

n° 92057

THE DETERMINATION OF A CAPITAL STOCK SERIE FOR DEVELOPING COUNTRIES : SOME COMMENTS TO BE SURE WHAT ONE GETS IS A REAL CHEASHIRE CAT AND NOT ITS GRID

Nathalie TROUBAT (OECD) Marie-Paule VERLAETEN (DIAL)

July 1992

TABLE OF CONTENTS

SUMMARY

INTRODUCTION

I. THE PERPETUAL INVENTORY MODEL (P.I.M.)

- 1. The equation set up for the PIM at the end of a year(t)
- 2. The procedure of estimation
- 3. The choice of the beginning year
- 4. The equation set up for the PIM at the beginning of a year(t)
- 5. Focusing on an explicit production function when departing from the PIM
- 6. A capital stock serie instead of a capital services one

II. A METHOD IMPLYING THE KNOWLEDGE OF A DEPRECIATION RATE SERIE

- 1. The rectangular survival curve
- 2. The exponential survival curve

III. A METHOD REFERRING TO A PRODUCTION FUNCTION FRAMEWORK

- 1. The CD production function
- 2. The CES production function
- 3. Extensions of the CD and CES functions
- 4. Other production functions

IV. APPLICATION OF THE PIM TO A SET OF SELECTED AFRICAN COUNTRIES

CONCLUSION

TABLE 1.

Application of the PIM to a set of selected African Countries

NOTES

REFERENCES

1

SUMMARY

In the paper we present comments on methods to estimate the capital stock in developing countries. The focus is on consistent but simple ones for in these countries there is a lack of statistics specially the quality ones. The survey covers (i) the Perpetual Inventory Model, (ii) a method implying the knowledge of a depreciation rate serie and (iii) a method referring to a production function framework. The result is that difficulties to estimate capital stock series in the framework of developing countries have not to be exaggerated. The PIM being the easiest method to use has been applied to a set of selected African countries. The conclusion is that the PIM can be applied leading to plausible results from the viewpoint of some economic rationality.

INTRODUCTION

1. It now has been admitted that supply in nature, quality and diversity is at the core of development. This is even more true in the case of developing countries specially when they are under adjustment processes. Indeed, these ones cover a sample of macro measures aiming at gradually improving the supply of these countries in the sense of increased efficiency and rentability of their economies. Furthermore, it also has been accepted that there are links between efficiency and justice particularly from the viewpoint of income distribution and, finally, social stability. Therefore, supply analysis and related policy simulation have become more and more important. Unfortunately, both analysis and simulation cannot be performed or are rendered difficult at the moment in most developing countries because of a lack of relevant data particularly the quality ones. In this respect, the lack of a capital stock serie has gradually challenged both supply analysis and policy simulation. Therefore, economists have turned to the estimate of such an aggregate. Doing so has led them to encounter a lot of economic and econometric problems which most generally bias the estimations. In this paper, we would like to clarify some issues departing from methods which may be used to compute a capital stock serie. The focus is on consistent but simple ones for the emphasis is on developing countries where statistics lack specially the quality ones. So, we will focus on :

(i) the Perpetual Inventory Model (PIM);

(ii) a method implying the knowledge of a depreciation rate serie ;

(iii) a method referring to a production function framework.

I. THE PERPETUAL INVENTORY MODEL (P.I.M.)

1) The equation set up for the P.I.M. at the end of a year (t)

2. One very **crude** but rather **vicious** method is to depart from an equation linking net productive capital stock (i.e. the means of production) at the end of a year (t) to gross investment of the same period via a depreciation rate and past values for the capital stock. Then, one gets the Perpetual Inventory Model (P.I.M.) of which the equation set up is :

 $K = K (1 - \delta) + I$ t t-1 t P.I.M. (1) where :

 K_t,K_{t-1} : the net capital stock at the end of year (t) and (t-1) respectively and at some base year prices;

 I_t : gross investment during year (t) at the same base year prices where agriculture, government, dwellings and domestic servants have been omitted (as it is also the case with Kt and Kt-1);

 δ : a constant depreciation rate (%).

The net capital stock represents the cumulated "depreciated" value of the existing gross stock of capital. The depreciation rate permits to make an allowance for the fact that some of the services originally embodied in the capital assets have expired because of retirement or deterioration of some part of the capital stock or this one getting older or obsolete. Depreciation is normally measured during the physical life of the capital assets. So, it presumes to be able to appreciate what proportion of capital produced in a given year is deemed to be still available t years later. Depreciation allowances are usually calculated on an original cost basis, some sort of accelerated declining balance method of write-off is usually used.

3. Equation (1) is based on the following assumptions :

(i) net capital stock at the end of year (t) equals the sum of all previous net investment, i.e.

$$K = \Sigma (I - IR)$$
(2)
t r=o t-r t-r

where :

 IR_t : replacement investment of year (t). Thus, it is assumed that the capital stock is replaced when it gets depreciated.

(ii) : depreciated capital stock is assumed to be distributed geometrically over time with a constant depreciation rate (δ). This rate equals one divided by the lifespan of each past investment. Here, the lifespan is constant amongst all past investments :

$$IR = \delta I + \delta (1-\delta)I + \delta (1-\delta)^2 I + ... + (3) t t-1 t-2 t-3$$

So, substituting (3) for IR_t into (2) leads to

$$K = \Sigma \begin{bmatrix} I & -\delta I & -\delta(1-\delta) & I & -\cdots \end{bmatrix}$$

and also,

$$\begin{array}{c} (1-\delta) \ K \\ t-1 \end{array} = \begin{array}{c} (1-\delta) \ \Sigma \\ r=0 \end{array} \begin{bmatrix} I \\ t-r-1 \end{array} - \begin{array}{c} \delta \ I \\ t-r-2 \end{array} - \begin{array}{c} \delta(1-\delta) \ I \\ t-r-3 \end{array} - \begin{array}{c} \cdots \end{bmatrix} (4) \\ \end{array}$$

so,

$$K - (1-\delta) K = \Sigma \begin{bmatrix} I & -\delta I \\ t & t-1 & r=0 \end{bmatrix} = I$$

$$t = t$$

Therefore :

$$K = I + (1-\delta) K$$

t t t-1 (5

2) The procedure of estimation

4. The preceding given one needs a procedure to estimate K_t which is missing by assumption or, more precisely, is not available on the basis of national accounts statistics. One may proceed as follows : first, to estimate or to calibrate a relation linking known variables so as to get an initial value for the capital stock serie, say K0 and second, to use equation (1) to compute the Kt serie departing from the initial value K0. This later value may be computed on the basis of a rough production function where capital is assumed to be the restricting factor of production. This is relevant in the case of developing countries where development strategies favour the industry sector compared to the agriculture one. The production function is :

$$GDP = f (I, I, ..., I, u)$$
(6
t t-1 t-2 t-i t

where ut is an error term. If data used to estimate the production function are annual ones, then, it can be assumed that a substantial part of the new investment (It) will contribute to GDPt, so that one can substitute It for I_{t-1} in equation (6a). So, one writes :

