain way to (sub)cultures.

As a research field, Ethnomathematics may be defined as the *cultural anthropology of mathematics and mathematical education*, or in the formulation of D'Ambrosio in 1977: "*Ethnoscience* as the study of scientific and, by extension, technological phenomena in direct relation to their social, economic and cultural background" (1987, p. 74).

*Ethnomathematical movement*

The scholars who are engaged in ethnomathematical research are normally socially actively engaged. In this sense I should like to speak of an *ethnomathematical movement*, that might be characterized by the following features, among others:

- *'Ethnomathematicians'* use a broad concept of mathematics, including, in particular, counting, locating, measuring, designing, playing, explaining (23);
- *'Ethnomathematicians'* emphasize and analyse the influences of socio-cultural factors on the teaching, learning and development of mathematics;
- *"Ethnomathematicians"* draw attention to the fact that mathematics (its techniques and truths) is a *cultural product*; they stress that every people – every culture and every subculture – develops its own particular mathematics. Mathematics is considered to be a *universal, pan-human activity*. As a cultural product mathematics has a history. Under certain economic, social and cultural conditions, it emerged and developed in certain directions; under other conditions, it emerged and developed in other directions. In other words, the development of mathematics is *not unilinear* (24);
- *"Ethnomathematicians"* emphasize that the school mathematics of the transplanted, imported "curriculum" is *apparently* alien to the cultural traditions of Africa, Asia and South America. Apparently this mathematics comes from the outside of the "Third World". *In reality*, however, a substantial part of the contents of this "school mathematics" is of African and Asian origin (25);
- *"Ethnomathematicians"* try to contribute to the knowledge of the mathematical realizations of the formerly colonized peoples. They look for culture elements, that survived colonialism and that reveal mathematical and other scientific thinking. They try to *reconstruct* these mathematical thoughts;
- *"Ethnomathematicians"* in "Third World" countries look for mathematical traditions that survived colonization and for mathematical activities in people's daily life and analyse ways to incorporate them into the curriculum;
- In the educational context *"ethnomathematicians"* generally favor a critical mathematics education that enables the students to reflect about the reality they live in and empowers them to develop and use mathematics in an emancipatory way. The influence of the well-known "Third World" pedagogue P. Freire is visible (26).

## Ethnomathematical research in Africa and in Mozambique in particular

The leadership of the African Mathematical Union is aware of the importance of research in ethnomathematics and on the history of mathematics in Africa. Its chairman, A.Kuku, expressed his concern in the following way:

"...mathematics is a very useful subject with firm roots in African history. For thousands of years Africa played a leading role in the development of mathematics in the world. The mathematical heritage of our continent, has to be recovered and valued. It should be 'incorporated' into mathematics education in order to contribute to the popularization of mathematics. When children and adults view mathematics as something belonging to African culture, they will become more selfconfident and learn more easily the mathematics Africa needs urgently for its development" (Kuku, 1991, p. 1).

In 1986 the African Mathematical Union (AMU) created its Commission on the History of Mathematics in Africa (AMUCHMA), which publishes since 1987 a newsletter (27). In the following, a short overview of ethnomathematical research in Africa south of the Sahara will be presented (28).

*Africa south of the Sahara*

During the past few years, a whole series of research projects on spoken and written numeration systems in Africa has been embarked upon concerning, e.g.:

- counting in traditional Ibibio and Efik societies (University of Calabar, Nigeria);
- numeration among the Fulbe (Fulani) (Ahmadu-Bello-University, Nigeria);
- pre-Islamic ways of counting (Bayero University, Nigeria);
- pre-colonial numeration systems in Burundi (University of Burundi);
- learning of counting in Côte d'Ivoire (29);
- numeration systems used by the principal linguistic groups in Guinea (University of Conakry);
- spoken numeration systems of West-Atlantic groups and of the Mandé (30);
- counting among the various ethnic groups in Kenya (Kenyatta University, Nairobi);
- number and pattern in Uganda (E.Segujja-Munagisa);
- numeration systems and popular counting practices in Mozambique (Instituto Superior Pedagógico, Maputo / Beira);
- traditional counting in Botswana (University of Botswana).

From the surviving San hunters in Botswana, Lea and her students at the University of Botswana have collected information. Her papers describe counting, measurement, time reckoning, classification, tracking and some mathematical ideas in San technology and craft. The San developed very good visual discrimination and visual memory as needed for survival in the harsh environment of the Kalahari desert (31). The Faculty of Education of the Ahmadu-Bello-University (Zaria, Nigeria) has been very dynamic in stimulating ethnomathematical research, e.g. on the mathematics used by unschooled children and adults in daily life, and the possibilities to embed this knowledge in mathematics education (32). S. Ale does research on the mathematical heritage of the Fulbe (Fulani) and the possibilities to construct a curriculum that builds upon this heritage and fits the needs of the Fulbe (Fulani) people (33). G.Aznaf studies on ethnomathematics in Ethiopia. D. Mtetwa (University of Zimbabwe) started a research project on "Mathematical thought in aspects of Shona culture". D. Seka (Tanzanian Institute for Curriculum Development) experiments with the tradition of story telling as a didactical means in mathematics teaching.