GDP = f (I, I, ..., I, ..., u)t t t-1 t-i t (6b)

assuming that each investment vintage was fully utilized to get the related product.

The production function may be explicitly written as follows :

 $GDP = \sum_{i=0}^{\infty} b(i) I(t-i) + ut$ (7)

where b(i) are parameters accounting for equipments getting depreciated as time is running; they are positive and decreasing. Assuming that depreciation is geometrically distributed over time, one can write differently the b(i) parameters :

 $b(i) = b a(i) \quad a < b \quad and \quad o < a < 1 \tag{8}$

Therefore, the production function may be re-written as follows :

$$GDP = \Sigma ba(i) I + u \qquad (9a)$$

t i=0 t-i t

or,

$$GDP = bI + \Sigma ba(i) I + u \qquad (9b)$$

t t i=1 t-i t

Then,

$$GDP = bI + a \Sigma ba(i-1) I + u$$
(9c)
t t i=1 t-i t

leads to :

$$GDPt = b It + a GDPt-1 + (ut - aut-1)$$
(9d)

And so, one gets :

$$GDPt = a \ GDPt-1 + b \ It + Vt$$
(10a)

where $V_t = u_t - a_{u_t-1}$ (10b)

In this respect, one has to account for a capital output ratio identity so as to identify the "a" and "b" parameters of equation (10a) in terms of equation (1) one wants to use to get a capital stock serie. So, one writes :

$$GDP_t = K_t / k_t$$
(11)

where kt is the capital output ratio at the period "t". Here it is worth mentioning that kt used in a production function framework is treated analogously to a technical coefficient linking capital to its product. So, this presumes that the capital stock is fully utilized in the production process. Combining relations (10a) and (11) leads to :

$$k_t$$

 $K_t = a ----- K_{t-1} + b k_t I_t + V_t k_t$ (12a)
 k_{t-1}

This equation is equation (5) or (1), when :

(i)
$$k_t = k_{t-1} = k_0$$
 (12b)

```
(ii) b k_t = 1
```

and

(iii) ut is an error term such that $E(u_t) = 0$ when $k_t = k_0$

Then, in the light of equation (10a), the "a" and "b" parameters obtained from the regression analysis may be viewed as estimations of the δ and k parameters related to equation (1).

a = $(1-\delta)$ δ = 1-a (° symbol for estimation) (12c) b = 1/k₀ k₀ = 1/b \hat{k}_0 = K₀/GDP₀ K₀ = k₀.GDP₀

when one runs the following model to estimate a capital stock serie.

$GDP_{+} = (1-\delta) GDP_{+-1} + (I_{+} / k_{0}) + V_{+}$	Model I t : 1n	(13)
$Kt = (1-\delta) K t-1 + Tt$	• • • • • • • • • • • • • • • • • • • •	(20)
$Vt = ut - (1-\delta) u t-1$		

5. The features of the estimation depend on the **methods of estimation** and these ones on the **structure of the error term**. That means one should estimate equation (10a or model I) using OLS and then subject the residuals to a Box Jenkins analysis to discover wether they follow a MA, an AR, both or neither process. It is worth indicating that the selection of a model using this method depends strongly upon individual judgement : two people facing the same data may come up with very different models. This could occur in case of residuals follow an ARMA process. So residuals not well identified would bias the parameters.

(i) if Vt follows a MA(1) process(1) then ut is a white noise and Model I can be estimated using the maximum likelihood or OLS with in both cases a grid search method on the rate of depreciation (δ). OLS can be applied after re-writting equation (10a) by combining equations (7), (8) and (12c). This leads to :

$$GDP_{t} = \frac{1}{k0} \sum_{i=0}^{\infty} (1-\delta)^{i} I + u \qquad (14a)$$
and
$$GDP_{t} = \frac{1}{k0} \sum_{i=0}^{t-1} \sum_{i=0}^{i} \frac{1}{1-\delta} \sum_{i=0}^{\infty} (1-\delta)^{i} I + \frac{1}{k0} \sum_{i=0}^{\infty} (1-\delta)^{i} I + u \qquad (14b)$$

$$GDP_{t} = \frac{1}{k0} \sum_{i=0}^{t-1} \sum_{i=0}^{\infty} (1-\delta)^{i} I + \frac{1}{k0} \sum_{i=1}^{\infty} (1-\delta)^{i} I + u \qquad (14b)$$

Then,

$$GDP = --- \Sigma (1-\delta) I + --- (1-\delta) \Sigma (1-\delta) I + u$$

t ko i=0 t-i ko i=0 -i t

and

* 1 1 t GDP GDP I $(1-\delta)$ (14d) u t ko t ko 1 t where t-1 Σ $(1-\delta)$ Ι t-i i=o GDP GDP u 1 1 1

It can be constructed using different values for δ and GDPt can be assimilated to a parameter. For each value of δ equation (14d) can be estimated via OLS while a grid search method is conducted to find the value of δ which results in the minimum sum of squared residuals of equation (14d)(2). The procedure is interesting although the criteria used is not a very optimal one (see remark at the end of paragraph 9).