Langdon (1989, 1990) (34) describes the symmetries of "adinkra" cloths (Ghana) and explores possibilities for using them in the classroom. In a similar perspective, M.Harris (1988) (35) describes and explores not only the printing designs on plain woven cloths from Ghana, but also symmetries on baskets from Botswana and "buba" blouses from the Yoruba (Nigeria).

Interest in the mathematical aspects of traditional riddles and dilemma tales is growing. The following is an example of an "arithmetical puzzle" from the Valuchazi (eastern Angola and northwestern Zambia), recorded and analysed by Kubik (36):

"This ... dilemma tale is about three women and three men who want to cross a river in order to attend a dance on the other side. With the river between them there is a boat with the capacity for taking only two people at one time. However, each of the men wishes to marry all the three women himself alone. Regarding the crossing, they would like to cross in pairs, each man with his female partner, but failing that any of the other men could claim all the women for himself. How are they crossing?"

In order to solve the problem or to explain the solution, auxiliary drawings are made in the sand. S.Doumbia conducts pedagogical experiments with traditional verbal games (Côte d'Ivoire) (37). At the regional conference *"Mathematics, Philosophy, and Education"* (Yamoussoukro, Côte d'Ivoire, Jan. 1993), S. Doumbia (Côte d'Ivoire) and P. Gerdes (Mozambique) conducted jointly a workshop on the didactical uses of traditional African games, drawings and craftwork.

*Mozambique*

Ethnomathematical research started in Mozambique in the late 1970s. As most "mathematical" traditions that survived colonization and most "mathematical" activities in the daily life of the Mozambican people are not explicitely mathematical, i.e. the mathematics is "hidden", the first aim of this research was to "uncover" the "hidden" mathematics. The first results of this "uncovering" are included in the book *"On the awakening of geometrical thinking"* (1985) (38) and slightly extended in *"Ethnogeometry: cultural-anthropological contributions to the genesis and didactics of geometry"* (1992) (39). In *"Fivefold Symmetry and (basket)weaving in various cultures"* (40), it is shown how fivefold symmetry emerged quite "naturally" when artisans were solving some problems in (basket)weaving. The examples chosen from Mozambican cultures range from the weaving of handbags, hats, and baskets to the fabrication of brooms.

In the papers *"On culture, mathematics and curriculum development in Mozambique"* (1986) (41) and *"On culture, geometrical thinking and mathematics education"* (1988) (42) we summarized our experimentation with the incorporation of traditional African cultural elements into mathematics education. The second paper presents alternative constructions of euclidean geometrical ideas developed from the traditional culture of Mozambique. As well as establishing the educational power of these constructions, the paper illustrates the methodology of *"cultural conscientialization"* in the context of teacher education. In the papers *"A widespread decorative motif and the Pythagorean Theorem"* (1988) (43) and *"How many proofs of the Pythagorean Proposition do there exist?"* (44) and further elaborated in the book *"African Pythagoras. A study in culture and mathematics education"* (1992, 1994) (45) he shows how diverse African ornaments and artifacts may be used to create a rich context for the discovery and the demonstration of the so-called Pythagorean Theorem and of related ideas and propositions. A series of ethnomathematical papers are included in the books *"Ethnomathematics: Culture, Mathematics, Education"* (46) and *"Ethnomathematics and education in Africa" (47)*.

In *"SONA Geometry: reflections on the tradition sand drawings in Africa south of the Equator"* (1993, 1994) (48), Gerdes tried to reconstruct mathematical components of the Tchokwe drawing-tradition (Angola) (49) and to explore their educational, artistic and scientific potential. In an earlier article *"On possible uses of traditional Angolan sand drawings in the mathematics classroom"* (1988) (50) Gerdes had analysed already some possibilities for educational incorporation of this tradition. In the paper *"Find the missing figures"* (1988) (51) and in the book *"Lusona: geometrical recreations of Africa"* (1991) (52) mathematical amusements are presented that are inspired by the geometry of the sanddrawing tradition. For children (age 10-15) the booklet *"Living mathematics: drawings of Africa"* (1990) (53) has been elaborated. Experimentation with the use of 'sona' in teacher education is described in *"Exploring Angolan sand drawings (sona): stimulating cultural awareness in mathematics teachers"* (54). An overview of this research is given in *"On mathematical elements in the Tchokwe 'sona' tradition"* (1990) (55).