(ii) if Vt follows an AR(1) process estimates are not consistent when OLS are used. Then, Vt is : $V_t = \xi t + \beta V_{t-1}$, where ξ_t is a white noise. In this case, Vt and GDP_{t-1} are highly correlated. One way to estimate such a model is to use the Hildreth-Lu's grid search method, the Maximum Likelihood or an instrumental variable for GDP_{t-1} before running the regression. This instrumental one is not easy to discover for one has to find a variable highly collinear to GDP_{t-1} at the opposite to Vt. A solution to both problems is to estimate parameters of model I using principal components. Then, there are two possibilities :

(i) a complete factorization of the explanatory variables i.e these ones being replaced by their principal components. Then, the new estimators have all the properties of the OLS ones;

(14c)

(ii) an incomplete factorization of the explanatory variables i.e. these ones being replaced by their (m) principal factors i.e those for which the corresponding characteristic root has a relatively high value, only. In this case there is an indetermination problem for there is "m" equations to estimate "n" unknown parameters . Nevertheless, there exists a rather complex procedure which issues biased estimators. The importance of the bias is a function of how far the product F1'F (where F1 is the matrix of the most important factors (m < n)) departs from the unit matrix F'F one would get if all factors had been retained).

(iii) if Vt follows neither a MA nor an AR process, then OLS can be an appropriate technique. This possibility can occur if ut follows an Ar(1) process : $u_t = \rho u_{t-1} + \epsilon_t$, where ϵ_t is a white noise, and if $\rho = 1-\delta$. Since we will have $v_t = (\rho + \delta - 1) u_t + \epsilon_t = \epsilon_t$, vt will also be a white noise.

6. Before using the estimators to generate "a" Kt serie one has to choose a beginning year for which $K_{t-1} = \hat{K_0}$ and $\hat{K_0} = GDP_0.\hat{k_0}$. In this respect, one has to mention that the δ and k_0 parameters are biased for the capacity utilization rate. Indeed, practically, the capital stock is rarely fully utilized in the production process. So, when one estimates capital parameters departing from production data, one gets capital utilization rate biased parameters. The preceding indicates that it is only when the capacity of production is permanently fully utilized that the capital parameters are not biased for the capacity utilization rate. In this framework, GDP equals potential output.

3. The choice of the beginning year

7. When no additional information exist on how to choose the year t = to there is a feasible solution which is to select the beginning year i.e. that for which $k_t = k_0$ via measures of central tendancy. The most common ones are the arithmetic mean, the median, the mode, the geometric mean and the harmonic mean. For instance, the beginning year can be the median one of the period on which one estimates model I. The preceding indicates there would be several Kt series departing from equation (1) because there are several possible $K_{t=t0}$. In this respect, if one wants to get the same value for $K_{t=t0}$ for the price base year both in real and current prices, one should select this base year as the beginning one. The preceding given the Kt serie one selects is far from being perfect. Indeed, it has been measured through an equation (a model) not correctly specified because kt has been accounted for as a parameter. Therefore, the vector of disturbances accounts for the distribution of kt and as such is correlated with the sample of explanatory variables. In this case OLS estimators are biased and not consistent. Further, the base (price) year has been chosen as the beginning year, as if it was the mean of the Kt serie.

4) The equation set up for the P.I.M. at the beginning of a year (t)

8. The method already discussed can also be applied departing from

$$K_t = K_{t-1} (1-\delta) + I_{t-1}$$
 (15)

where :

 K_t , K_{t-1} : net productive capital stock at the beginning of year (t), and (t-1) respectively, at some base year prices ;

 I_{t-1} : gross investment during year (t-1) at the same base year prices where agriculture, government, dwellings and domestic servants have been omitted (as it is the case for Kt and Kt-1).

Equation (15) results from the following set-up :

$$K_{t} = \sum_{r=1}^{\infty} (I_{t-r} - IR_{t-r})$$
(16)

$$GDP_t = f(I, \dots, I, \dots, u) = \Sigma ba I + u$$
$$t-1 t-i t i=1 t-i t$$

Then one gets

where :

under the same conditions as previously (12b).

~

In this case, one gets a capital stock serie at the **beginning** of each period "t". It is worth indicating that both for model I and II, there is collinearity between investment and the implicit error term for this one accounts for the distribution of k_t . Thus, OLS hypothesis have been violating. Collinearity is much more severe for model II compared to model I because it is reinforced via national accounts computed GDP_{t-1} which is linked to I_{t-1} . One solution would be to use an instrumental variable for GDP_{t-1} before running the regression.

5) Focussing on an explicit production function when departing from the PIM

9. Some economists have tried to use a Cobb-Douglas production function in the framework of the PIM(3). This one is of the form :

$$GDP_t = \theta_0 K_t \theta_1 L_t \theta_2 \tag{19}$$

where :

 K_t , L_t : measures of the aggregate capital stock and employment respectively;

 $\theta_0, \theta_1, \theta_2$: coefficients to be estimated.

Then, they developed a specific way to estimate the K_t serie which consists in an estimation of the CD with a grid search method on the depreciation rate. One has to proceed as follows :

(i) to re-write the CD production function in its log form :

$$Log GDP_t = Log \theta_0 + \theta_1 Log K_t + \theta_2 Log L_t$$
 (20a)

(ii) to estimate the Kt scrie from the PIM written in its log form and the departing log form of the CD function :

- Log K_t = Log
$$\begin{pmatrix} t-1 & i & t \\ \Sigma & (1-\delta) & I & + (1-\delta) & K \\ i=0 & t-i & o \end{pmatrix}$$
 (20b)

where K_0 is the initial stock of capital. Then, a first order approximation leads to the new expression of K_t

(20c)

-
$$\log K_t \simeq \log 2 + \frac{1}{2} \log \begin{bmatrix} t-1 & i \\ \Sigma & (1-\delta) & I \\ i=0 & t-i \end{bmatrix} + \frac{t}{2} \log (1-\delta) + \frac{1}{2} \log K$$

Equation (19) to be estimated can be re-written as follows :

$$Log GDPt = \theta'o + \theta 1 K't + \theta 2 Log Lt$$
(21)

where :

$$\theta_1$$

 $\theta_0 = \log \theta_0 + \frac{\theta_1}{2}$ (22a)

$$K't = Log2 + --- Log \Sigma (1-\delta) I + --- Log (1-\delta)$$
(22b
2 i=0 t-i 2

It leads to an estimation of the Kt serie corresponding to that value of the depreciation rate (δ) which maximizes the R² in equation (21). With this method one obtains **a** capital stock serie on the basis of equation (22b), then there is no need to seek for a K₀ value to compute the Kt serie. However, from an econometric viewpoint the method is not really optimal since one **just** maximizes the related R². Indeed, searching for a high R² or a high adjusted R² runs the real danger of finding, through perseverance, an equation that fits the data well but is incorrect because it captures accidental features of the particular data set at hand (called "capitalizing on chance") rather than the true underlying relationship. With this method, it is also assumed that production is adjusted by capital at the opposite of employment. This is relevant in the case of developing countries where financial constraint on investment and then production is the major one. The reasoning is illuminated a little more at paragraph 12.

6) A capital stock serie instead of a capital services one

10. Apart from econometrical difficulties (which may turn to be severe because they cumulate) the P.I.M. yields theoretically a capital stock serie which is a capital services one, only when the capital stock is fully utilized. Therefore, the estimated capital stock serie cannot be used in a production function without knowing that in that framework (where the capital stock is rarely fully utilized) the related parameters would be biased and, further, inconsistent. If the capital stock were utilized at a constant rate (that of the beginning year) no difficulties would arise for capital services would be proportional to capital stock. Hence using capital stock in lieu of capital services would constitute a change in the units of measurement and would affect a scale constant in the production function. But here one has to account for a non constant rate and, therefore, the production function parameters one would estimate with the Kt serie would be biased and inconsistent. Practically, the problem just mentioned is a rather vicious one for the Kt serie one estimates is neither a pure stock one nor a real capital services one. Indeed, the stock is biased for the capacity utilization rate distribution influes on δ and k₀ and the analyst is unable to measure the bias. So, Kt is a misspecified variable in the framework of a production function. The outcome of running an OLS regression in this respect are as follows. When the misspecified variables are independent of some vector of disturbances, the specification bias is given by(4) :

$$b (\beta^*) = (P-I) \beta$$
(23a)

where :

 $\overline{\beta}^*$: estimate (-) coming from the regression run on the misspecified variables (X^{*})

 β : parameter to be estimated on the true variables (X)

$$P = (X^*' X^*)^{-1} X^{*'} X$$
(23b)

and the specification inconsistency by :

$$i(\bar{\beta}^*) = (M^{-1}**M^* - I)\beta$$
 (24a)

with:

 $M^{**} = \lim X^{*'}X^{*} / T \qquad M^{*} = \lim X^{*'}X / T \qquad (24b)$ T -> ∞

where :

M** : second-order moment matrix of the misspecified variables ;

M* : cross moment matrix between the misspecificied and true variables.

In case of misspecified variables not being independent of the vector of disturbances the specification bias does not hold except possibly asymptotically. The specification inconsistency holds in probability limits.

11. Now let us give an example of the consequence of misspecifying capital in a production function. Let :

$$X^* = (X1, x^*.n) \quad X = (X1, x.n)$$

(25)

where :

-54' - -

x*.n, x.n : vectors of observations on the misspecified and true variables (capital)

and :

X1 : the matrix of observations on the rest of the variables (n-1) correctly specified.

One can write the following :

$$X = (X1, x.n) = X * \begin{bmatrix} In - 1 & 0 \\ 0 & 0 \end{bmatrix} + (0, x.n) \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$
(26a)

$$(X*'X*)^{-1}X*'X = (X*'X*)^{-1}X*'X* \begin{bmatrix} In-1 & 0 \\ 0 & 0 \end{bmatrix} + (X*'X*)^{-1}X*'$$
(26b)
(26b)

$$(0, x.n) \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} In-1 & 0 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} 0, (X*'X*)^{-1} X*' x.n \end{bmatrix}$$
(26c)

Thus if :

$$\overline{\beta} \star = (\chi \star' \chi \star)^{-1} \chi \star' Y \tag{27a}$$

then :

$$b(\overline{\beta}^{*}) = \left[(X^{*}X^{*})^{-1} X^{*}X^{*} + \begin{bmatrix} 0\\0\\-1 \end{bmatrix} \right] \beta n \qquad (27b)$$

17

This means that even if a single variable is misspecified in a regression, the entire set of coefficient estimators is biased. Empirical works showed that in case of a Cobb-Douglas production function the labour parameter would be biased upward at the opposite of the capital one.

12. Specification bias and inconsistency occur in the production function framework because capital has not been specified as the other variables i.e. the other input (labour in a two factor case) and output. Thus if one specifies the other variables as the capital stock one the bias would disappear or at least reduce. The preceding means to run a regression of potential output (instead of output) on the estimated Kt and potential employment (instead of employment) to get an estimated production function or, analogously, to regress potential output on gross investment in model I or II. But, then, one should get measures of potential output and input which are as missing as the capital stock. A solution to the bias related to the capacity utilization rate (C.U.R.) problem is to run a regression either with model I or II by introducing a measure for this rate so as to remove its influence on the "a" and "b" parameters. When there are surveys related to this rate (generally conducted by some Central Bank's staff members) estimations improve assuming that all firms which have been questioned define their rate departing from a common definition which is not always the case. Indeed, there are different concepts of capacity (potential) output on the basis of which to compute a capacity utilization rate. For instance, departing from maximum output, normal, preferred... or minimum cost one. When there are not such surveys, one may use proxy. It may be either the ratio of GDP to some GDP fitted trend or the rate of employment i.e (100 - unemployment rate : u) or both. In case of a Keynesian disequilibrium i.e when firms and households are rationed it would be sensible (and sufficient) to introduce (100-u) to get a more plausible estimation for Kt. At the opposite, in case of a classical disequilibrium (rationed households, only), one should introduce the first aggregate i.e the ratio of GDP to some GDP fitted trend. In both cases one should also introduce a trend accounting for factor-augmenting technological change (F.A.T.C.) and some relative price ratio. Indeed, developing countries depend fundamentally on the ratio of export to import price (PX/PM). This ratio determines the amount of available cash-flow from operating and, therefore, that of investment that can be financed without increased debt. Rentable investment defines the capacity of production and the production. Then external balance adjusts (or not) supply to demand. To conclude one runs a regression of the following type to get a relatively consistent estimation of a capital stock serie either at the beginning or end of a year "t".