In recent years more lecturers and in particular young lecturers, who returned home after having studied abroad became interested in and started ethnomathematical research. So far, two collective works have been published. *"Numeration in Mozambique"* presents a reflection on culture, language and mathematics education. It includes studies on African systems of numeration (P. Gerdes & M. Cherinda), written and oral on spoken numeration systems in Mozambique, popular counting techniques (A. Ismael & D. Soares), comparative tables and maps on numeration (A. Mapapá & E. Uaila), and on spoken numeration and the learning of arithmetic (J. Draisma). In *"Explorations in Ethnomathematics and Ethnoscience"* (1994) is presented a collection of papers written by various lecturers at the *Instituto Superior Pedagógico* (transformed in April 1995 into the *Universidade Pedagógica*), both based in Maputo situated in the South of Mozambique and in Beira in the central Sofala Province. The ethnomathematical papers reflect on some mathematical ideas involved in basket and mat making in the North of the country (A.Ismael), on languages and mental calculation (J. Draisma), on popular counting practices all over Mozambique (D. Soares & A. Ismael), symmetries on gratings in Maputo city (A. Mapapá) and on decorations of spoons in Sofala (D. Soares) and the southeastern Inhambane Province (M. Cherinda). A paper on the traditional interpretation of lightning and thunder in the southern Catembe and physics teaching is a first publication in the field of ethnophysics. Short articles on perspectives in ethnoscience related to physics, chemistry and biology conclude this collection. The book *"Women and Geometry in Southern Africa: Some suggestions for further research"* will be published in 1995 (56).

*Two examples of geometries in southern Africa*

Now I will briefly present two examples that I have been analysing: the originally female geometry of the ornamentation of *sipatsi* handbags in Mozambique's Inhambane Province and the male geometry of *sona* sand drawings mostly of Eastern Angola and North-Western Zambia.

*Sipatsi*

The making of *sipatsi* [singular: *gipatsi*] (57) is nowadays practicised by both men and women, having been formerly an exclusively women's activity. *Sipatsi* are straw woven handbags used for storing and transporting coins and documents. They are much appreciated for their ornamental beauty. Figure 1 gives examples of strip patterns with which the basketweavers decorate their *sipatsi*.

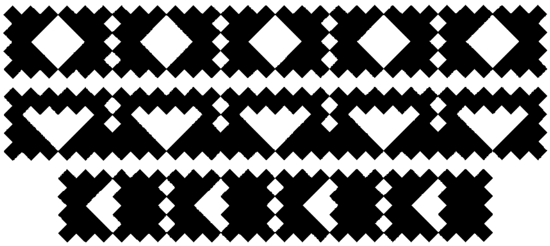

**Figure 1**

The examples display different symmetries. From a theoretical algebraic point of view there exist seven infinite "one-dimensional" symmetry groups. In other words, there are seven essentially distinct ways to repeat a motif on a strip. The creativity of the *sipatsi* weavers expresses itself also in the fact that they invented strip patterns belonging to all of the seven different theoretically possible symmetry groups.

When weaving their sipatsi, the basketmakers repeat their decorative motif in an oblique or diagonal position, as Figure 2 illustrates for the strip pattern in Figure 3. In this example, to generate one exemplar of the motif, 8 coloured strands are needed. We may say that the motif has 8 as its *period*.

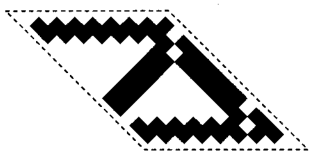

**Figure 2**

**Figure 3**

One of the conditions that a *gipatsi* must satisfy to be considered beautiful and of good quality is the following: in each ornamental strip, the motif must fit exactly a whole number of times around the *gipatsi*. This implies that the total number of coloured strands has to be a common multiple of each of the periods of the motifs used for its decoration. For instance, in the example shown in Figure 4, the total number of coloured strands has to be a multiple of 10 and 3.

For technical reasons related to the weaving process, the number of coloured strands must also always be a multiple of 4. So the basketweavers have to determine (mentally) the common multiple of 4 and of each of the periods of the motifs they have chosen before weaving their *sipatsi*. As sometimes the common multiple is too big to produce a *gipatsi* of an acceptable size, the weavers look for the best *approximate* solution within the acceptable limits.

**Figure 4**

The geometry of ornamentation of sipatsi raises many interesting questions for investigation. For example, how many ornamental strip patterns of the *gipatsi* type exist of given dimensions *p* and *d*, where *p* denotes the period and *d* the diagonal height of the decorative motif? In the case where the dimensions are 2 x 4, there exist six different strip patterns of the *gipatsi* type. Figure 5 illustrates these strip patterns together with motifs that generate them.