 $GDP_t = (GDP_{t-1}, I_t(t-1), proxy_t (CUR), trend_t (F.A.T.C.), PX_t/PM_t, w_t)$ (28)

where :

wt : is a vector of disturbances

The choice between model I or II depends on GDP_t . If this one is an end year measure as for the IMF one should select model I, the opposite being true when it is measured at the beginning of the year. It is worth indicating that one remains always with collinearity between explanatory variables using the P.M.I. As J. Johnston indicated⁽⁵⁾ the main consequences of multicollinearity are the following :

1- The precision of estimation falls so that it is not always possible to disentangle the relative influences of the explanatory variables. This loss of precision has three aspects :

- a) specific estimates may have very large errors ;
- b) errors may be correlated ;
- c) sampling variances of the coefficients will be very large.

2- Some variables may be dropped from the analysis because of their student -t statistics being low at the opposite of what would occur in reality;

3- Estimates of coefficients become very sensitive to particular sets of sample data, and the addition of a few more observations can sometimes produce dramatic shifts in some of the coefficients.

II. A METHOD IMPLYING THE KNOWLEDGE OF A DEPRECIATION RATE SERIE

13. Another method one may use to estimate a capital stock serie departs from a capital depreciation one : so, this one has to be available or at least be proxied. The departing hypothesis is that a piece of equipment is replaced when it fails, the depreciation rate being identified with the failure rate which can be known from a study of the survival curve of the equipment. The curve is the ratio between the number of surviving pieces of a batch at any given moment and the initial number of pieces in that batch. According to the type of equipment there exists several survival curves corresponding to various probability distributions. Economists generally thought to the rectangular and the exponential survival ones.

1. The rectangular survival curve

14. In case of the survival curve corresponding to the rectangular or uniform distribution :



where p(t) represents the probability that a given piece of equipment has age t and b the total length of life or the period of scrapping. The capital stock serie can be obtained through the following model :

$$K(t) = \Sigma t I_{r}$$

$$r=t-D+1$$
(30a)
$$At = 1/D K_{t}$$
(30b)

where K,I, and A are respectively the capital stock (unknown), gross investment and the capital consumption, the later data being obtained from national accounts statistics. D is the total lenght of life of the capital stock installed in each vintage. The model comprising equations (30a+b) contains two equations and unknowns (K and D). Equation (30b) indicates that every year the capital invested in period t–D is entirely scrapped. A graphical solution to the model can be found by plotting a curve of backward cumulated gross investment. The point where the 45° line and the cumulated investment curve cut each other gives simultaneously the capital stock at the end of period t and its lenght of life D. Algebraically, the solution is found by substituting (30a) into (30b) giving :

$$A_{t} = 1/D \sum_{r=t-D+1}^{t} I_{r}$$
(30c)

where D is the only unknown. One problem with such a model is that even when the capital stock follows the expected depreciation law (29) the model is incorrect because A_t does not represent the replacement of worn out capital, but rather the depreciation allowances on the basis of the tax laws in force in the year t. As a result, the estimated capital stock will be biased.

2. The exponential survival curve

15. Another law of survival also used is the exponential one. The related equation set up is :

$$K_{t} = \Sigma t \text{ Ir } e^{\delta(r-t)}$$
(31a)
$$r = -\infty$$

$$At = \delta Kt$$
(31b)

where δ is the depreciation rate. Substituting (31a) into (31b) one gets :

$$A_{t-\delta} \sum_{r=-\infty} t r e^{\delta(r-t)} = 0$$
(31c)

expression which can have several real roots in the depreciation rate, even in the interval between zero and one. One difficulty with such an equation is that it involves a rather long serie of past investments (to 1900 in several cases). In this case, there is a lack of data even for countries very well endowned with statistics **and** statisticians. So, very often one has to estimate a past investment serie before doing the same for the capital stock. This is a vicious circle that obliges to rule out the method, because there is no valid criterium to choose among the different estimated series of past investments.

16. It is worth indicating that both preceding methods deal with real terms series (Kt and It). On the basis of the P.I.M., this means that one focuses on equilibrium prices. Indeed, equations (1) to (5) are supply oriented ones while equations (13) and (17) are de facto demand focusing ones. The preceding indicates that the capital stock serie is also price biased for economies are not always at equilibrum. In this respect, what price index to select for the capital serie remains an open question i.e. one for which there is no general consensus among economists and/or statisticians. And finally, one has also to mention the problem of the definition given to capital. Through the P.I.M. it has been narrowly defined i.e. as produced means of production. But it can also include all or a large part of the factors of production in the economy. For instance, instead of being the sum at base year prices of equipment, structures, inventories it could also include land, consumer durables, human capital (i.e. the accumulated cost of education treated as investment) and accumulated expenditure on research and development. Now that environment challenges growth in many countries particularly the developing ones, capital could even include net natural resources expenditure. Then, a lot of new problems appear but we let these ones for other economists.

III. A METHOD REFERRING TO A PRODUCTION FUNCTION FRAMEWORK

17. It is also possible to get a capital stock serie departing from a production framework. In this case, one gets a desired capital stock serie and, therefore, the question which has to be adressed is how to get the unknown effective capital stock serie.

1. The CD production function

18. Measure of the desired capital stock has been popularized by D.W. Jorgenson. The starting point is a firm combining labour and capital to produce output in situations where prices of factors and goods are given in such a way as to maximize its net worth. Two constraints restrict the firm's behaviour : a putty-putty(6) Cobb-Douglas production function and an identity relating capital, investment and depreciation. In this case, the capital productivity relation leads to a desired capital stock serie which is given by the following equation assuming GDP equals potential output (Y) :

$$\begin{bmatrix} Kd = \alpha p Y/c \end{bmatrix} \leq \frac{\text{in case of a CD}}{\alpha + \beta = 1} \qquad (32)$$

where :

K^d : desired capital stock

 α : elasticity of output with respect to capital (to be estimated or proxied by the profit share in GDP)

p : output price

Y : GDP in real terms.

c : cost of capital. It represents the fictive charges paid by the firm for using its equipment (to be computed)

2. The CES production function

19. The Cobb-Douglas function restricts the substitution between factors of production such that the elasticity of substitution is always unity. That is its main weakness. Therefore, the economists turned to other sorts of production functions. For instance, one obtains :

$$\begin{bmatrix} \sigma \\ K^{d} = (\alpha p/c)^{\sigma} \end{bmatrix} \text{ in case of a C.E.S. } \begin{bmatrix} Y = \left[\alpha K^{-\theta} + (1-\alpha)L^{-\theta} \right]^{(-1/\theta)} \end{bmatrix}$$

i.e. a function which does allow the elasticity of substitution (σ) to differ from unity; that elasticity can be estimated from a regression linking labour productivity (Y/L) and the wage rate (w).