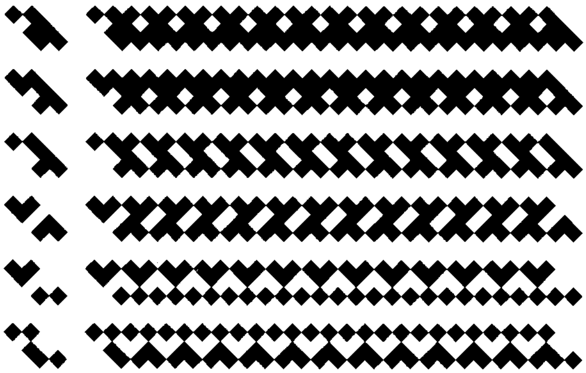

**Figure 5**

The strip patterns on *sipatsi* may serve as a source of inspiration not only for mathematicians but also for architects, painters, and other artists; they could be reproduced on clothes, walls, stamps, etc. There are many possibilities for an educational exploration of *sipatsi*, contributing to the improvement of mathematics teaching by embedding it into the cultural context of pupils and teachers. The confidence of the youth of Southern Africa, in particular that of girls, in their mathematical capacities and potential may increase as they explore the mathematics of *sipatsi*.

*Sona*

The *sona* tradition [singular: *lusona*] (58) was highly developed among the Tchokwe of Northeastern Angola and related peoples. The Tchokwe culture is well known for its decorative art that ranges from ornamentation on woven mats and baskets, iron works, ceramics, sculpture and engravings on calabash to tattooing, paintings on the walls of houses, and *sona* sand drawing. Each boy learnt the meaning and execution of the easier *sona* during their period of intensive schooling, the initiation rites. The more complicated *sona* were transmitted by drawing experts to their male descendents. These drawing experts were at the same time the story tellers who used the sand drawings as illustrations referring to proverbs, fables, games, riddles, animals. The drawings were executed in the following way: After cleaning and smoothing the ground, the drawing experts first set out with their fingertips an orthogonal net of equidistant points and then they draw a line figure that embraces the points of the network. The experts execute the drawings swiftly. Once drawn, the designs are generally extinguished. Figure 6 presents some examples of *sona*.

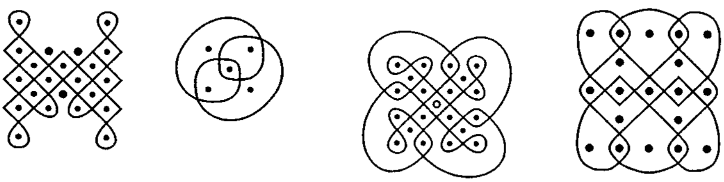

**Figure 6**

Colonial penetration and occupation provoked a cultural decline and the loss of a great deal of knowledge about *sona*. On the basis of an analysis of *sona* reported by missionaries, colonial administrators and ethnographers, I tried to contribute to the reconstruction of mathematical elements in the *sona* tradition.

As the examples in Figure 6 suggest, symmetry and monolinearity played an important role as cultural values: most Tchokwe sona are symmetrical and/or monolinear. Monolinear means composed of only one (smooth) line.

Figure 7 displays two monolinear sona belonging to the same class in the sense that, although the dimensions of the underlying grids are different, both *sona* are drawn applying the same geometric algorithm.

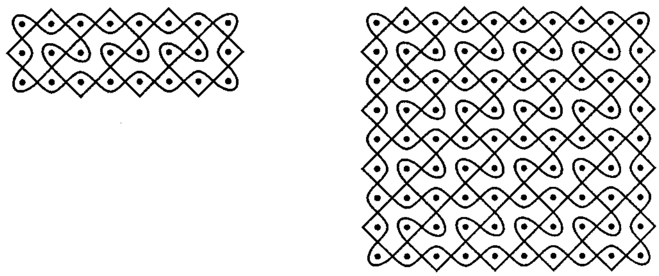

Figure 7

The drawing experts discovered various rules for constructing monolinear *sona*. Figure 8 shows three monolinear *sona*. They are similar to each other: each presents a basic pattern of triangular form.

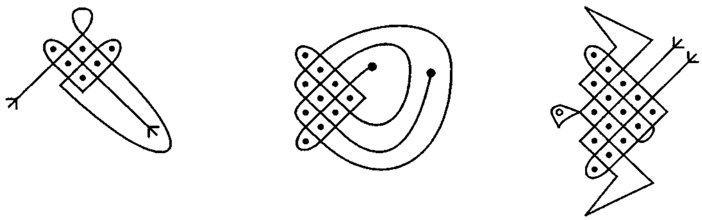

Figure 8

I suppose that the drawing experts who invented these *sona* began with triangular patterns and transformed them into monolinear patterns with the help of one or more loops (see the example in Figure 9). The monolinear patterns so obtained were adapted topologically so that they could express the ideas the drawers wanted to transmit through them.

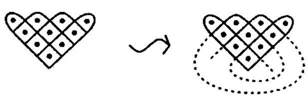

Figure 9

*Sona* experts also discovered various rules for chaining monolinear *sona* to form bigger monolinear *sona*. Figure 10 displays a chain rule: it indicates how the appearance of the monolinear drawing in Figure 10c may be explained on the basis of the monolinearity of the two patterns in Figure 10a and the way they have been chained together (see Figure 10b).

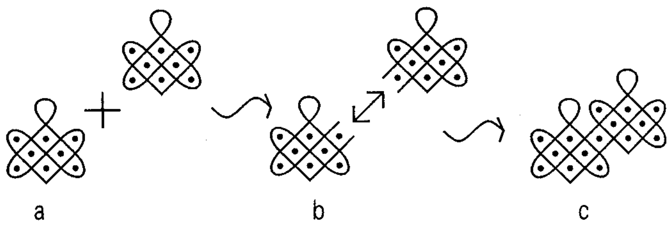


**Figure 10**

The monolinearity of the *lusona* in Figure 11a may be explained on the basis of another chain rule: When one joins square grids (in the example: two grids of dimensions 2 x 2) to a rectangle with dimensions which are relative prime (in the example: 3 x 5), then the resulting grid leads to a monolinear drawing if one applies the same geometric algorithm as used to draw a *lusona* with the rectangular grid ("plaited mat" algorithm).

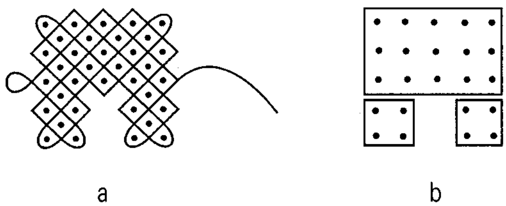


**Figure 11**

The study of the reconstructed mathematical elements in the old Southern African *sona* tradition may serve as a source of inspiration for research mathematicians and mathematics educators.

## Perspectives

So far ethnomathematical research projects in Africa are mostly based at institutions of higher learning, including those of teacher education as in the case of Mozambique. Reflections on and experiments with the educational "incorporation" of the African mathematical heritage and of living mathematical practices have been concentrated in secondary and, in particular, teacher education. From the new teachers and the student teachers who became interested in ethnomathematics during their studies important initiatives may be expected in the coming years. Certainly they will raise fundamental questions.

# NOTES

1) Oxford University Press, Oxford, 1990, 325 pp.

2) UNESCO-UNICEF, Dakar / Abidjan, 1990, 193 pp.

3) J. Ki-Zerbo, *Educate or Perish: Africa's Impasse and Prospects,* UNESCO-UNICEF, Dakar / Abidjan, 1990, 109 pp.

4) S. Doumbia, Mathematics in traditional African games, in: C. Keitel *et al.* (ed.), *Mathematics, Education, and Society,* UNESCO, Paris, 1989, 174-175; many interesting examples are given in the book: S. Doumbia & J. Pil, *Les jeux de cauris,* IRMA, Abidjan, 1992, 74 p.

5) L. Shirley, *Historical and ethnomathematical algorithms for classroom use,* ICME VI, Budapest, 1988, 12 p.; Anzenge, H. *et al., Indigenous mathematical algorithms,* B. Ed. project, Ahmadu Bello University, Zaria, 1988.

6) J. Draisma, Arithmetic and its didactics, in: *Report of the first year of the Master's Degree Program in Mathematics Education for Primary Schools (August 1991-July 1992),* Mozambique's Instituto Superior Pedagógico, Beira, 1992, p. 89-129.

7) See P. Gerdes, *Survey of curent work on Ethnomathematics,* paper presented at the Annual Meeting of the American Association for the Advancement of Science, Boston, 14.2.1993.

8) Cf. U.D'Ambrosio, *Etnomatemática: raízes socio-culturais da arte ou técnica de explicar e conhecer,* UNICAMP, Campinas, 1987, p.80

9) Cf. e.g. U. D'Ambrosio, *Socio-cultural bases for mathematics education,* UNICAMP, Campinas, 1985a, 103 p.; Ethnomathematics and its place in the history and pedagogy of mathematics, *For the Learning of Mathematics,* Montreal, 1985b, Vol. 5, no. 1, 44-48; *Etnomatemática: arte ou técnica de explicar e conhecer,* Editora Ática, São Paulo, 1990, 88 p.