In $(Y/L)_t = a0 + alln w_t + t$ (34a) $\hat{a}l = \sigma$

In this framework one gets the estimation of the parameter θ which reflects the extent of substitution by putting :

$$\sigma = 1/(\theta+1) \tag{34b}$$

The elasticity of output to capital can be proxied as in the CD case.

3. Extensions of the CD and CES functions

20 The C.E.S. has a number of drawbacks. A very restricting one is the constancy of the parameters over the whole range of output with as result the constancy of the marginal productivities. Another one is that the C.E.S. does not allow the elasticity of substitution to vary with the factor proportions criticism also related to the CD function. Therefore, there have been extensions of the CD and C.E.S. functions. The C.E.S., for instance, was extended to produce the V.E.S. production function which exhibits a variable elasticity of substitution. Given values of its parameters the V.E.S. shows increasing marginal productivities up to an optimal ratio of labour to capital and then decreasing ones beyond this optimal point. V.E.S. production function are difficult to handle econometrically therefore economists prefer to use CD extensions. In that framework one gets :

 K^{d} = (α + δ V1⁻¹) (p^{Y}/c)

in case of a quasi C.D.

$$Y = \cap K\alpha L(1-\alpha) e^{-\alpha} (L/K)$$

(35)

(22)

where :

$$V1 = (K/L)d = \frac{\forall + \alpha (w/c) + \left[\left[\forall - \alpha (w/c) \right]^{2} + 4 \forall (w/c) \right]^{2}}{2(1-\alpha)}$$
(36)

That function exhibits increasing marginal productivities up to an optimal ratio of labour to capital and then decreasing ones beyond this optimal point. It has a variable elasticity of substitution.

Analogously, one gets :

$$K^{d} = (\alpha - \chi V_2^{-1}) (pY/c)$$
 in case of a quasi C.D. (37)

$$Y = \bigcap K\alpha L(1-\alpha) e - \delta (K/L)$$

where :

$$V2 = (K/L)^{d} = \frac{2 \alpha (w/c)}{\left[(1-\alpha) + \forall (w/c)\right] + \left[(1-\alpha) + \forall (w/c) + 4 \forall (w/c)\right]}$$
(38)

Here, one has to find a way to estimate \mathfrak{F} which remains unknown.

4. Other production functions

21. Efforts of economists to produce plausible functions led also to the linear elasticity of substitution production function. On its basis, one gets :

$$K^{d} = \frac{(1-\alpha\theta) pY}{c - (\theta-1)w}$$
(39a)

departing from :

$$Y = \bigcap K(1-\alpha\theta) \left[L + (\theta-1) K \right] \alpha_{\theta}$$
(39b)
$$\bigcap > 0$$

$$0 < \alpha < 1$$
 $0 \leq \alpha \quad \theta \leq 1$

$$(L/K) > \begin{bmatrix} 1-\theta \\ 1-\alpha\theta \end{bmatrix}$$

This function is a generalization of the C.D. to which it reduces when $(\theta = 1)$. It also includes as particular cases the fixed coefficients production function when $(\theta = 0)$ and the linear production function when $\theta = (1/\alpha) > 1$. To get an estimation of θ -1 one runs the following regression.

$$(----)t = a0 + a1 (w/c)t$$
 (40a)

then

22. As it has made clear in the light of the preceding paragraphs there is a lot of production functions one can use to compute a desired capital stock serie. But then one has to know how to measure the effective capital stock departing from the desired one. A sensible procedure is to substitute GDP for potential output in the framework of some selected production function fitted on data related to **developed countries**. Doing so means to assume that parameters which link potential output and desired capital stock remain unchanged in the case of the relationship between GDP and the effective capital stock. And further, that supply analysis related to developed countries is relevant in the framework of the developing ones. It is obvious to indicate that on the basis of the preceding procedure one gets a capital stock serie which depends on the selected production function. Further let us indicate that one gets a **capital services** serie given one uses GDP which results from the capacity utilization rate of production factors. Further, it is biased for the demand prices.

IV. APPLICATION OF THE PIM TO A SET OF SELECTED AFRICAN COUNTRIES

23. The PIM being the easiest method to use has been applied in the framework of a set of African countries which signed the Lome Convention. The aim was to discover if they would share some common features regarding their capital (k_0) and depreciation (δ) coefficients. Indeed, very often, these countries have pursued analogous development strategies departing from analogous comparative advantages. As a result, their integration to the world markets has revealed a basket of common items mainly grouped as either mineral or agricultural raw materials or both. The most well known are : iron, copper, other non-ferrous metals, uranium, precious stones, oil, coffee, cocoa, tea, sugar, tobacco, cotton, oilseeds, spices, timber, and hides and skins. With regard to manufacturing, the African contribution has been very limited up to now. It has concerned mainly clothes, clock making, toys, wood products, fertilizers, fruit juices, processed fruits, fish and meat. Further, it has featured a limited number of countries. For instance, Zambia, Niger, Mauritius, Zimbabwe, Zaïre, Benin, Gambia, Senegal, Ghana, Nigeria, Tanzania, Madagascar, Cameroon, the Ivory Coast. Within the preceding group, only Madagascar and Mauritius have traded rather sophisticated manufactured goods.

24. The preceding indicates that despite the Lome Convention supply diversification within African countries through increased manufactured exports has been rather weak. This has been mainly the result of two factors. The first one is the utilization of Heckscher-Ohlin advantages to get growth. As indicated by Fantu Cheru (12) on page 498 this is part of a larger historical phenomenon : "After the partition of Africa in 1884, the Western European powers established the rules by which Africa would participate in the world economy. Simply, Africa was to produce raw materials and agricultural goods to meet the needs of Europe's industries and consumers. Thus, Kenya would plant coffee and tropical fruits, Sudan would grow the cotton needed in Manchester, Ivory Coast would grow bananas and pineaple, Ghana would produce cocoa and Senegal the groundnuts needed to make margarine. A luxury beverage and fruit cocktail economy was thus created. This pattern of commodity dependence has changed very little since the era of independence. The second is the fact that African supply could easily meet EC's demand of raw materials. As a result African countries could get foreign exchange needed to pay for their imports. In the 1980s, the decrease of raw materials prices has limited strongly and even rendered impossible any further diversification. As a result manufactured export goods got long-run declining share in almost all African countries trying to promote manufacturing. All that preceeds indicates that there is some economic rationality which could explain that the coefficients (k_0, δ) would share common values between groups of African producers which signed the Lome convention.

11

25. A regression analysis of the sort indicated at paragraph 12(7) was thus run on 1968-88 for a set of African suppliers covered by the Lome Convention. These countries were selected to be able to utilize results of one of our previous studies illuminating the development strategies of these countries since 1970(8). This permits a rather consistent interpretation of their capital coefficients (k_0 and δ). Results are shown at table 1. Within table 1 African countries have been grouped according to their **dominant** (i.e. at long run) export products. Other ones also important to explain their capital coefficients have been put into brackets. Indeed, other activities could also be relevant explanatory variables of the capital coefficients, for instance, when a country decides or is forced to modify its development strategy or acts on different product fronts. Since the aim is not to pretend to have discovered African structural parameters comments on table 1 just refer to big trends. Comments are the following :

1. As normally expected, mineral raw materials suppliers tend to get higher capital output ratios than agricultural raw materials ones. This is particularly obvious for iron suppliers (k_0 of about 5) but these ones get also the lowest depreciation coefficients (from 3 to 6%). So, their high capital output ratios could also be affected by a low level of depreciation allowances which overvalues their capital stocks.

2. Depreciation rates of mineral raw materials suppliers fluctuate more widely than those of agriculture raw materials ones (from 2% to 22% compared to from 3% to 15%). This is a consistent result since depreciation is much more specific to each raw materials activity than it is the case within agricultural raw materials ones.

3. Supply diversification through the promotion of manufacturing reduces the consumption of capital within the minerals suppliers group. This again is a consistent result since manufacturing (also called light industry) generally consumes less capital than heavy industry. In the case of Nigeria, this is more a reflection of a deep economic crisis since the 1980s.

4. The low k_0 of Sierra Leone and the high value for RCA are due to a modification of their development strategies : SL tends to pass from iron and non-ferrous metals to precious stones and RCA from cotton and coffee to precious stones and uranium. The depreciation coefficients of these countries are then influenced by the changes in their growth strategies (increased depreciation for SL, low level for RCA).

5. Agricultural raw materials suppliers get k_0 values rather homogeneously distributed. They seem to fluctuate between 1.4 to 2.0. This is again a consistent result since African suppliers produce a set of agriculture raw materials which compete with each other for land, labour input, and credit. For instance, cocoa with coffee, cotton with coffee and groundnuts, tobacco with yams, cassava and maize. Depreciation rates are also more narrowly distributed which is also consistent with crops orientations. Coffee seems to be a rather capitalistic good compared to cotton or oilseeds. It could get a capital output ratio of about 2 under normal circumstances while oilseeds and cotton could have it meaningfully below 2.

6. The k₀ values of Madagascar (2.77), Sudan (0.88), Ghana (0.7 to 0.9) and Togo (5.5) are the result of modifications within their development strategies : Madagascar tends to launch clothing activities ; Ghana to reduce its bauxite sector stop its high capital intensive projects and promote agriculture production (this explains why its depreciation rate varies strongly) ; Togo to substitute phosphate for agriculture production and Sudan, hides and skins after failing to develop sugar, textiles industries and in general infrastructure such as irrigation to support the "bread basket" strategy of the 1970s. Further, the country has also been affected by civils wars and recurrent drought. All this has led to a decrease of the capital stock through increased depreciation.

7. The Ethiopian k_0 results not only from the launching of hides and skins activities, but more fundamentally from the decrease of depreciation of the capital of state's industrial firms as a result of civil war and increased indebtness. This has overvalued the Ethiopian stock of capital.

8. Amongst coffee producers, Kenya has benefitted from an investment surplus influencing positively its k_0 via revenue from tourism in the framework of decreased coffee prices.

9. At the opposite of raw materials suppliers, manufacturing to be promoted in conjunction with agricultural raw materials activities seems to lead to an increase of the capital output ratio. This is true for Madagascar and Zimbabwe via clothes and Togo via phosphate. This could also be true for Benin, Burkina–Faso via hides and skins, although the k_0 of Sudan cannot be used as reference value being somewhat low as for Mali.

10. The highest is the manufacturing impact, the lowest was the initial capital level. That would be the case for Somalia ($k_0 = 4.35$) trying to launch hides and skins (departing from fruits) compared to Kenya, Ethiopia, Mali, Burkina-Faso or Sudan. Hides and skins is a rentable investment opportunity. That is the reason why some African countries tend to penetrate on its market. Obviously, the manufacturing impact on k_0 values would also depend on the profitability trend of the main activity (or activities) as on its their share in the export basket of a given country. Further, the role of the State's support has also to be accounted for.

11. Evidence coming from industrial surveys or information from missions to African countries suggest that all k_0 related to industrial activities (the mineral raw materials and manufacturing ones) could be upward biased indicators of true k_0 . Indeed, African countries lack from funds for depreciation allowances. Therefore, their stock of capital is not the real productive one. It is overvalued.

12. The concluding remark is that the PIM although being featured by a lot of economic and econometric weaknesses leads to plausible analysis.

CONCLUSION

26. In this paper, we made comments on some methods available to compute a capital stock serie in the framework of developing countries. In these countries, there is a lack of data related to the capital stock. Therefore, it is rather difficult to perform supply analysis and the related policy simulation. That is rather unfortunate for most of the developing countries are under adjustment processes i.e. pursue policy aiming at improving supply gradually. One of the main conclusions of the paper is that the capital stock serie one gets through the P.I.M., or a known capital depreciation serie, or in the framework of some selected production function has a lot of problems. In this framework, we would like to say that in the light of what has occured in developed countries when no data related to capital stock were available, these problems have not to be exaggerated. Indeed, trends issued from the surveyed estimation methods were correct i.e. got plausibility after data appeared. The PIM which is the casiest method to apply has been used in the framework of some selected African countries. It has led to results consistent from the viewpoint of some economic rationality. All this pleas for further studies on the capital stock of developing countries.