10) Cf. e.g. the studies of G. Joseph, Foundations of Eurocentrism in Mathematics, *Race & Class,* London, 1987, Vol. XXVIII, No. 3, 13-28; Eurocentrism in mathematics: the historical dimensions, in: C. Keitel, *et al.* (eds.), *Mathematics, Education, and Society,* UNESCO, Paris, 1989, 32-35; *Crest of the Peacock: non-European Roots of Mathematics,* Tauris, London, 1991; cf. S.Anderson, Worldmath curriculum: fighting eurocentrism in mathematics, *Journal of Negro Education,* 1990, Vol. 59, no. 3, pp. 348-359.

11) J. Gay & M. Cole, *The new mathematics and an old culture, a study of learning among the Kpelle of Liberia,* Holt, Rinehart & Winston, New York, 1967, 100 p.

12) D.Lancy (ed.), The Indigenous Mathematics Project, *Papua New Guinea Journal of Education,* 1978, Vol. 14, 217 p.

13) C.Zaslavsky, *Africa Counts: number and pattern in African culture* (1973), Lawrence Hill Books, Brooklyn, NY, 1979, 328 p.

14) J.Posner, The development of mathematical knowledge in two west African societies, *Child Development,* Chicago, 1982, Vol. 53, 200-208 (based on her Ph. D. thesis, 1978).

15) U. D'Ambrosio, *Mathematics for rich and for poor countries,* CARIMATH, Paramaribo, 1982, 10 p.

16) T.Carraher, D.Carraher & A.Schliemann, Na vida, dez; na escola, zero: os contextos culturais da aprendizagem de matemática, *Cadernos de Pesquisa,* São Paulo, 1982, Vol. 42, p. 79-86.

17) E. Kane, *Les systèmes de numération parlée des groupes ouest-atlantiques et Mande. Contribution à la recherche sur les fondaments et l'histoire de la pensée logique et mathématique en Afrique de l'Ouest,* Lille.

18) P. Gerdes, *Mathematics for the benefit of the people,* CARIMATH, Paramaribo, 1982, 13 p.

19) P. Gerdes, Conditions and strategies for emancipatory mathematics education in underdeveloped countries, *For the Learning of Mathematics,* Montreal, 1985, Vol. 5, no. 3, p. 15-20.

20) M.Harris, An example of traditional women's work as a mathematics resource, *For the Learning of Mathematics*, Montreal, 1987, Vol. 7, no. 3, p. 26-28.

21) S.Mellin-Olsen, Culture as a key theme for mathematics education. Postseminar reflections, in: *Mathematics and Culture, a seminar report*, Caspar Forlag, Radal, 1986, p. 99-121.

22) E.Ferreira, Por uma teoria da etnomatemática, *BOLEMA*, Rio Claro, 1991, Vol. 7, p. 30-35.

23) See the chapter "Environmental activies and mathematical culture" in: A. Bishop, *Mathematical enculturation, a cultural perspective on mathematics education*, Kluwer, Dordrecht/Boston, 1988, 195 p.

24) Cf. Ascher & Ascher, Ethnomathematics, 1986, p.139-140.

25) Cf. e.g. A. Bishop, Cultural imperialism and western mathematics: the hidden persuader, in: Mangan, J.A. (ed.), *Education, Europe and Imperialism*; Manchester, 1989; A.Bishop, Western Mathematics: the secret weapon of cultural imperialism, *Race & class*, 1990, Vol. 32, no. 2.

26) Cf. e.g. M. Frankenstein, Critical mathematics education: an application of Paulo Freire's epistomology, *Journal of Education*, Boston, 1983, Vol. 165, 315-339; M.Frankenstein & A.Powell, Toward liberatory mathematics: Paulo Freire's epistemology and ethnomathematics, in P. McLaren & C. Lankshear (Eds.), *The politics of liberation: Paths from Freire*, Routledge, London, 1994, 74-99; E. Ferreira, *A matemática no pensamento de Paulo Freire*, UNICAMP, Campinas, 1992, 8 p.

27) To receive the AMUCHMA-Newsletter, contact the editor: P.O. Box 915, Maputo, Mozambique.

28) For a more detailed overview, see P.Gerdes, On mathematics in the history of subsaharan Africa, *Historia Mathematica*, New York, 1994 (in press).

29) R. Zepp, *L'apprentissage du calcul dans les langues de Côte d'Ivoire*, Institut de Linguistique Appliqueé, Université d'Abidjan, 1983, Vol. 99, 121 pp.; G.Tro, *Étude de quelques systèmes de numération en Côte d'Ivoire*, Abidjan, 1980, 191 p.

30) See Kane o.c. (note 17).