TABLE 1.

•

\$

APPLICATION OF THE PIM TO A SET OF SELECTED AFRICAN COUNTRIES Results from period 1968--88

Main activity	Countries	Capital– output ratio:k0	Depreciation rate : δ
A. Mineral raw materials			
1. Iron	Mauritania (fishes) Liberia (rubber) Sierra Leone (bauxite, precious and semi-	5.26 5.15	5 to 6% 3 %
	precious stones)	0.9	14 %
2. Non-ferrous metals	Zambia (Cu, manufac- tured goods) Zaïre (Cu, others, coffee, manufactu-	2.00	10 %
	red goods) Niger (Uranium, ma-	1.51	7 %
	nufactured goods)	1.43	22 %
3. Oil	Libya (9) Algeria (9) Nigeria (2000, ma	2.78 2.78	8 % 10 %
	nufactured goods) Congo (timber, manu-	1.45	12 %
	factured goods)	3.84	10 %
4. Precious stones	R.C.A. (cotton, coffee)	4.54	2 %
B. Agricultural raw materials			
5. Coffee, tea, tobacco	Madagascar (clothes, spices) Burundi Ethiopia (Wideo and	2.77 2.00	6 % 5 %
	skins, manufacturing) Tanzania (tobacco,	2.52	2 %
	tured goods) Kenva (tea Hides and	2.05	6 %
	skins, tourism)	2.00	8%
	cotton, clothes)	2.27	7%
	+ I	i	

Main activity	Countries	Capital- output ratio:k0	Depreciation rate : δ
6. Cocoa	Ghana (timber, bauxite)	0.7 to 0.9	3 to 8 %
7. Coffee+cocoa+others	7. Coffee+cocoa+others Ivory Coast (fruits, timber, manufactured goods) Cameroon (oil, timber, Aluminium manufactu	1.51	9%
	red goods) Togo (cotton,	1.51	8 %
	phosphate)	5.5	4 %
8. Cotton, other vegetal raw materials Sudan (Hides and Skins, oilseeds) Benin (palmoil, groundnuts, manu- factured goods) Burkina-Faso (oil-	0.88	15 %	
	factured goods) Burkina-Faso (oil-	2.04	3 %
	Skins)	1.42	12 %
9. Oilseeds, fishes	Senegal (Manufactured goods) Gambia (Manufactured	1.39	10 %
	goods) G.Bissau (timber)	1.51 1.69	9% 11%
10. Breeding	Mali (Hides and Skins, cotton) Botswana (precious	1.45	9 %
	stones)	2.00	10 %
C. Hides and Skins	Somalia (fruits, sugar, cotton)	4.35	6 %
D. Clothes	Mauritius (sugar, honey, clock-making, toys)	2.00	9 %

•14

NOTES

1) Paragraph quoted from :

"Simultaneous estimation of production functions and capital stocks for developing countries", K-M. Dadkhah and F. Zahedi, Review of Economics and Statistics – Vol. LXVIII, August 1986, n° 3, pp. 444–5. 2) Johnston (6b) pp. 368–370.

3) "A macroeconometric model for developing countries", N.U. HAQUE, K. LAHIRI, P-J. MONTIEL, IMF Staff Paper – Vol. 37, n° 3) (september 1990).

4) The following notes are quoted from Ph. J. Dhrymes : "Econometrics-Statistical Foundations and Applications", Harper International Edition, NY, 1970, pp 227-8.

5) Johnston (6a), p. 160.

6) With a putty-putty model the equipment can be modified at any moment and factor proportions can be adjusted instantaneously according to price variations.

7) Except that the residuals (V_t) were not subjected to a Box-Jenkins analysis since we just assumed that U_t followed an AR(1) process with $\rho = 1 - \delta$. This leads us to assimilate V_t to a white noise (see paragraph 5 iii) 8) For a detailed analysis, refer to M-P. Verlaeten [11].

9) These countries have been included in the table as reference ones, the Nigerian estimations being biased.

REFERENCES

1. J.L. BRIDGE

Applied Econometrics, North-Holland Pub. Company, 1971

2. K.M. DADKHAH AND F. ZAHEDI

Simultaneous estimation of production functions and capital stocks for developing countries, Review of Economics and Statistics, vol LX VIII, August 1986, n° 3

3. PH. J. DHRYMES

Econometrics Statistical Foundations and Applications, Harper Int. Edition, NY, 1970

4. T. DE BIOLLEY

A New Class of neo-classical production functions with corresponding investment behaviour, Proefschrift ter verkrijging van de graad van doctor in de economische wetenschappen, Rotterdam, 1972

5. THE ECONOMIST

Economic Growth : Explaining the mystery, Jan 4, 1992, pp 17-20

6. J. JOHNSTON

a- Econometrics Methods (2nd Edit.), Mc Graw-Hill Kogakusha, Ltd Tokyo, 1972, p. 160 b- Econometric Methods (3rd Edit), MC Graw-Hill, New-York, (1984)

7. L. R. KLEIN

A Textbook of Econometric, Evanston : Row, Peterson, 1953, p. 121

8. E. MALINVAUD

Méthodes statistiques de l'Econométrie, Dunod, 1983, chapitre 10

9. O E C D

The Measurement of Capital : The Methodology of Capital Stock Estimates in OECD Countries by M. Ward, 1976

10. D. USHER

The Measurement of Capital, The University of Chicago, Press, 1980

11. M-P. VERLAETEN

Eclairage de la politique de LOME avant LOME IV par l'analyse structurelle des exportations ACP destinées à la C.E.E. sur longue période 1970-1986, DIAL, april 92, 35 p.

12. WORLD DEVELOPMENT (April 92)

F. CHERU : "Structural Adjustment, Primary Resource Trade and Sustainable Development in Sub-Suharan Africa", p. 497 to 512.