31) See e.g. H. Lea, Traditional mathematics in Botswana, *Mathematics Teaching*, London, 1987, Vol. 119; Informal mathematics in Botswana, *Proceedings of the 41st CIEAEM Meeting of the International Commission for the Study and Improvement of Mathematics Teaching*, Brussels, 1989, 43-53; *Mathematics in a cultural setting*, University of Botswana, Gaberone, 1989, 10 p.; *Informal Mathematics in Botswana: Mathematics in the Central Kalahari*, University of Botswana, Gaberone, 1990, 9 pp.; *Informal Mathematics in Botswana: Spatial concepts in the Kalahari*, University of Botswana, Gaberone, 1989, 9 pp.; Spatial concepts in the Kalahari, *Proceedings of the 14th International Conference on Psychology of Mathematics Education*, Oaxtepec, Mexico, 1990, Vol. 2, p. 259-266.

32) For an survey of these research projects, see: L.Shirley, History of mathematics in Nigerian mathematics classrooms: values and problems, *Abacus, Journal of the Mathematical Association of Nigeria*, Ilorin, 1986, Vol. 12, 123-133; *Ethnomathematics and the history of African mathematics*, paper presented at the 2nd Pan-African Congress of Mathematicians, Jos, 1986, 8 p.(mimeo); *Historical and ethnomathematical algorithms for classroom* use, Ahmadu Bello University, Zaria, 1988, 12 p. (mimeo).

33) S. Ale, Mathematics in rural societies, in: C.Keitel *et al.* (ed.), *Mathematics, Education, and Society*, UNESCO, Paris, 1989, p. 35-38.

34) N. Langdon, Cultural starting points for mathematics: a view from Ghana, *Science Education Newsletter*, British Council, London, 1989, No. 87, 1-3; *Cultural starting points*, paper presented at SEACME 5, Brunei, 19 p. (mimeo).

35) M. Harris *et al.*, *Cabbage, Mathematics in Work*, Institute of Education, University of London, 1988.

36) G. Kubik, Visimu vya mukatikati – dilemma tales and "arithmetical puzzles" collected among the Valuchazi, *South African Journal of African Languages*, Pretoria, 1990, Vol. 10, No. 2, p. 59-68.

37) S. Doumbia, *Les jeux verbaux au Côte d'Ivoire*, paper presented at ICME7, Montreal, 1992.

38) *Zum erwachenden geometrischen Denken*, Maputo / Dresden, 1985, 260 p. Cf. also Gerdes, P.: How to recognize hidden geometrical thinking? A contribution to the development of anthropological mathematics, *For the Learning of Mathematics*, Montreal, 1985, Vol. 6, No. 2, 10-12, 17. In the following two papers it is analysed why basketweavers "prefer" certain symmetries: P.Gerdes, Fivefold Symmetry and (basket)weaving in various cultures, in: I. Hargittai (ed.), *Fivefold symmetry in a cultural context*, World Scientific Publishing, Singapore, 1992, 243-259, and P. Gerdes, On ethnomathematical research and symmetry, *Symmetry: culture and science*, Budapest, 1990, Vol. 1, No. 2, p. 154-170.

39) Gerdes, P.: *Ethnogeometrie. Kulturanthropologische Beiträge zur Genese und Didaktik der Geometrie*, Verlag Franzbecker, Bad Salzdetfurth /Hildesheim (Germany), 1992, 360 pp.

40) P. Gerdes, Fivefold Symmetry and (basket)weaving in various cultures, in: I. Hargittai (ed.), *Fivefold symmetry in a cultural context*, World Scientific Publishing, Singapore, 1992, p. 243-259.

41) S. Mellin-Olsen & M.J.Hoines (eds.), *Mathematics and Culture, a seminar report*, Caspar Forlag, Radel, 1986, p.15-42.

42) *Educational Studies in Mathematics*, Dordrecht /Boston, 1988, Vol.19, No. 3, 137-162; and reproduced in: Bishop, A. (ed.), *Mathematics Education and Culture*, Kluwer Academic Publishers, Dordrecht / Boston, 1988, p.137-162.

43) *For the Learning of Mathematics*, Montreal, 1988, Vol. 8, No. 1, p. 35-39.

44) *Namnären*, Göteborg (Sweden), 1988, Vol. 15, No. 1, p. 38-41.

45) P. Gerdes, *Pitágoras Africano: Um estudo em cultura e educação matemática*, Mozambique's Instituto Superior Pedagógico, Maputo, 1992, 102 pp. (English translation, 1994).

46) P.Gerdes, *Etnomatemática: Cultura, Matemática, Educação*, Mozambique's Instituto Superior Pedagógico, Maputo, 1991, 110 p.

47) P. Gerdes, *Ethnomatics and education in Africa*, University of Stockholm Institute of International Education, 1995, 184 pp.

48) P. Gerdes, *Geometria Sona: Reflexões sobre uma tradição de desenho em povos da África ao sul do Equador*, Instituto Superior Pedagógico, Maputo, Vol. 1, 1993, 200 pp.; Vol. 2, 1993, 156 pp.; Vol. 3, 1994, 120 pp. English language edition of volume 1, Instituto Superior Pedagógica, Maputo, 1994, 200 pp. French language edition: *Une tradition géométrique en Afrique – Les dessins sur le sable*, L'Harmattan, Paris, 1995, 3 volumes, 594 pp.

49) Cf. also M.Ascher, Graphs in cultures (II): a study in ethno-mathematics, *Archive for History of Exact Sciences*, Berlin, 1988, Vol.39, no.1, 75-95 and chapter 2 in the aforementioned book *"Ethnomathematics"*.

50) First version, in: *Abacus, the Journal of the Mathematical Association of Nigeria*, Ilorin (Nigeria), Vol.18, No.1, 107-125; second more elaborated version in: *Educational Studies in Mathematics*, Dordrecht / Boston, Vol.19, p.3-22.

51) *Namnären*, Götenborg (Sweden), Vol.15, No.4, p.51-53.

52) Mozambique's Instituto Superior Pedagógico, Maputo, 120 p.

53) P. Gerdes, *Vivendo a matemática: desenhos da África*, Editora Scipione, São Paulo (Brazil), 64 p.

54) Radical Teacher, Boston, 1993, Vol. 43, p.18-24.

55) *For the Learning of Mathematics*, Montreal, Vol. 10. No. 1, 31-34, and slightly adapted versions in: *Afrika Matematika*, Benin City, 1991, Series 2, Vol. 3, 119-130, and in: *Discovery and Innovation*, Nairobi, 1991, Vol. 3, No. 1, p. 29-36.

56) P. Gerdes, *Women and Geometry in Southern Africa*, Universidade Pedagógica, Maputo, 1995, 200 pp. The translation into French *Femmes et Géométrie en Afrique Australe* will be published by L'Harmattan, Paris.

57) See the book: P. Gerdes & G. Bulafo, *Sipatsi: Technology, Art and Geometry in Inhambane*, [English translation: P.Gerdes & A. Powell] Instituto Superior Pedagógico, Maputo, 1994, 102 pp.

58) On *sona* I have written the following:
- *Vivendo a matemática: Desenhos da África* [Living mathematics: drawings of Africa], Editora Scipione, São Paulo, 1990, 64 pp. [children's book, English translation by A.Powell is forthcoming];
- *Lusona: Geometrical recreations of Africa*, Instituto Superior Pedagógico, Maputo, 1991, 117 pp.;
- *Geometria sona: Reflexões sobre uma tradição de desenho em povos da África ao sul do Equador* [*Sona* geometry: Reflections on the tradition of sand drawings in Africa south of the Equator], Instituto Superior Pedagógico, Maputo, 1993, Vol. 1 [reconstruction of mathematical elements in the *sona* tradition], 200 pp.; Vol. 2 [exploration of the educational and scientific potencial of the *sona* tradition], 160 pp.; Vol. 3 [comparison with drawing traditions in Ancient Egypt, Mesopotamia, India and other regions of Africa), 1994, 120 pp. French translation: *Une tradition géométrique en Afrique – Les dessins sur le sable*, L'Harmattan, Paris, 1995, 3 volumes, 594 pp.

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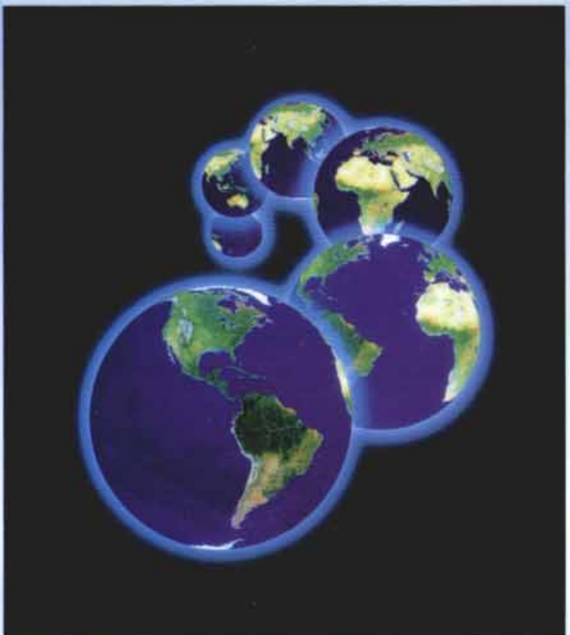

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